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BLTP Bogoliubov Laboratory of Theoretical Physics

EMPIRICAL SYSTEMATICS OF LONG-RANGE ALPHA-PARTICLE EMISSION PROBABILITIES IN FISSION OF HEAVY AND SUPERHEAVY NUCLEI

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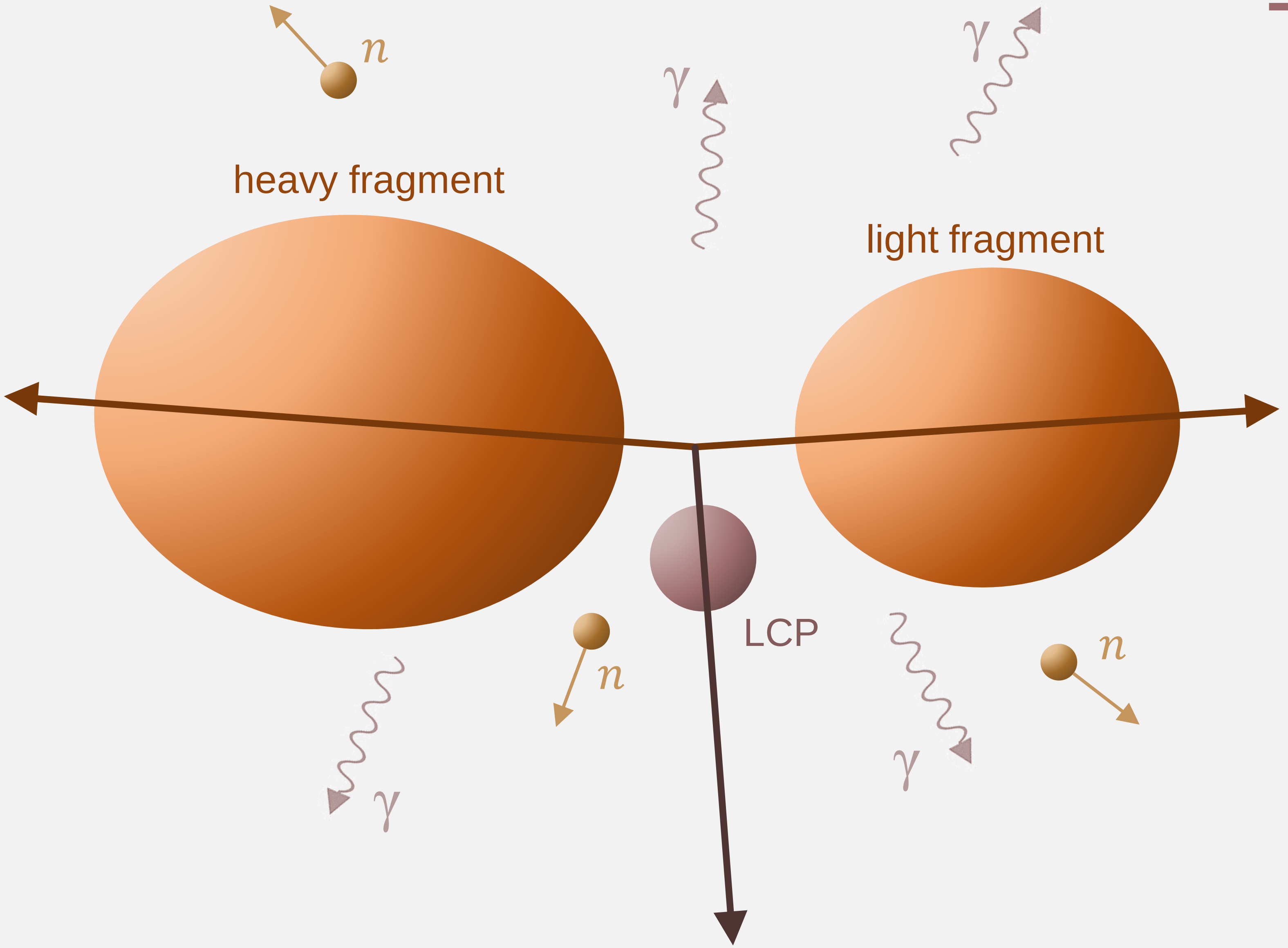
Light Charged Particle-Accompanied Fission = Ternary Fission



- Rare fission mode: **~0.3%** of actinide SF
- LCP typically emitted perpendicularly to the fission axis
- Charge/mass numbers vary from p $(Z = 1)$ to Ar $(Z = 18)$
- Most common LCP (>90%) is ^{4}He – **Long-Range Alpha (LRA)**

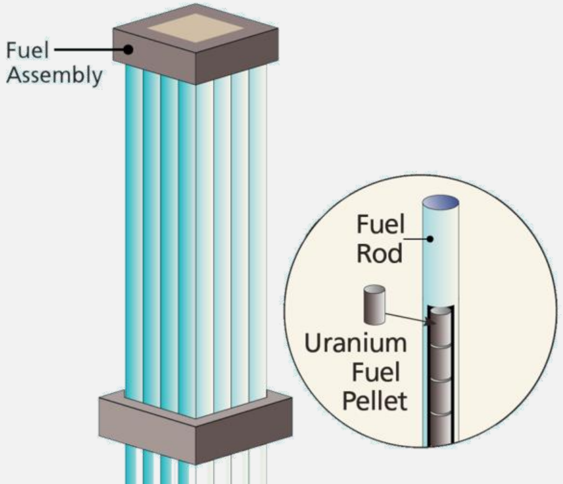
The probability of LRA emission

$$P_{\mathrm{LRA}} = \frac{N_{\mathrm{F1F2}\alpha}}{N_{\mathrm{F1F2}}}$$

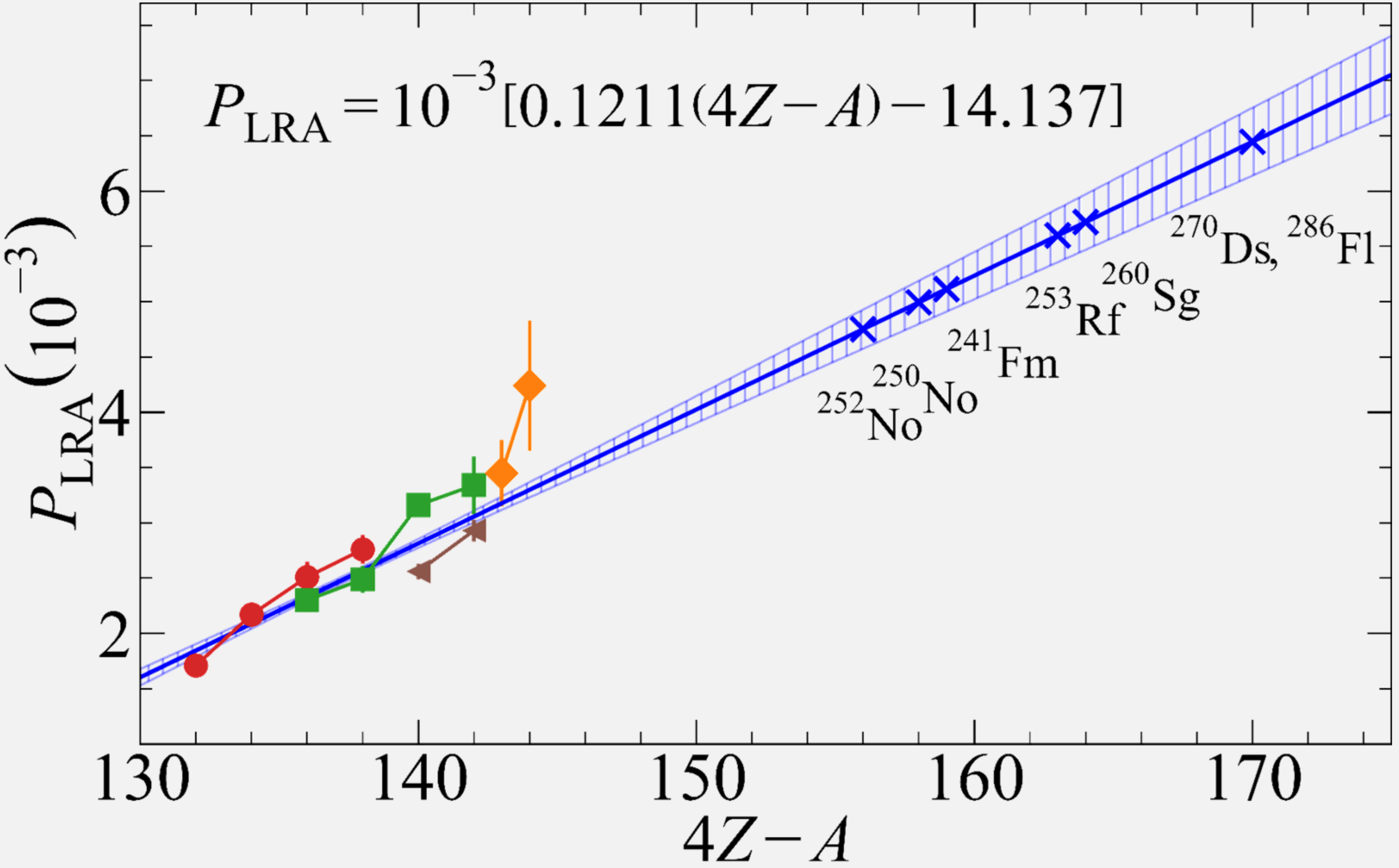
$N_{\mathrm{F1F2}\alpha}$ is the number of fission events in which two fission fragments accompanied by LRA are detected

N_{F1F2} is the number of events in which only two fission fragments are detected

- Represents **~90% of all TF events**
- **~1** LRA emission event per 300-500 binary fission events
- Affects reactor isotopic composition (^{4}He, ^{3}H)



P_{LRA}: function of charge and mass numbers combination

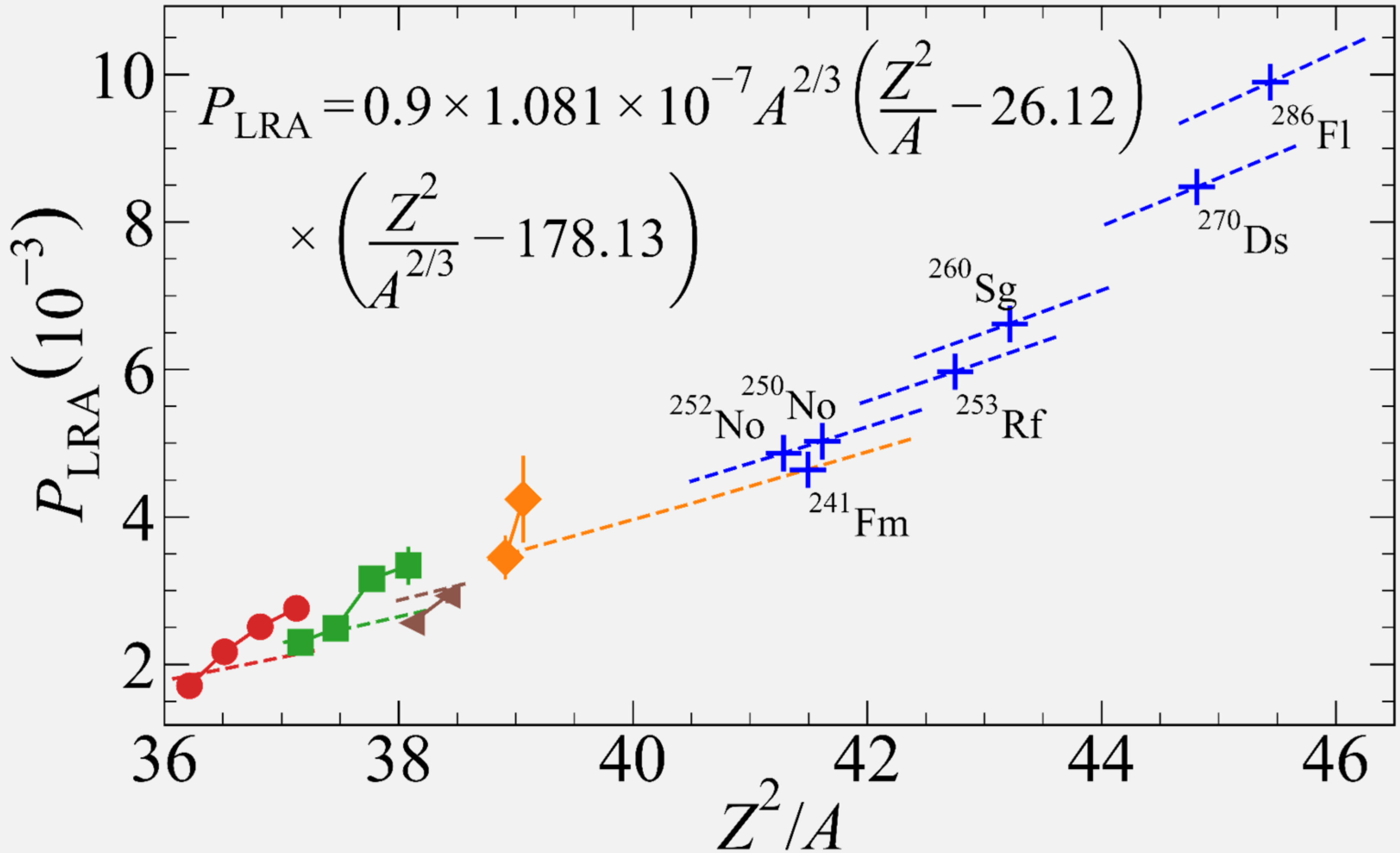


[I. Halpern, Annu. Rev. of Nucl. Sci. **21**, 245 (1971); J. A. Coleman, A. W. Fairhall, and I. Halpern, Phys. Rev. **133**, B724 (1964); W. D. Loveland, A. W. Fairhall, and I. Halpern, Phys. Rev. **163**, 1315 (1967)]

- Linear dependence on Z and A
- P_{LRA} increases with Z
- P_{LRA} decreases with A (for a given element)
- *Limitation:* predicts the same LRA emission probabilities

P_{LRA} (solid line) as a function of $4Z - A$. Shaded areas indicate the limits of the fit within the confidence interval of 1σ. Cross symbols represent predicted P_{LRA} values for the indicated actinides and SHN. Experimental data for isotopes of the same element are shown as closed symbols, , , and connected by solid lines.

P_{LRA}: function of fissility parameter

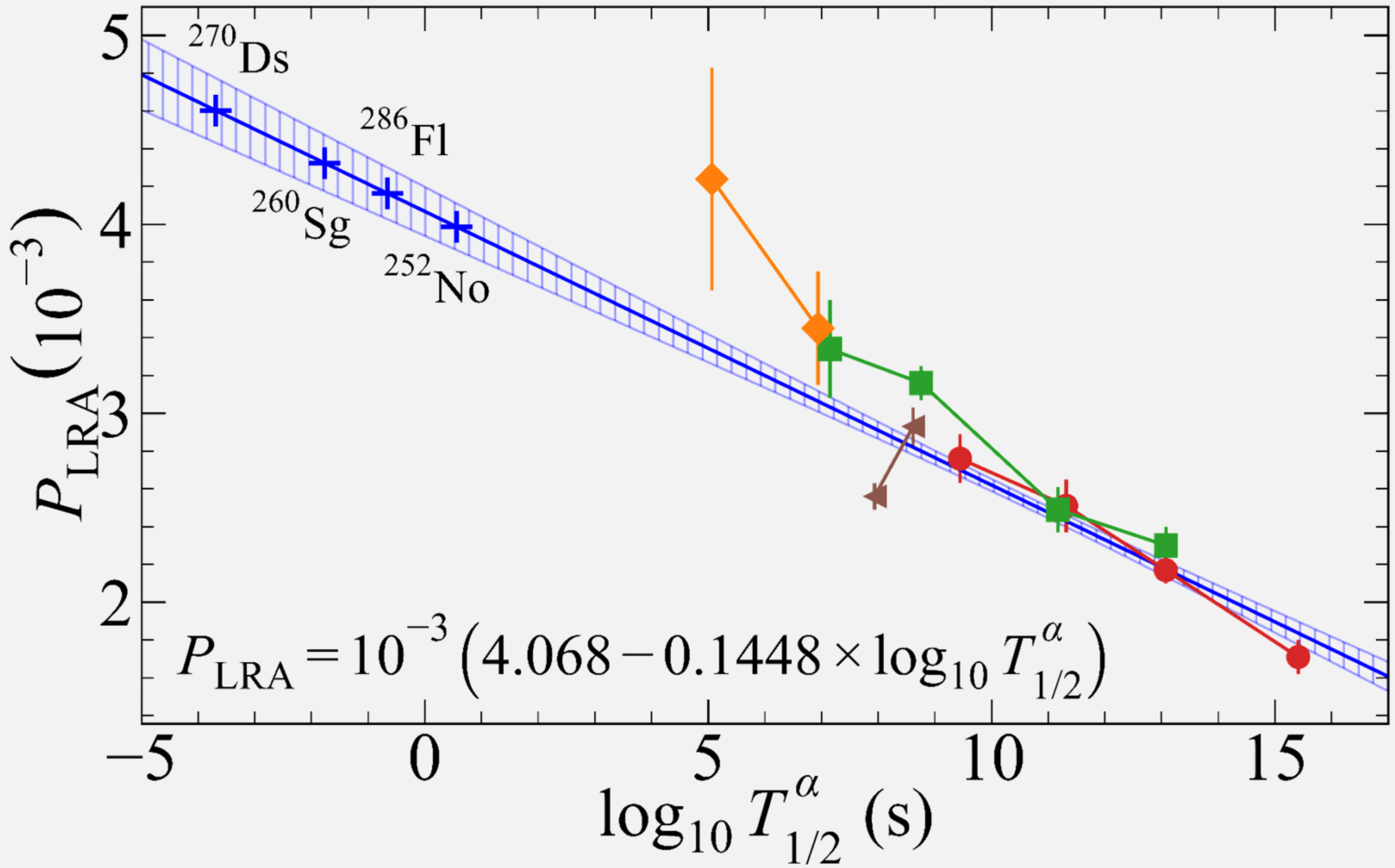


[V. A. Rubchenya and S. G. Yavshits, Zeitschrift für Physik A Atomic Nuclei **329**, 217 (1988)]

- Linear dependence on Z^2/A
- Favors LRA emission in neutron-deficient isotopes
- Predicts significant P_{LRA} (~1%) for SHN

P_{LRA} (solid line) as a function of fissility parameter Z^2/A. Shaded areas indicate the limits of the fit within the confidence interval of 1σ. Cross symbols represent predicted P_{LRA} values for the indicated actinides and SHN. Experimental data for isotopes of the same element are shown as closed symbols connected by solid lines.

P_{LRA}: function of α-decay half-life

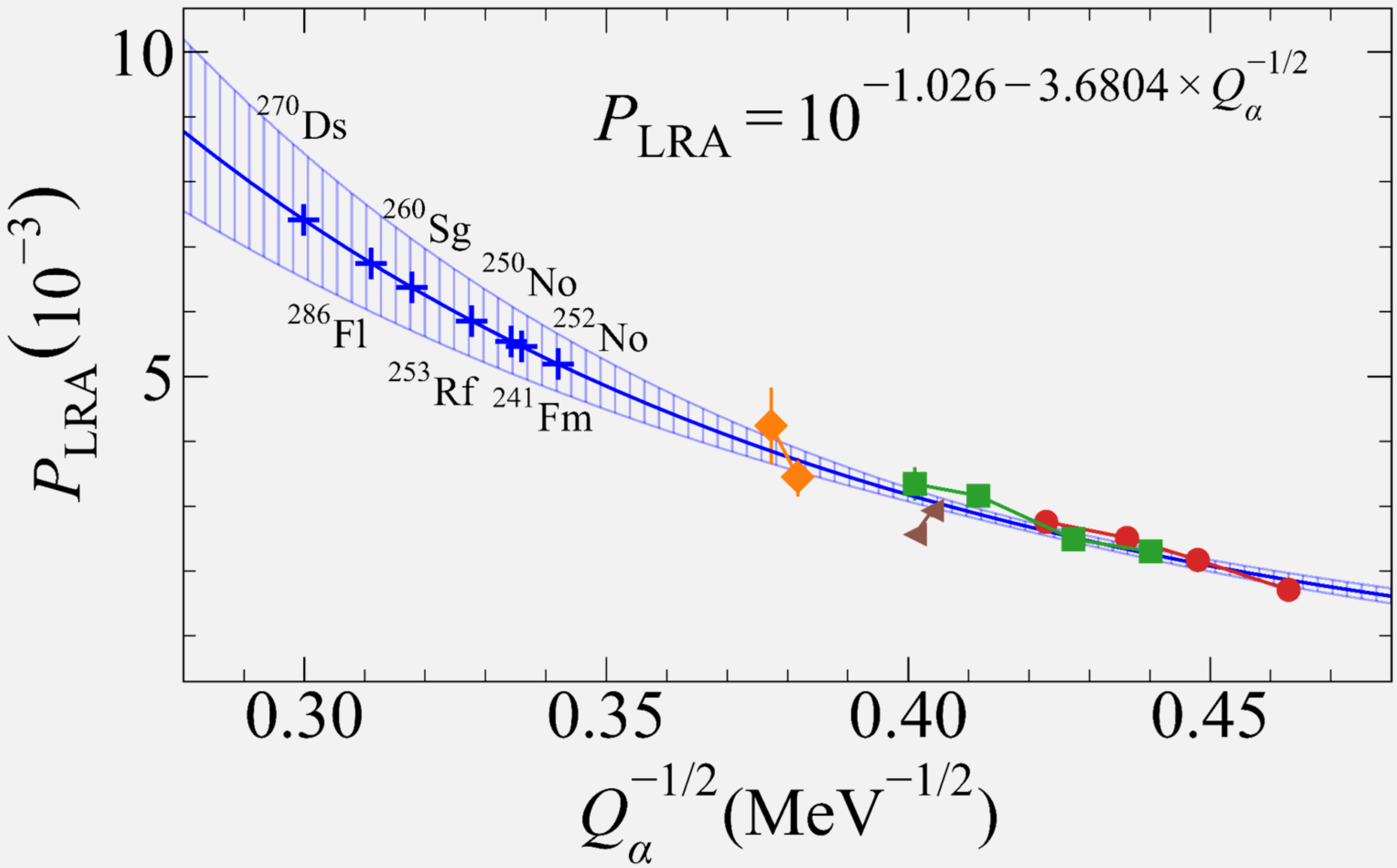


[C. Wagemans et al., Phys. Rev. C **33**, 943 (1986); J. Khuyagbaatar, Phys. Rev. C **110**, 014311 (2024)]

- Increased LRA emission for neutron-deficient nuclei
- Predicts different LRA emission probabilities for SHN
- *Limitation:* $\log T^{\alpha}_{1/2}$ are not known for all nuclei

P_{LRA} (solid line) as a function of alpha-decay half-live $\log T^{\alpha}_{1/2}$. Shaded areas indicate the limits of the fit within the confidence interval of 1σ. Cross symbols represent predicted P_{LRA} values for the indicated actinides and SHN. Experimental data for isotopes of the same element are shown as closed symbols connected by solid lines.

P_{LRA}: function of energy release of α decay

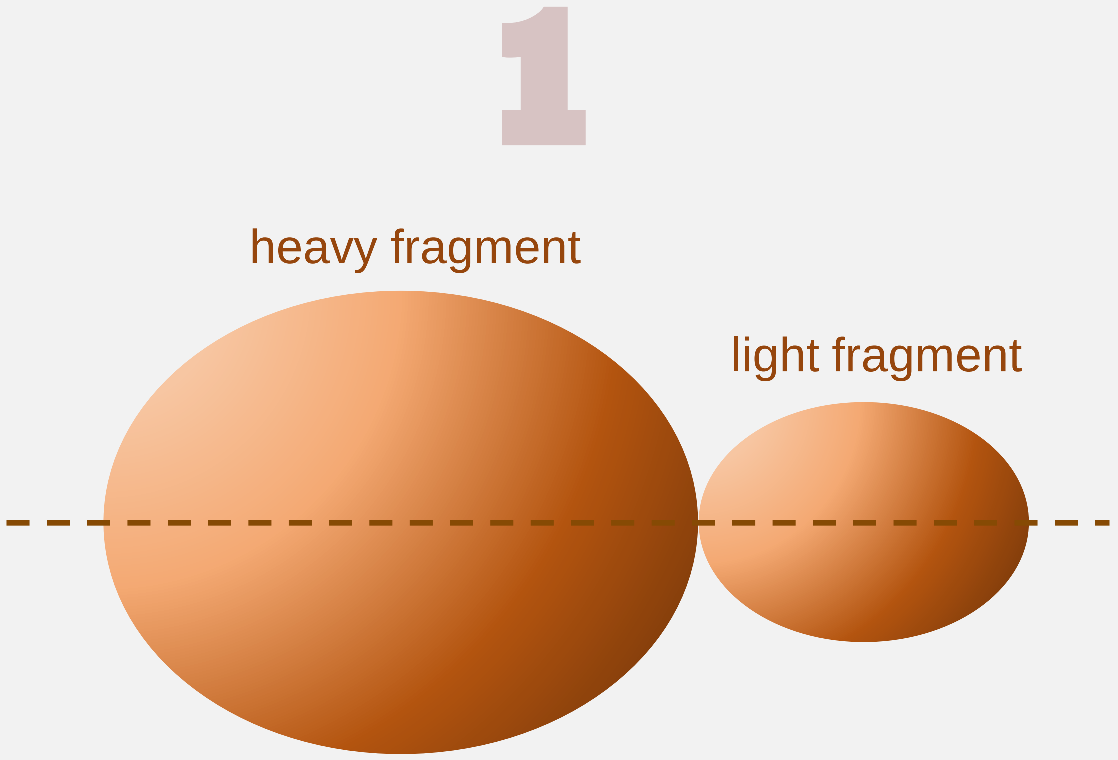


[J. Khuyagbaatar, Phys. Rev. C **110**, 014311 (2024)]

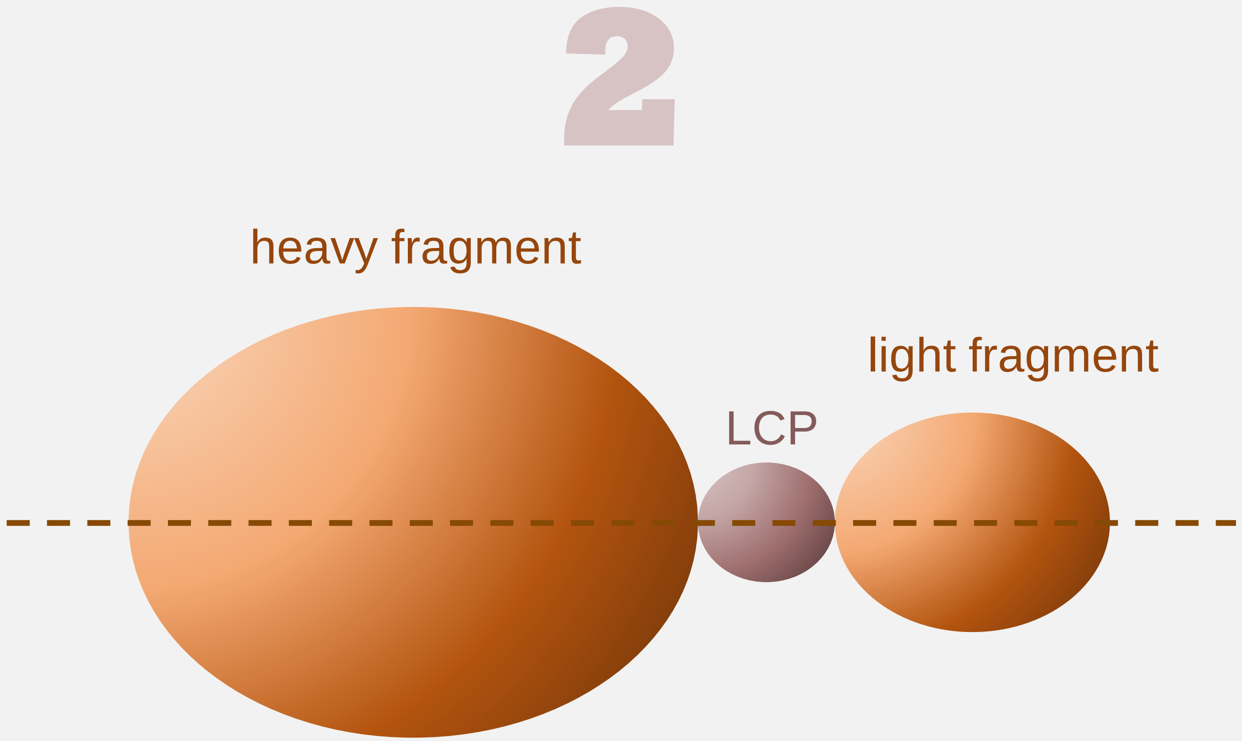
- Based on energy release of α decay
- Can use estimated Q_α values when experimental data is missing

P_{LRA} (solid line) as a function of $Q_\alpha^{-1/2}$. Shaded areas indicate the limits of the fit within the confidence interval of 1σ. Cross symbols represent predicted P_{LRA} values for the indicated actinides and SHN. Experimental data for isotopes of the same element are shown as closed symbols connected by solid lines.

Ternary fission as a two-step process



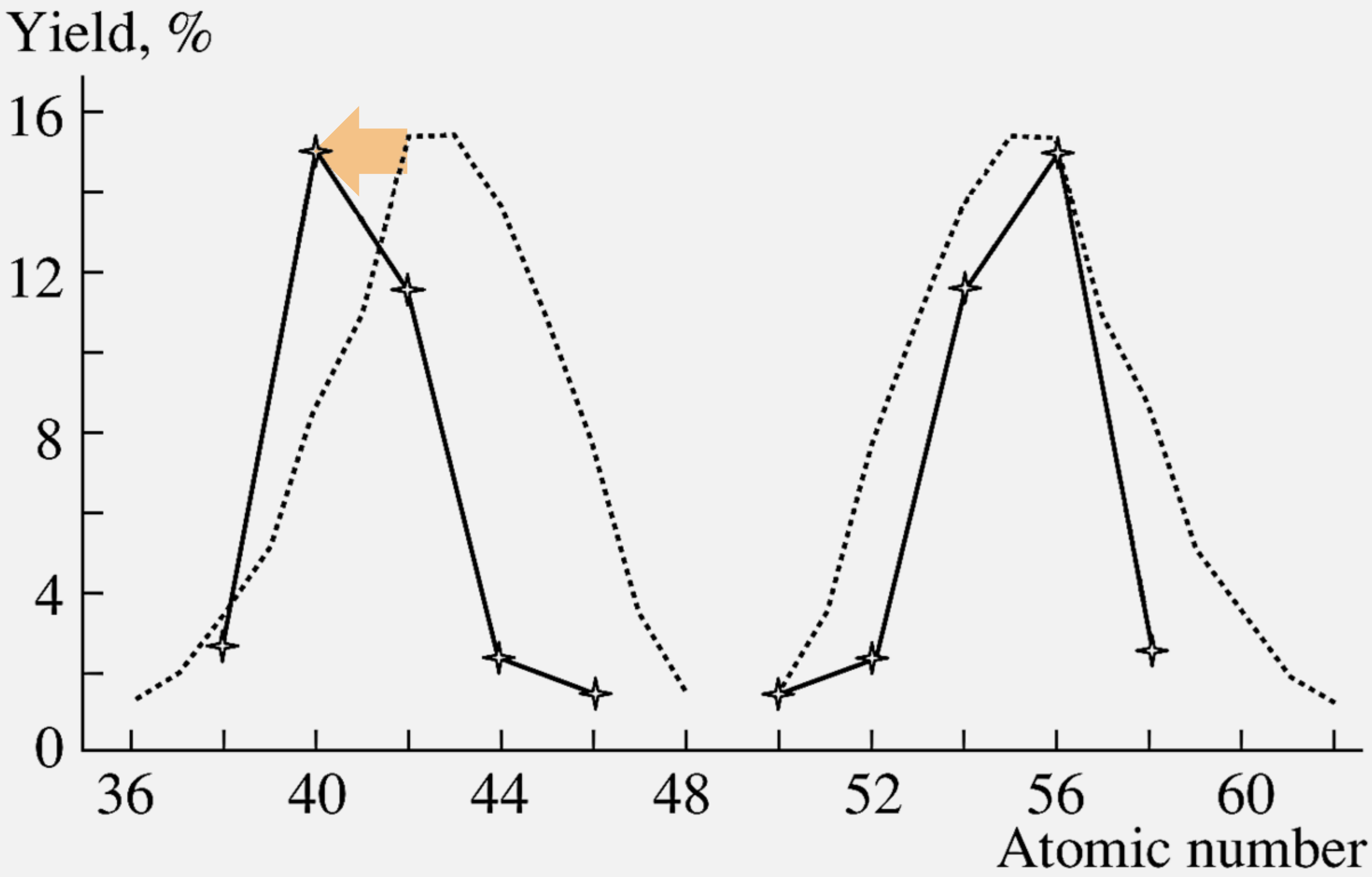
Scission configuration of the binary system



Scission configuration of the ternary system

[A. V. Andreev, G. G. Adamian, N. V. Antonenko, S. P. Ivanova, S. N. Kuklin, and W. Scheid, Eur. Phys. J. A **30**, 579 (2006)]

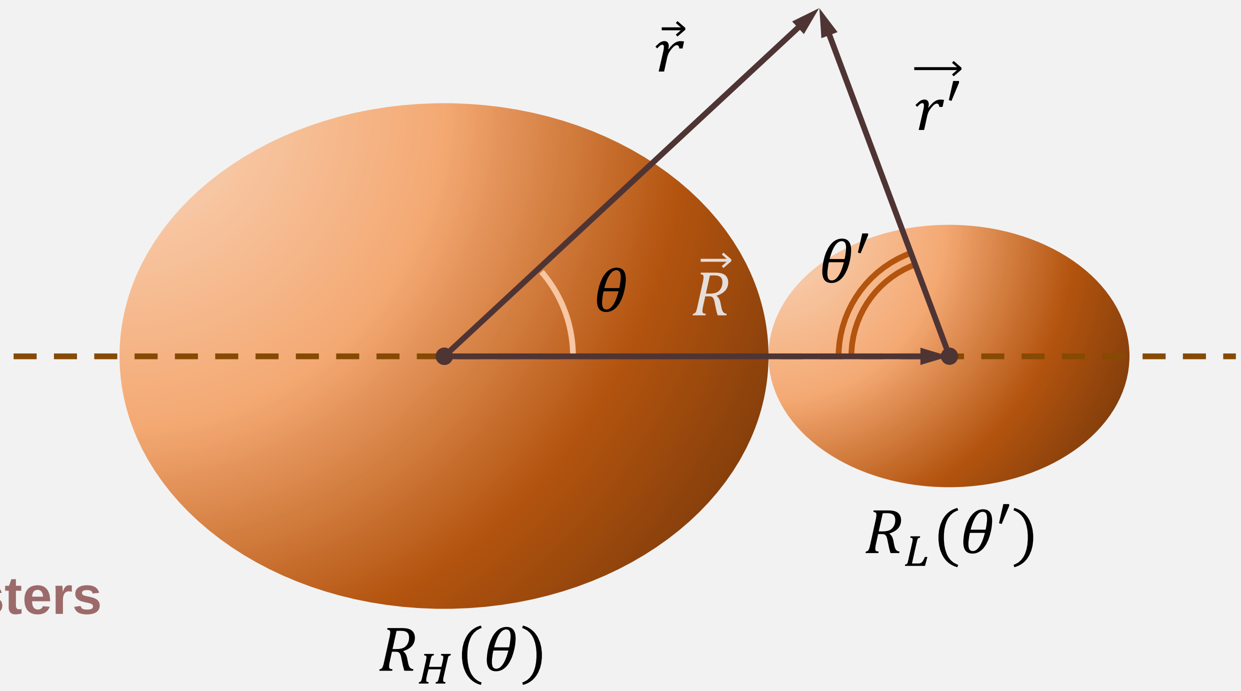
[G. Ter-Akopian et al., Physics of Atomic Nuclei **67**, 1860 (2004)]



The charge distributions obtained for main fragments emitted in the ternary fission of ^{252}Cf accompanied, respectively, by He (stars) LCPs. The dotted lines show the charge distribution known for the binary fission of ^{252}Cf.

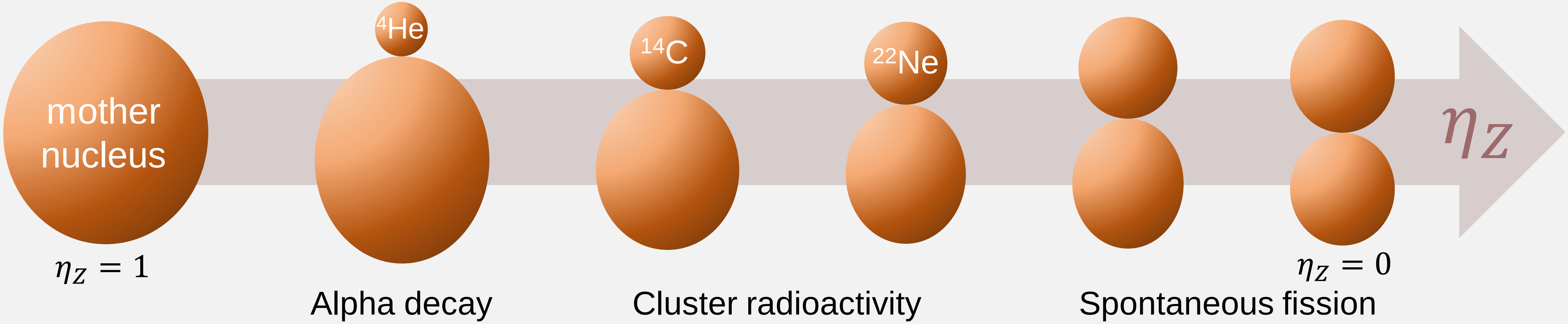
DNS

Schematic representation of the DNS
in the case of axially symmetric mutual arrangement of clusters



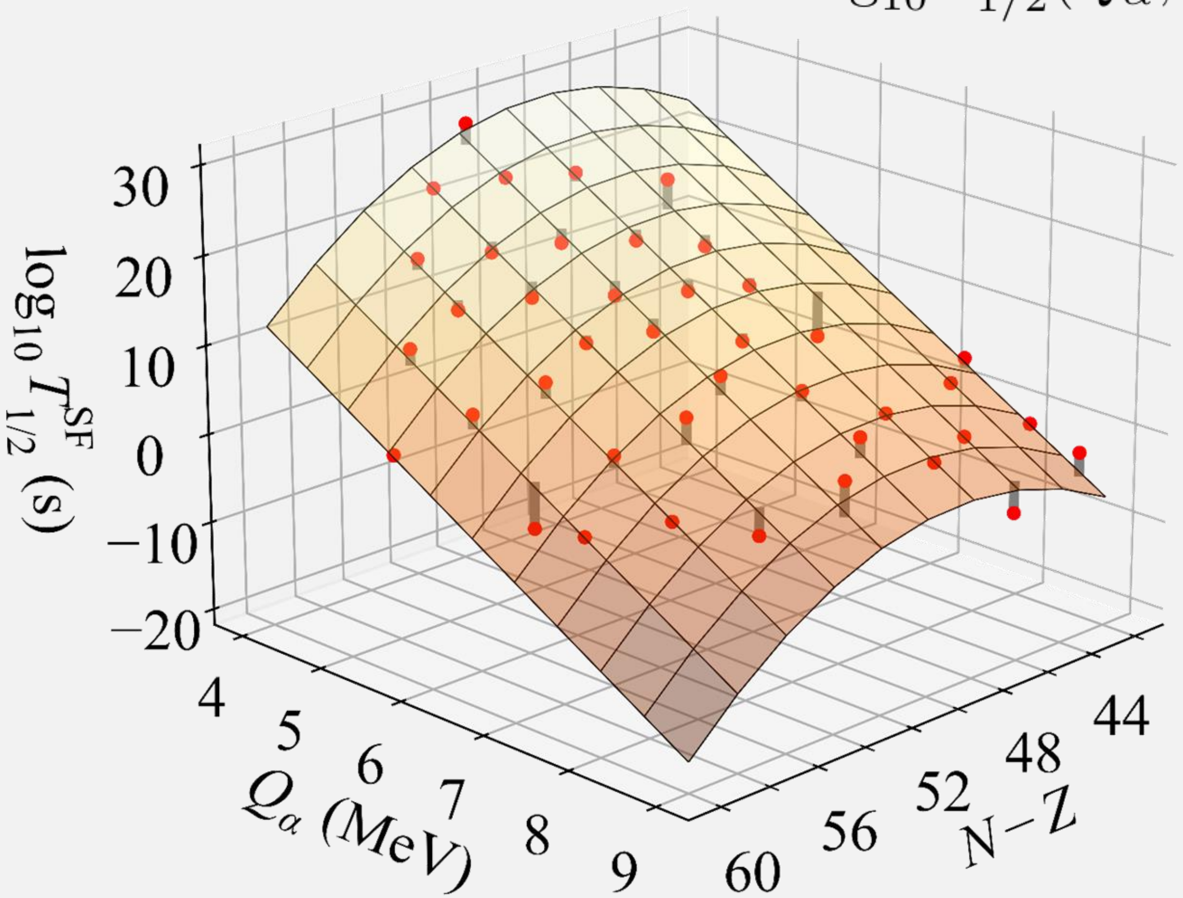
Characteristics of DNS:

- relative **distance $\vec{R}$ between clusters**
- **Euler angles**
- **charge asymmetry** $\eta_Z = \frac{Z_H - Z_L}{Z_H + Z_L}$, Z_H and Z_L are charge numbers of the heavy and light clusters



[G. Adamian, N. Antonenko, and W. Scheid, Clustering effects within the dinuclear model, in Clusters in Nuclei, Vol.2, edited by C. Beck (Springer Berlin Heidelberg, Berlin, Heidelberg, 2012) pp. 165–227.]

NEW EMPIRICAL FORMULA
FOR SPONTANEOUS FISSION HALF-LIVE



$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha, N-Z) = c_0 + c_1(N-Z) + c_2(N-Z)^2 + c_Q Q_\alpha + \begin{cases} 0, & \text{for even-even,} \\ h, & \text{for odd-}A, \\ 1.575h, & \text{for odd-odd.} \end{cases}$$

$90 \leqslant Z \leqslant 102$:

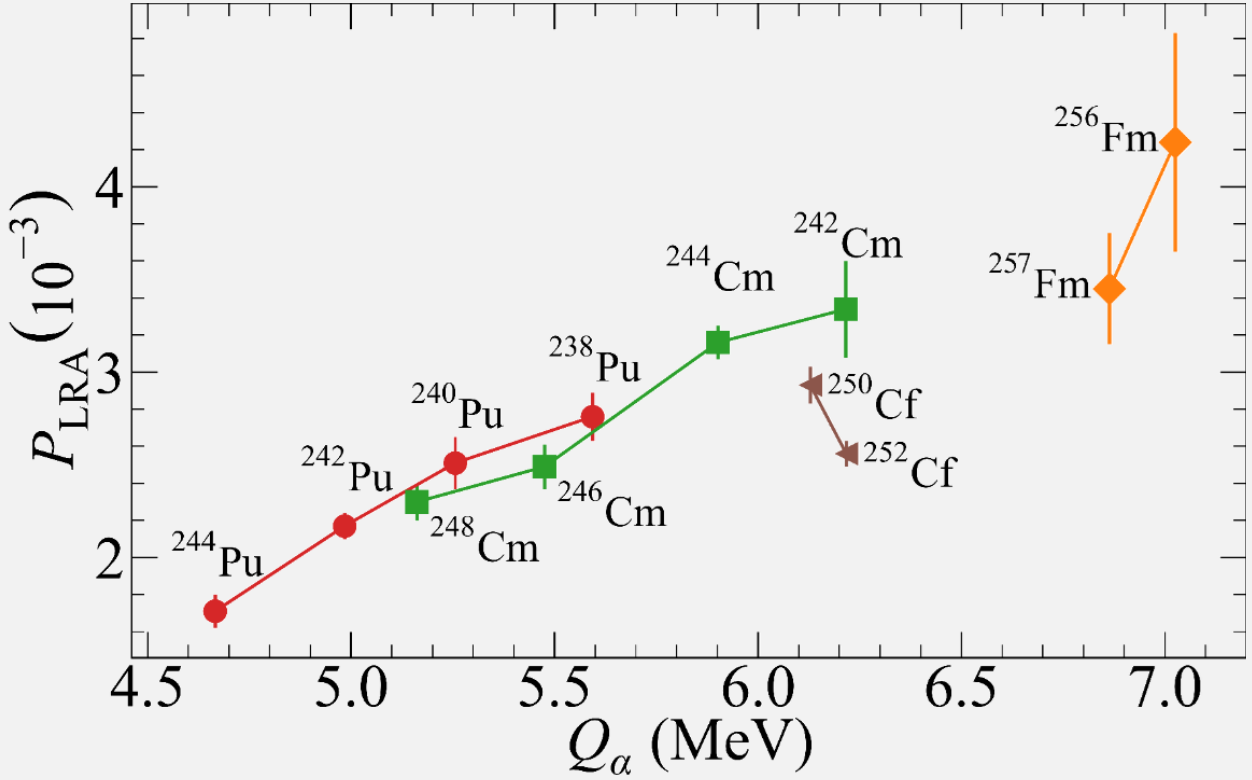
$c_0 = -244.845, \quad c_1 = 12.2843, \quad c_2 = -0.12658,$
$c_Q = -6.1023, \quad h = 4.309,$

$Z \geqslant 103$:

$c_0 = -0.414, \quad c_1 = 0.1379, \quad c_2 = 0,$
$c_Q = -0.9097, \quad h = 2.327.$

Decimal logarithm of the spontaneous fission half-life of even-even isotopes with atomic numbers $90 \leq Z \leq 102$, plotted as a function of Q_α and $N - Z$. The surface is based on calculations using proposed formula. Experimental values [F. Kondev et al., Chin. Phys. C **45**, 030001 (2021)] are shown as red circles. Gray segments visualize the deviations of the experimental data from the calculated surface.

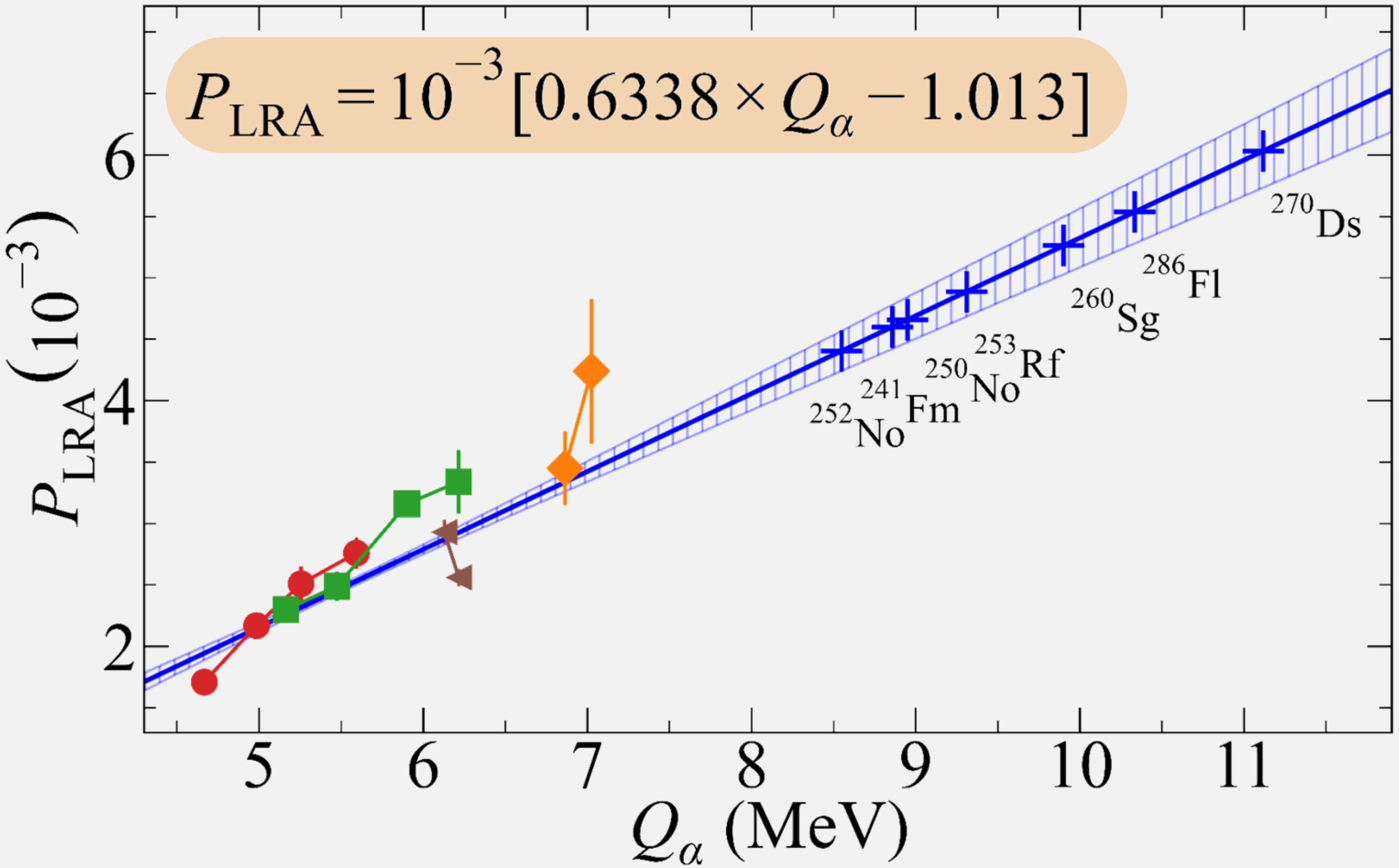
Experimental values of P_{LRA} in various spontaneous TF



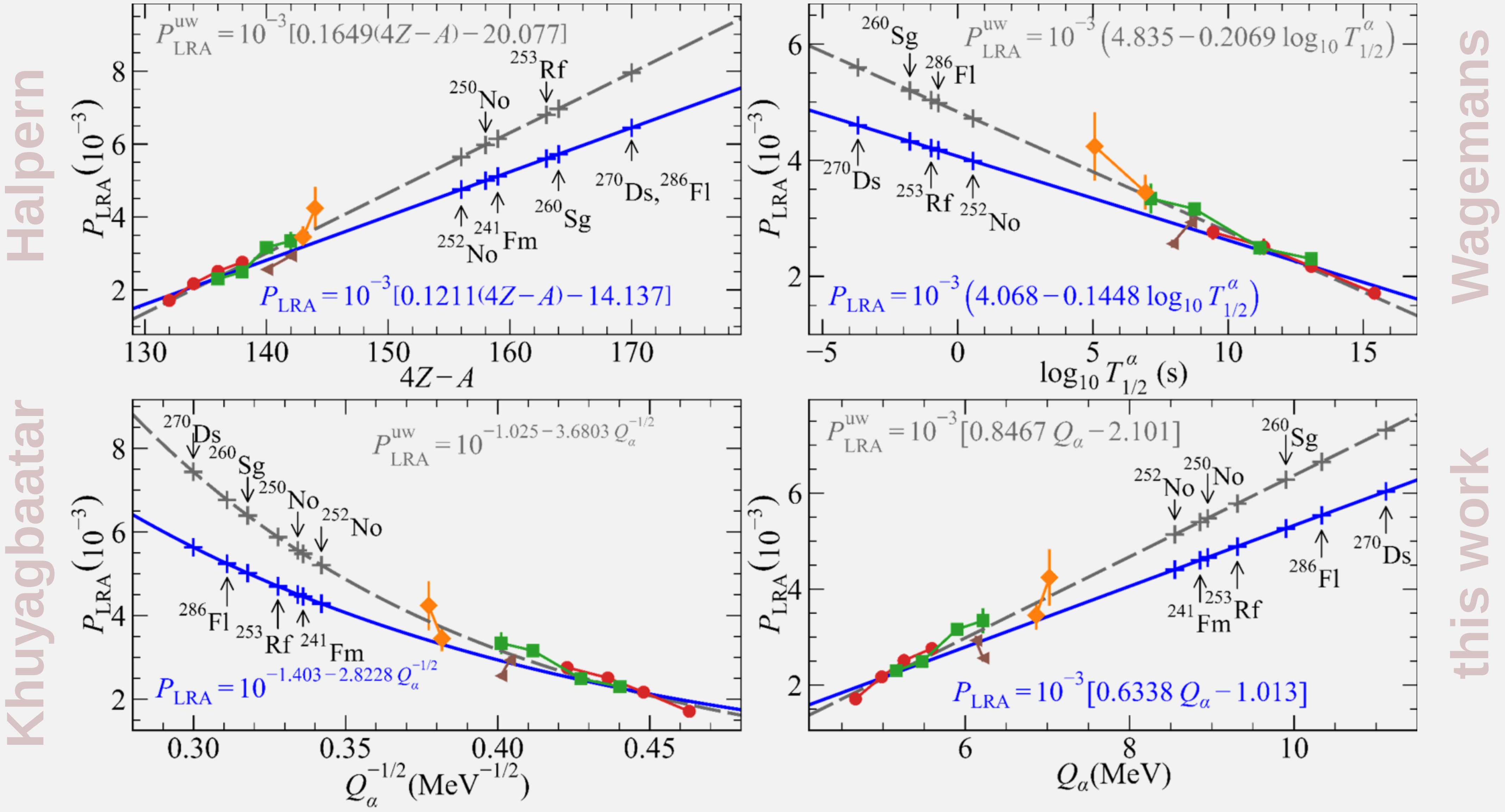
Experimental values of LRA emission probabilities P_{LRA} as a function of Q_α of the fissioning nucleus. For isotopes of the same element, symbols are connected by solid lines.

Isotope	Z	N	$P_{\mathrm{LRA}}(10^{-3})$
^{238}Pu	94	144	2.76 ± 0.13
^{240}Pu	94	146	2.51 ± 0.14
^{242}Pu	94	148	2.17 ± 0.07
^{244}Pu	94	150	1.71 ± 0.09
^{242}Cm	96	146	3.34 ± 0.26
^{244}Cm	96	148	3.16 ± 0.09
^{246}Cm	96	150	2.49 ± 0.12
^{248}Cm	96	152	2.30 ± 0.10
^{250}Cf	98	152	2.93 ± 0.10
^{252}Cf	98	154	2.56 ± 0.07
^{256}Fm	100	156	4.24 ± 0.59
^{257}Fm	100	155	3.45 ± 0.30

NEW EMPIRICAL FORMULA FOR PROBABILITY OF LRA EMISSION

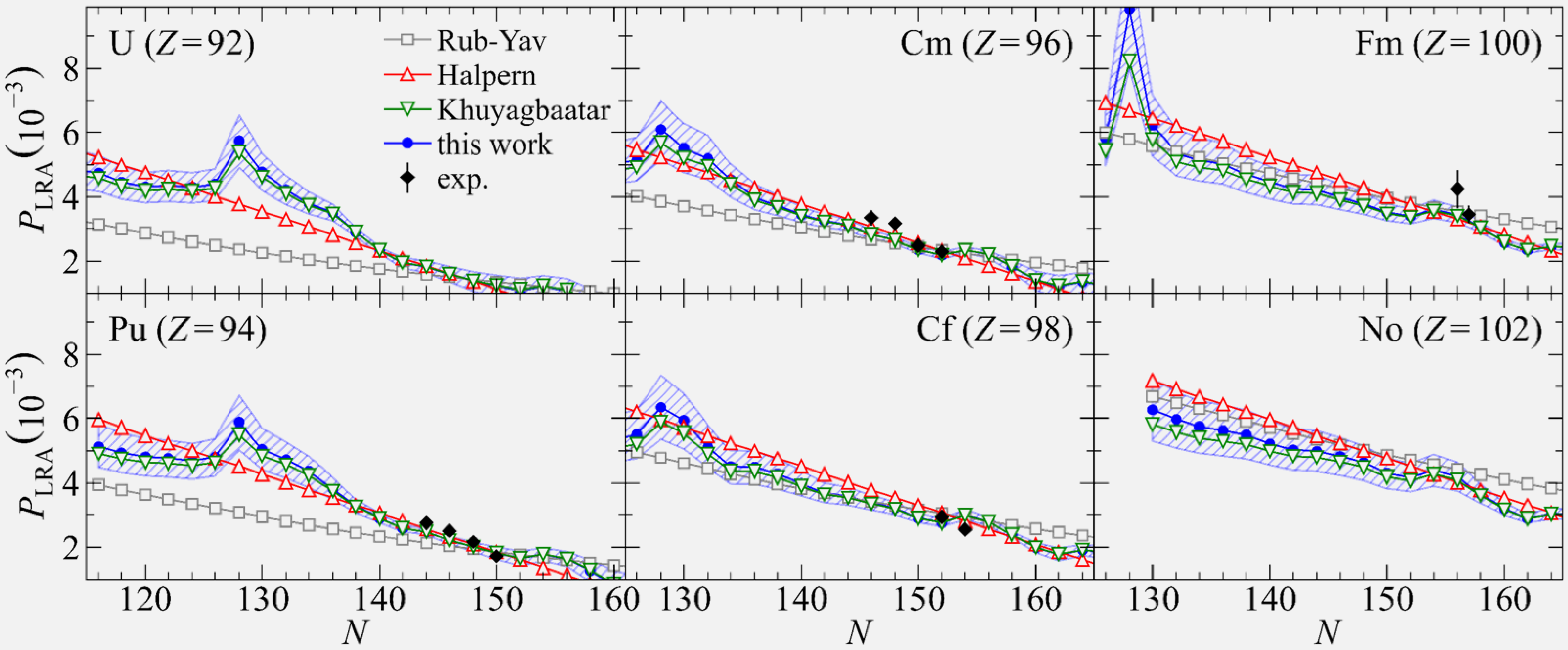


P_{LRA} (solid line) as a function of Q_α. Shaded areas indicate the limits of the fit within the confidence interval of 1σ. Plus symbols represent predicted P_{LRA} values for the indicated actinides and SHN. Experimental data for isotopes of the same element are shown as closed symbols connected by solid lines.



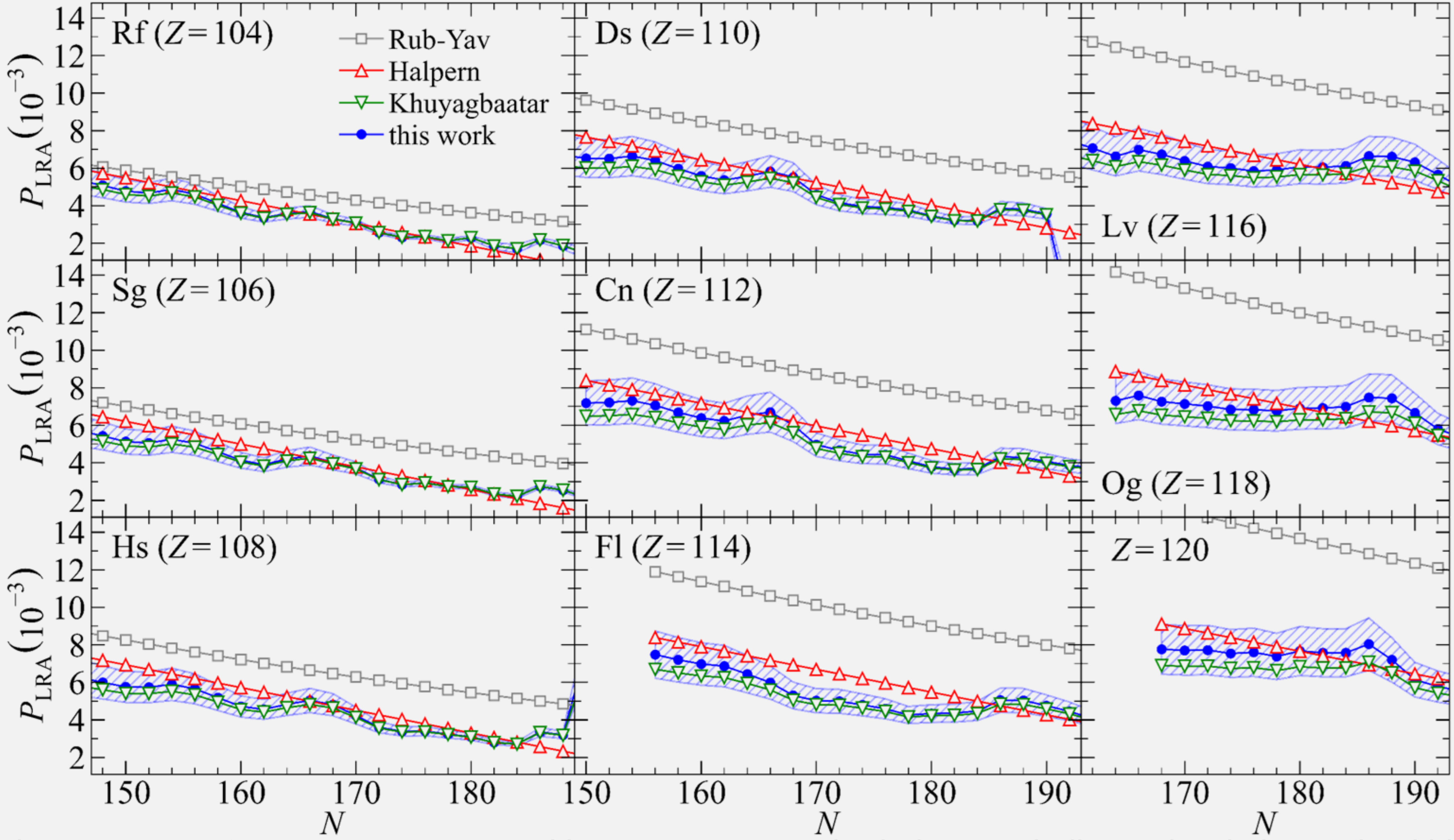
Impact of experimental uncertainties of $P_{\rm LRA}$ on the fit and predictive power. LRA emission probabilities $P_{\rm LRA}$ (solid and dashed lines) are plotted as functions of $4Z-A$, $\log_{10} T_{1/2}^{\alpha}$, $Q_\alpha^{-1/2}$, and Q_α. The gray dashed lines show the fit obtained without taking into account experimental uncertainties (unweighted fit), and the solid blue lines illustrate the fit when experimental uncertainties are included (weighted fit). Symbols , , , and represent experimental data.

COMPARISON OF THE PREDICTIONS: Actinide Region $(92 \leq Z \leq 102)$



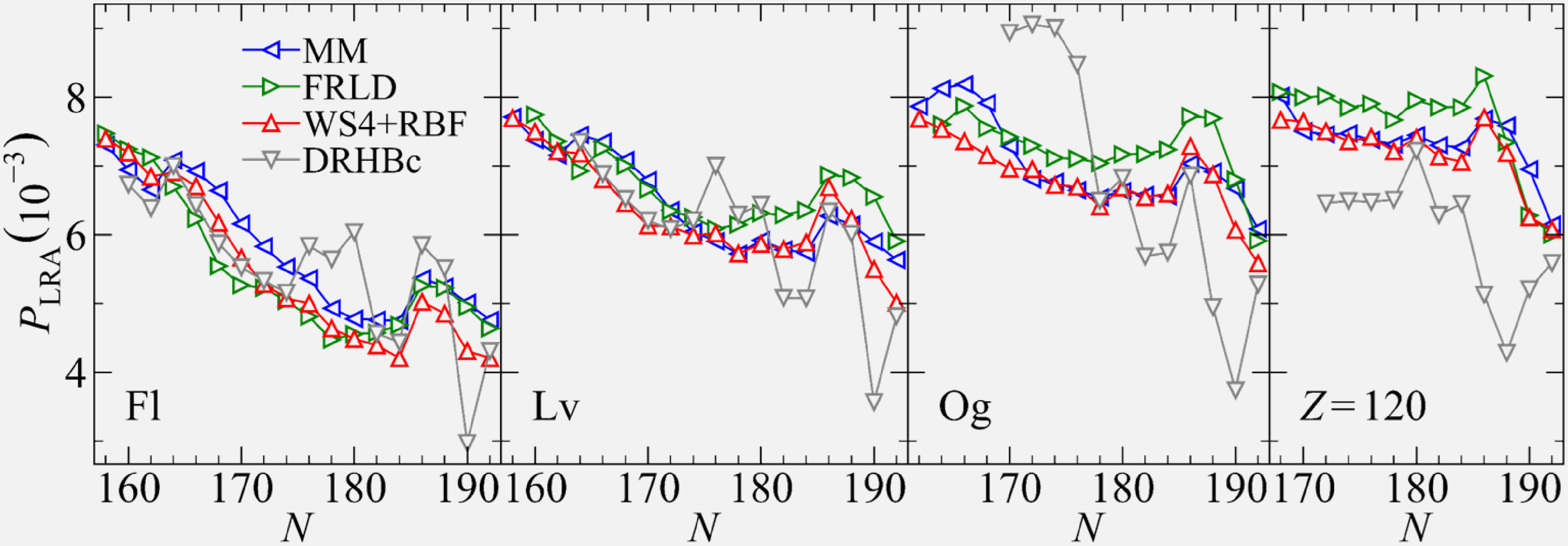
Predicted P_{LRA} values for even-even heavy nuclei with charge numbers $Z = 92$–102 based on different formulas. Shaded areas indicate the limits of fit within 3σ with the new formula. Q_α values are taken from [P. Möller et al., At. Data Nucl. Data Tables **59**, 185 (1995)]. Experimental data are shown as solid symbols.

COMPARISON OF THE PREDICTIONS: Superheavy Region $(104 \leq Z \leq 120)$



Predicted P_{LRA} values for even-even SHN with $Z = 104$–120. Shaded areas indicate the limits of fit within 3σ with proposed formula. Q_α values are taken from [P. Möller et al., At. Data Nucl. Data Tables **59**, 185 (1995)].

Sensitivity to Mass Table Choice



Effect of the choice of atomic mass model on predicted P_{LRA} for even-even SHN with charge numbers $Z = 114$–120, calculated using proposed formula.

THEORETICAL MODELS

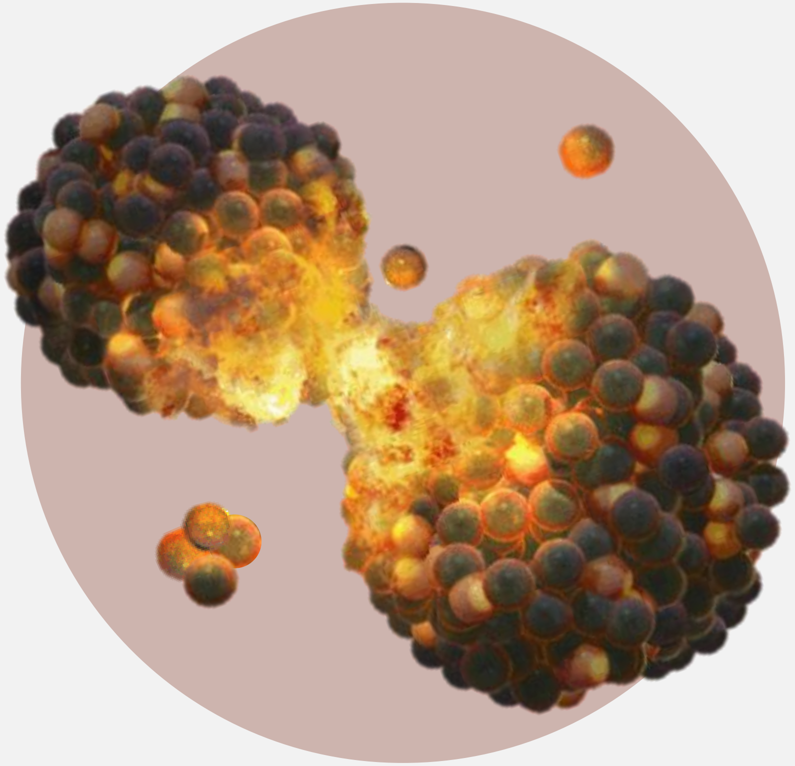
macroscopic-microscopic:

- deformed Woods-Saxon single-particle potential with the macroscopic Yukawa-plus-exponential energy (MM) [P. Jachimowicz, M. Kowal, and J. Skalski, Atom. Data Nucl. Data Tables **138**, 101393 (2021)]
- finite range liquid-drop model (FRLDM) [P. Möller et al., Atom. Data Nucl. Data Tables **109**, 1 (2016)]
- Weitzäcker-Skyrme models with a radial basis function (WS4+RBF) [N. Wang et al., Physics Letters B **734**, 215 (2014)]

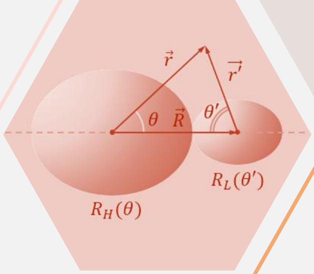
microscopic:

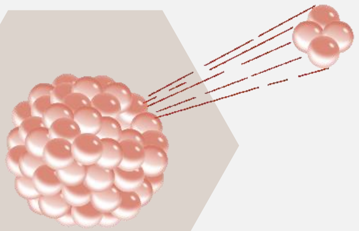
- the deformed relativistic Hartree-Bogoliubov theory in the continuum (DRHBc) [K. Zhang et al., Phys. Rev. C **102**, 024314 (2020), Atom. Data Nucl. Data Tables **144**, 101488 (2022); Guo et al., Atom. Data Nucl. Data Tables **158**, 101661 (2024)]

CONCLUSIONS



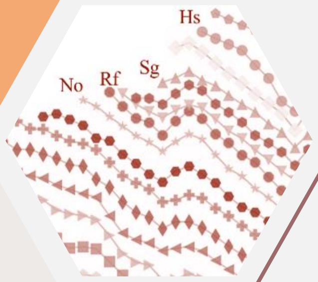
- **correlations** between **LRA emission probabilities** and the fissility parameter Z^2/A, the linear combination of proton and mass numbers $4Z - A$, and the inverse square root of the energy release $Q_\alpha^{-1/2}$ **were analyzed**
- **semi-empirical formula** for the LRA emission probability **was proposed** as a linear function of the alpha-particle energy release Q_α
- linear dependence helps to **reduce prediction errors** for neutron-deficient nuclei
- **maxima in LRA emission probability**, attributed to the **influence of shell e**ffects on Q_α, are discussed
- all methods consistently predict an **increase in** P_{LRA} **towards SHN**, almost approaching 1%, which indicates the importance of considering their LRA emission
- predicted P_{LRA} values for heavy neutron-deficient nuclei decrease by about (23–32)% when the **experimental error bars are included** in the fit





THANK YOU
FOR ATTENTION

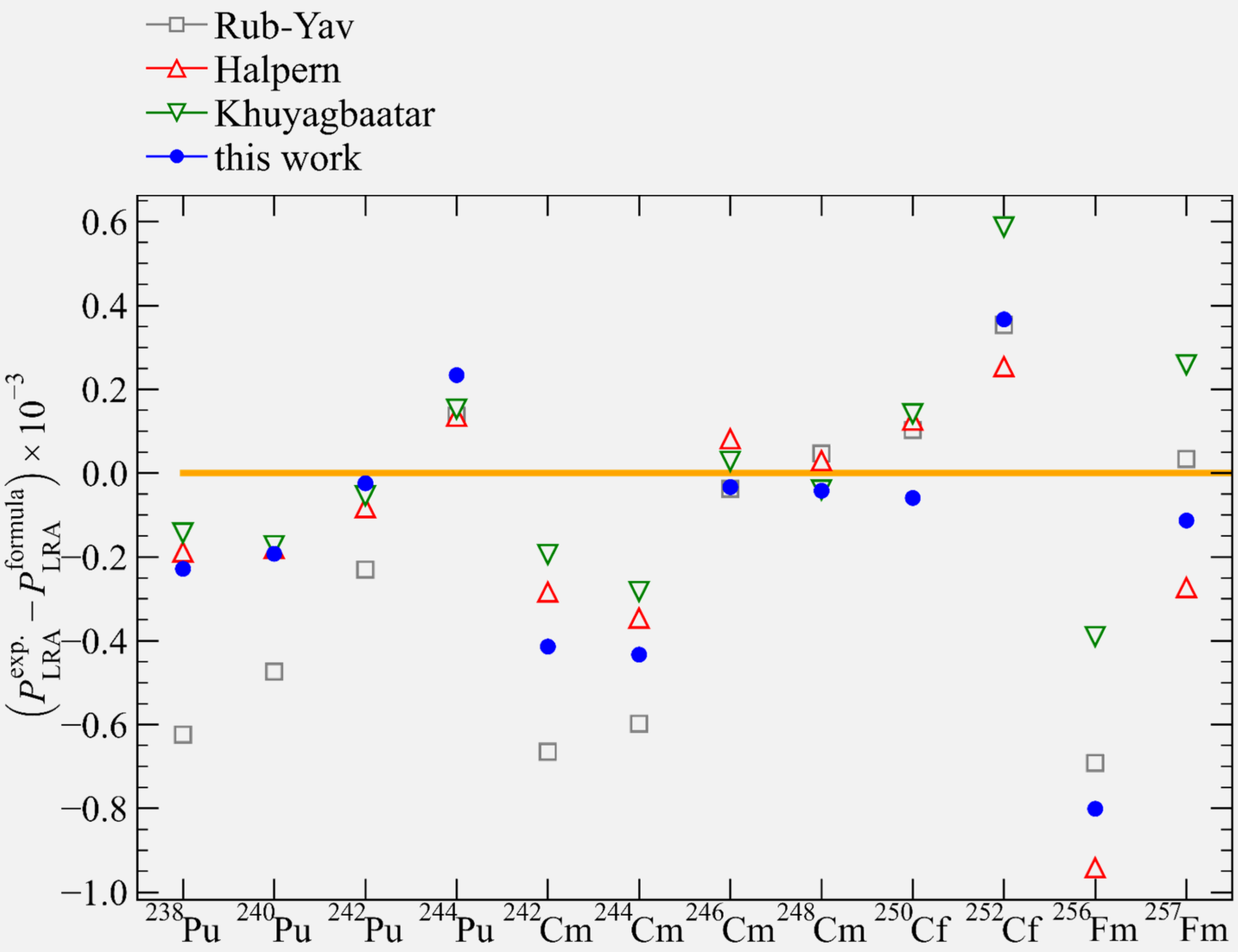
Nikita Moiseev



BACKUP SLIDES

Experimental values of P_{LRA} in various spontaneous TF

Isotope	Z	N	$P_{\mathrm{LRA}}(10^{-3})$	Refs..
^{238}Pu	94	144	2.76 ± 0.13	[O. Serot and C. Wagemans, Nuclear Physics A **641**, 34 (1998)]
^{240}Pu	94	146	2.51 ± 0.14	[O. Serot and C. Wagemans, Nuclear Physics A **641**, 34 (1998)]
^{242}Pu	94	148	2.17 ± 0.07	[O. Serot and C. Wagemans, Nuclear Physics A **641**, 34 (1998)]
^{244}Pu	94	150	1.71 ± 0.09	[O. Serot and C. Wagemans, Nuclear Physics A **641**, 34 (1998)]
^{242}Cm	96	146	3.34 ± 0.26	[O. Serot and C. Wagemans, Nuclear Physics A **641**, 34 (1998)]
^{244}Cm	96	148	3.16 ± 0.09	[S. Vermote et al., Nuclear Physics A **806**, 1 (2008)]
^{246}Cm	96	150	2.49 ± 0.12	[S. Vermote et al., Nuclear Physics A **806**, 1 (2008)]
^{248}Cm	96	152	2.30 ± 0.10	[S. Vermote et al., Nuclear Physics A **806**, 1 (2008)]
^{250}Cf	98	152	2.93 ± 0.10	[S. Vermote et al., Nuclear Physics A **837**, 176 (2010)]
^{252}Cf	98	154	2.56 ± 0.07	[S. Vermote et al., Nuclear Physics A **837**, 176 (2010)]
^{256}Fm	100	156	4.24 ± 0.59	[O. Serot and C. Wagemans, Nuclear Physics A **641**, 34 (1998)]
^{257}Fm	100	155	3.45 ± 0.30	[O. Serot and C. Wagemans, Nuclear Physics A **641**, 34 (1998)]



Deviations of P_{LRA} predictions with different formulas from the experimental values for known nuclei.

Formula	$\langle\delta\rangle$	δ_{rms}	δ_{max}
Rub-Yav	0.33	0.42	0.69
Halpern	0.24	0.33	0.94
Khuyagbaatar	0.20	0.26	0.59
this work	0.24	0.33	0.80

Mean absolute deviation $\langle\delta\rangle$, rms deviation δ_{rms}, and maximum absolute deviation δ_{max} of P_{LRA} calculated with different formulas from the experimental data.

P_{LRA}: function of charge and mass numbers combination

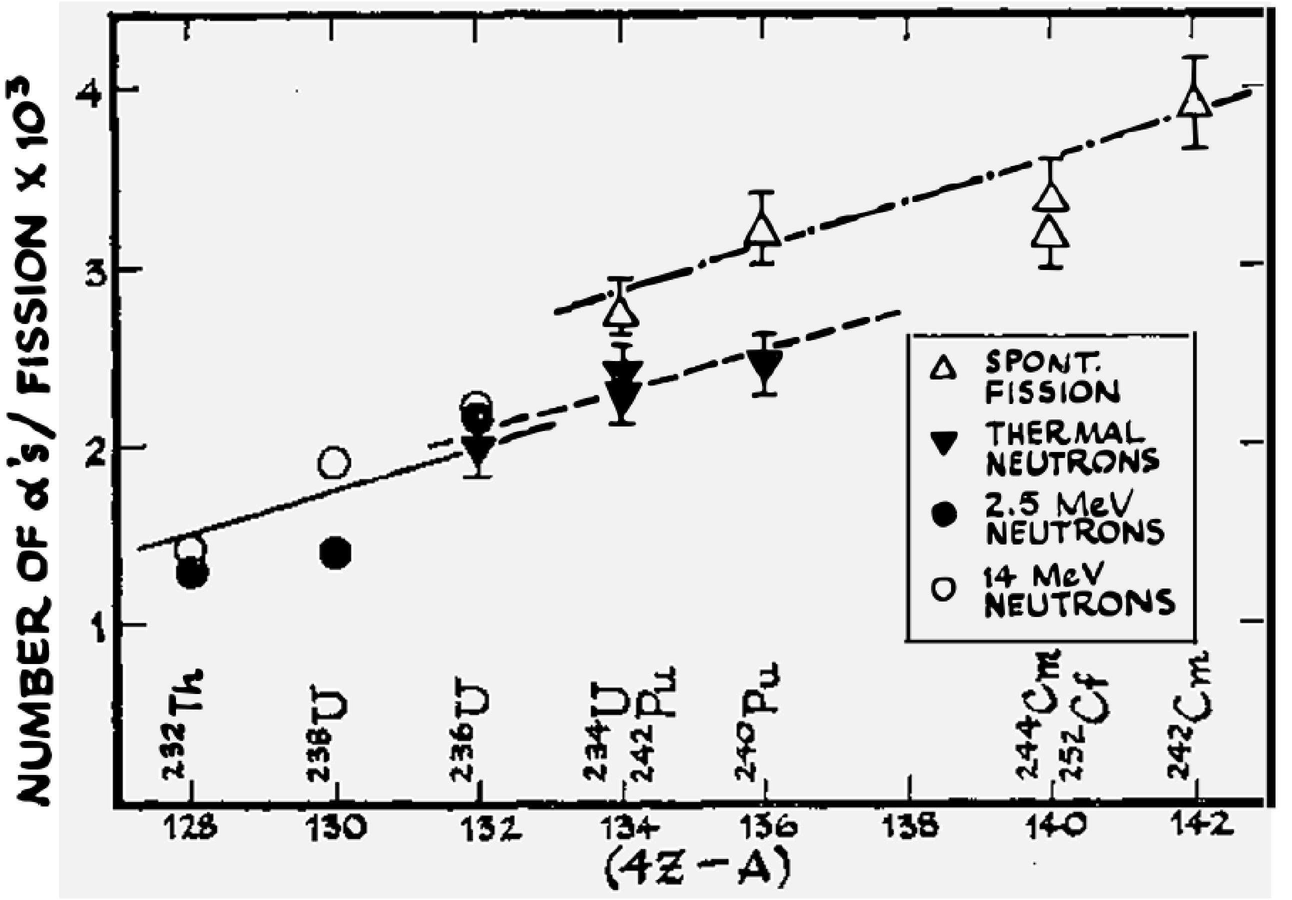


FIGURE 7. The dependence of the α-particle production rate in fission upon Z and A of the fissioning nucleus. It is found empirically that the single variable $\xi = 4Z - A$ organizes the measured yields into reasonable straight lines for the different excitation energies. The largest yields are for spontaneous fission (62). It is seen that there is only a weak dependence of the yields (suitably normalized) upon the energy of the incident neutrons between thermal energies and 14 MeV (4, 54). The statistical uncertainties of the data given by circles are about as large as the circles themselves.

[I. Halpern, Annu. Rev. of Nucl. Sci. **21**, 245 (1971)]

Expantion of the yield about some central point Z_0, A_0:

$$Y(Z, A) \simeq Y(Z_0, A_0) + (\partial Y/\partial Z)_0 \Delta Z + (\partial Y/\partial A)_0 \Delta A$$

$$\xi \equiv (\beta \Delta Z + \Delta A)$$

$$\beta = (\partial Y/\partial Z)_0/(\partial Y/\partial A)_0$$

$$\beta = -4$$

$$(4Z - A)$$

$(\partial Y/\partial Z)_0 = 5 \times 10^{-4}$ and $(\partial Y/\partial A)_0 = -1.2 \times 10^{-4}$

$$Y = [0.12(4Z - A) - 13.3] \times 10^{-3}$$

P_{LRA}: function of fissility parameter

[V. A. Rubchenya and S. G. Yavshits, Zeitschrift für Physik A Atomic Nuclei **329**, 217 (1988)]

- Ternary fission is assumed to be a rapid process governed by the dynamics of neck rupture.
- Semiclassical estimates are used for the rupture time and neck lifetime.

Ternary fission occurs when two statistically independent neck ruptures arise within a short time interval Δt, approximately equal to the typical rupture time τ_{sc}. The portion of the neck between the ruptures is considered LCP.

(1) The ternary fission probability P_t is proportional to the ratio of the neck rupture time τ_{sc} to the average neck lifetime τ_{neck}. The coefficient c accounts for corrections like LCP capture, incomplete statistical independence of ruptures, etc.

$$P_t = c\,\tau_{sc}/\tau_{\text{neck}}$$

(2) The neck rupture time τ_{sc} is estimated as the time needed for the neck with radius R_{neck} to be traversed at the Fermi velocity v_F.

$$\tau_{sc} = 2\,R_{\text{neck}}/v_F$$

(4) The average neck lifetime τ_{neck} is related to the neck length L, the friction coefficient μ, and the conservative force (F) acting on the neck.

$$\tau_{\text{neck}} = L\mu/F$$

(5) The conservative force F is the sum of the Coulomb and nuclear forces.

$$F = b\,Z^2/A^{2/3} - F_{\text{nucl.}}$$

(9) The dissipative energy E_{diss} is proportional to the fissility parameter Z^2/A minus a constant q.

$$E_{\text{diss}} = \varepsilon\,A^{2/3}(Z^2/A - q)$$

(8) The friction coefficient μ depends on the ratio of the critical energy E_{cr} to the dissipative energy E_{diss}.

$$\mu = \mu_0\,E_{cr}/E_{\text{diss}}$$

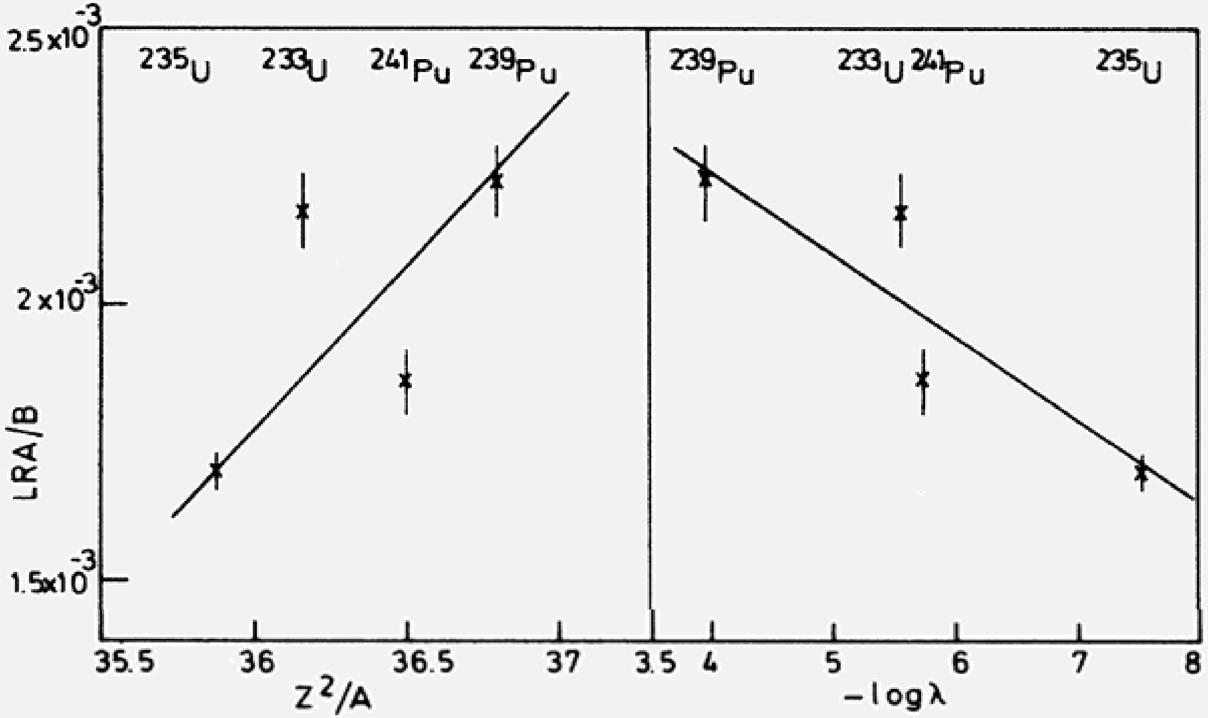
After simplification and renaming constants, we arrive at formula **(10)**

$$P_t = p\,A^{2/3}(Z^2/A - q)(Z^2/A^{2/3} - f)$$

p is a normalization constant
q is a parameter related to energy dissipation
f is a parameter related to nuclear forces and the neck radius

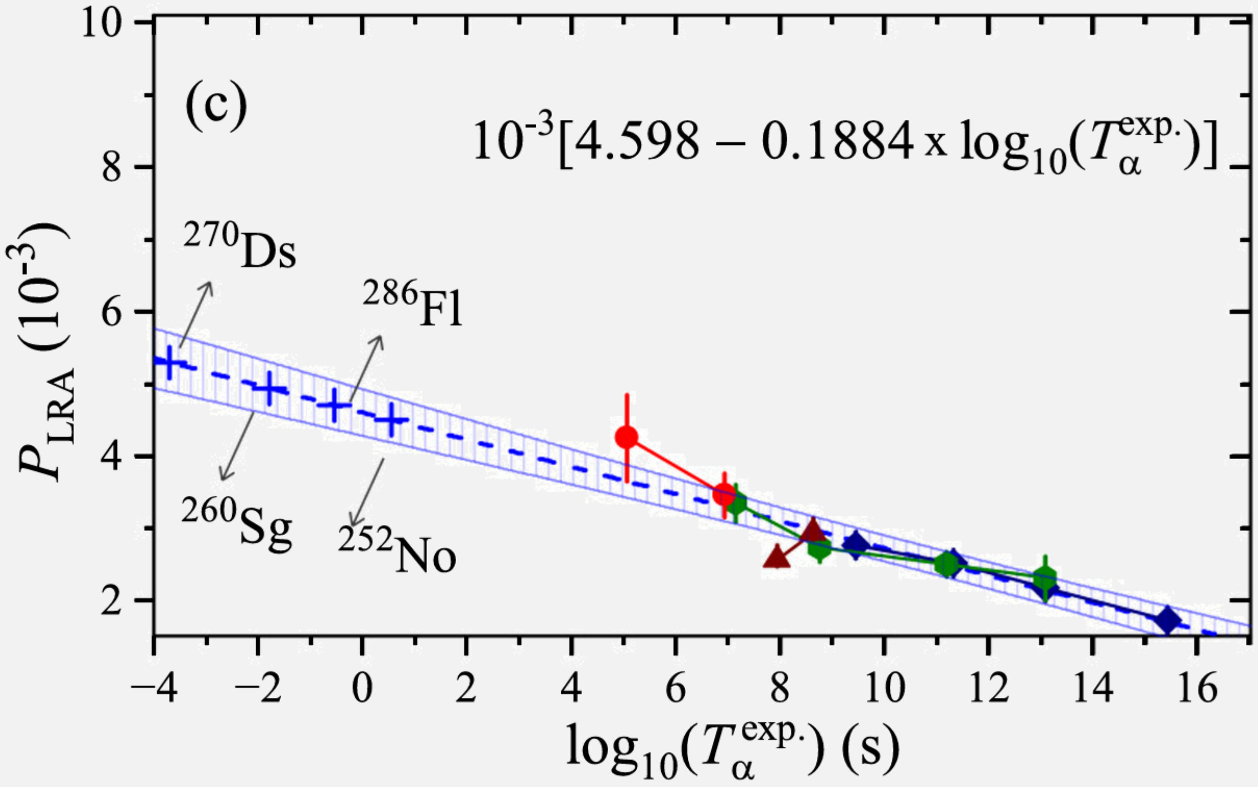
P_{LRA}: function of radioactive α-decay constant / α-decay half-life

[C. Wagemans et al., Phys. Rev. C **33**, 943 (1986)]



LRA/B as a function of Z^2/A and of $-\log\lambda$ (λ being the ground-state radioactive α-decay constant) of the fissioning system. Here λ, is given in y^{-1}.

[J. Khuyagbaatar, Phys. Rev. C **110**, 014311 (2024)]



P_{LRA} measured in the cases of the spontaneous fissioning nuclei are shown as functions of $\log\left(T_{\alpha}^{\mathrm{exp.}}\right)$.

P_{LRA}: function of energy release of α decay

[J. Khuyagbaatar, Phys. Rev. C **110**, 014311 (2024)]

Model [N. Cârjan, J. Phys. 37, 1279 (1976)]: LRA emission happens in two steps: 1) an alpha cluster forms within the nucleus, 2) it's emitted at the time of fission.

(2) This formula defines PLRA as the product of two probabilities: $P_{\mathrm{LRA}} = P_\alpha P_{\mathrm{s.c.}}$

- P_α is the probability of finding an alpha particle inside the nucleus.
- $P_{\mathrm{s.c.}}$ is the probability of emitting the alpha particle during fission (at a scission configuration).

(3) The alpha preformation probability is estimated as: $P_\alpha = T_\alpha^{\mathrm{theo.}} / T_\alpha^{\mathrm{exp.}}$

- $T_\alpha^{\mathrm{theo.}}$ is the theoretical alpha-decay half-life.
- $T_\alpha^{\mathrm{exp.}}$ is the experimental alpha-decay half-life.

(4) Substituting eq. (3) into eq. (2) yields: $\log_{10} P_{\mathrm{LRA}} = \log_{10} T_\alpha^{\mathrm{theo.}} - \log_{10} T_\alpha^{\mathrm{exp.}} + \log_{10} P_{\mathrm{s.c.}}$

This term can be described using the WKB approximation, where an alpha particle with energy Q_α penetrates a potential barrier.

$$\log_{10} T_\alpha^{\mathrm{theo.}} \sim Q_\alpha^{-1/2}$$

Experimental alpha-decay half-lives are consistent with the Geiger-Nuttall rule.

$$\log_{10} T_\alpha^{\mathrm{exp.}} \sim Z Q_\alpha^{-1/2}$$

This term represents the probability of alpha particle emission from a "potential well" at scission. *Assumption:* the term is weakly dependent on the properties of the fissioning nucleus.

Despite the speculative nature of these assumptions, the dependence of PLRA on $Q_\alpha^{-1/2}$ was explored, since the first two terms in equation (4) are primarily dependent on it.

$$\log_{10} P_{\mathrm{LRA}} \sim \log_{10} P_\alpha \sim Q_\alpha^{-1/2}$$

$$P_{\mathrm{LRA}} = 10^{-1.053 - 3.6286 Q_\alpha^{-1/2}}$$