

# Phase Diagram Structure of QCD under Critical Conditions

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# QCD Phase Diagram

- QCD Phase Diagram
  - Confinement/deconfinement
  - First-order phase transitions
- Holographic Quantum Chromodynamics (HQCD) Phase Diagram
  - Confinement/deconfinement under extreme conditions
  - First-order phase transitions under extreme conditions
    - high densities and magnetic fields
- i) Predictions of HQCD and
  - ii) the extent to which they can be experimentally tested
    - energy losses, scattering cross-sections, in particular, direct photons
- How to configure HQCD calculations to assess phase structure agreement with experiment via machine learning.

# Confinement in QCD

- Wilson criterion (Wilson '74):

$$\langle W(C) \rangle_{vac} \equiv \langle P e^{g \int A_\mu dx^\mu} \rangle \sim e^{-\sigma_{string} S(C)}$$

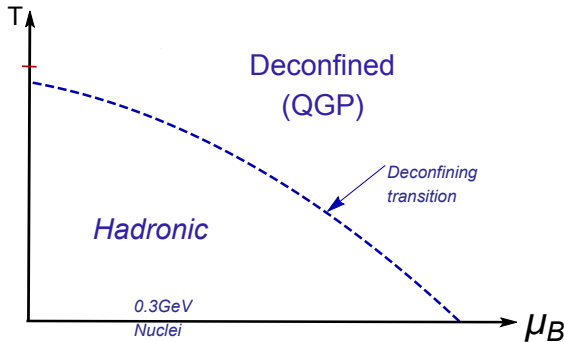
- D=4

- according to pert. theory - not satisfied
- on a lattice - satisfied (in strong coupling, Wilson '74);  
record calculations 96 · 48<sup>3</sup>
- D=2  $QCD_2$  - is satisfied ('t Hooft, Gross )

- On the lattice the main question  $\mu \neq 0$  -  $\mu_B = \frac{1}{3}[\mu_u + \mu_d + \mu_s]$
- The problem of the sign [Fugacity expansion]

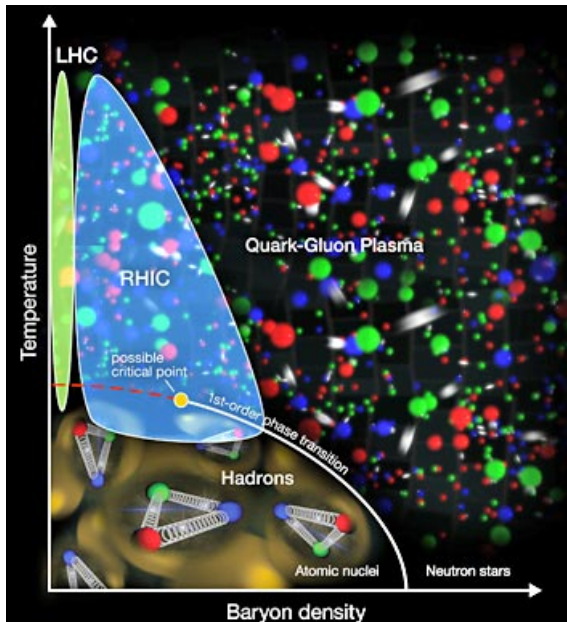
# QCD Phase Diagram: Early Conjecture

*Cabibbo and Parisi, 1975*

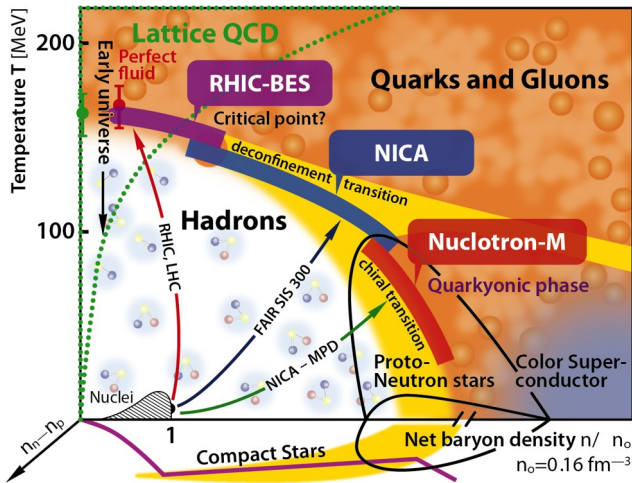


- $\mu$  a measure of the imbalance between quarks and antiquarks in the system

# QCD Phase Diagram - main expectations



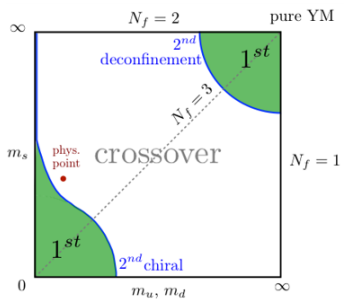
# Studies of QCD Phase Diagram - the main goal of new facilities



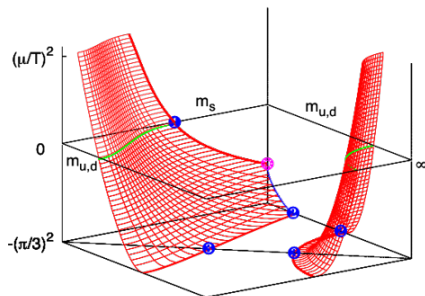
From: <https://nica.jinr.ru/physics.php>

# QCD Phase Diagram: Lattice

Dependence of phase diagram  
on quark mass



Main problem with  $\mu \neq 0$   
Imaginary chemical potential method



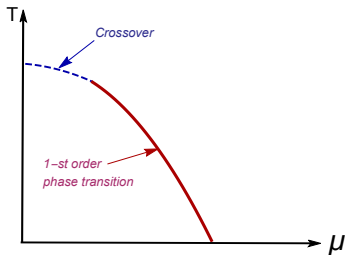
Columbia plot

*Brown et al., PRL (1990)*

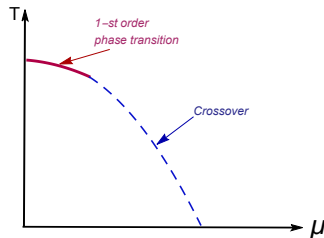
*Philipsen, Pinke, PRD (2016)*

# “Heavy” and “light” quarks from Columbia plot

Light quarks



Heavy quarks



$$T_c = 155 - 160 \text{ MeV}$$

$$\mu_{CEP} \sim 400 \text{ MeV}$$



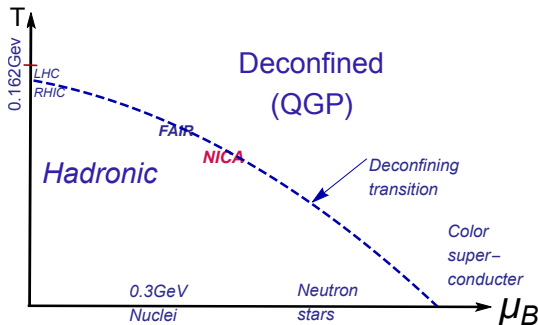
# QCD Phase Diagram: Experiments

- LHC, RHIC (2005);
- FAIR (Facility for Antiproton and Ion Research),

**NICA** (Nuclotron-based Ion Collider fAcility)

## Main goals

- search for signs of the phase transition between hadronic matter and QGP;
- search for new phases of baryonic matter



# Holographic QCD -phenomenological approach

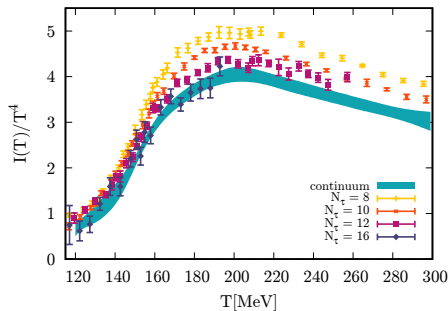
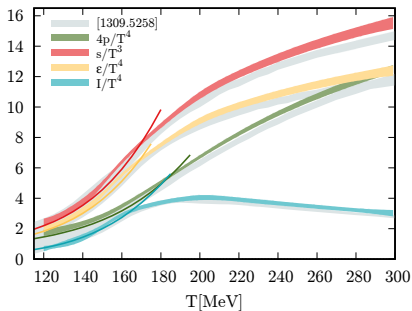
- Perturbation methods are not applicable to describe QCD phase diagram
- Lattice methods do not work, because of problems with the chemical potential.
- Holographic - phenomenological model(s)
- One of goals of Holographic QCD – describe QCD phase diagram
- **Requirements:**
  - reproduce the QCD results from perturbation theory at short distances
  - reproduce Lattice QCD results at large distances ( $\sim 1$  fm) and small  $\mu_B$

# Holographic QCD vs exact AdS/CFT

Maldacena, 1998

What is wrong with exact AdS/CFT applications to QCD:

- QCD is not conformal, conformal invariance is restored only in high energy



From: S. Borsanyi et al, arXiv:2502.10267

**Left.** Pressure (green), entropy (red), energy density (yellow) and **trace anomaly** (cyan) as functions of the temperature. EoS:  $\frac{p(T_0=185)}{T^4} = 1.371$ . The gray bands from

S. Borsanyi et al, arXiv: 1309.5258

**Right** Trace anomaly

# Holographic QCD vs exact AdS/CFT

Maldacena, 1998

What is wrong with exact AdS/CFT applications to QCD:

- QCD is not conformal, conformal invariance is restored only in high energy
- $T > T_{HP}$  for BHAdS (check for Poincare BH)

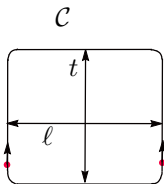
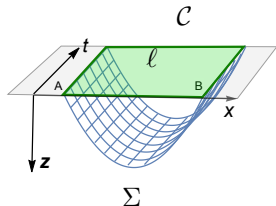
ADD Plots

# Holographic QCD vs exact AdS/CFT

Maldacena, 1998

What is wrong with exact AdS/CFT applications to QCD:

- QCD is not conformal, conformal invariance is restored only in high energy
- $T > T_{HP}$  for BHAdS
- No confinement in  $BHAdS_5$



$$W_R[C] \underset{t \rightarrow \infty}{\sim} e^{-V(\ell)t}$$

$$V(\ell) \underset{\ell \rightarrow \infty}{\sim} \frac{1}{\ell}$$

corresponds to  
conformal invariance

+ Remark about Sakai-Sugimoto model.

# Holographic QCD

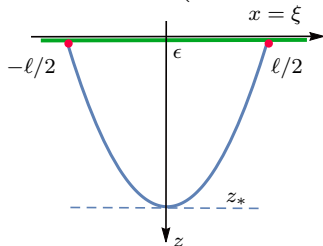
- Temperature in QCD  $\iff$  black hole temperature in (deform.)AdS
- Thermalization in QCD  $\iff$  formation of black hole in (**deform.**)AdS<sub>5</sub>

# Time-like Wilson loops in a deformed metric

O.Andreev, V.Zakharov, PRD'07, I.A., K.Rannu, JHEP'18 + P.Slepov, PLB'19.

Metrics in the "Einstein Frame":

$$ds^2 = \frac{b(z)}{z^2} \left( -g(z)dt^2 + d\vec{x}^2 + \frac{1}{g(z)}dz^2 \right) \quad \text{in "string frame"} \quad b_s = e^{\sqrt{\frac{2}{3}}\phi} b \quad (1)$$



String action "on the barn":

$$S = \mathfrak{t} \int_{-\ell/2}^{\ell/2} d\xi M(z(\xi)) \sqrt{\mathcal{F}(z(\xi)) + (z'(\xi))^2}, \quad (2)$$

$$M(z) = \frac{b_s(z)}{z^2}, \quad \mathcal{F}(z) = g(z). \quad (3)$$

**Lemma:** If there is a solution to the equation defining **the dynamic wall** ( $z = z_{DW}$ )

$$\frac{M'(z)}{M(z)} + \frac{1}{2} \frac{\mathcal{F}'(z)}{\mathcal{F}(z)} \Big|_{z=z_{DW}} = 0, \quad (4)$$

then when  $\ell \rightarrow \infty$   $S \sim \sigma_{DW} \ell$ , где  
this is confinement.

$$\sigma_{DW} = \frac{b(z_{DW})}{(z_{DW})^2} \sqrt{g(z_{DW})}$$

# Holographic model of an anisotropic plasma in a magnetic field at a nonzero chemical potential

I.A, K. Rannu, P.Slepov, JHEP, 2021

$$S = \int d^5x \sqrt{-g} \left[ R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_B(\phi)}{4} F_{(B)}^2 - \frac{1}{2} \partial_M \phi \partial^M \phi - V(\phi) \right]$$

$$ds^2 = \frac{L^2}{z^2} \left[ -g(z) dt^2 + dx^2 + dy_1^2 + e^{c_B z^2} dy_2^2 + \frac{dz^2}{g(z)} \right]$$

$$A_{(1),m} = A_t(z) \delta_m^0, \quad A_t(0) = \mu, \quad F_{(B)} = dx \wedge dy^1$$

*Giataganas'13; IA, Golubtsova'14; Gürsoy, Järvinen '19; Dudal et al.'19*

$$\mathbf{b}(z) = e^{2\mathcal{A}(z)} \Leftrightarrow \text{quarks mass} \quad \text{“Bottom-up approach”}$$

**Heavy quarks** (c, b, t):

$$\mathcal{A}(z) = -cz^2/4 \quad \text{Andreev, Zakharov'06}$$

$$\mathcal{A}(z) = -cz^2/4 + pz^4 \quad \text{IA, Hajilou, Rannu, Slepov, EPJ C (2023)83}$$

**Light quarks** (d, u, s)

$$\mathcal{A}(z) = -a \ln(bz^2 + 1) \quad \text{Li, Yang, Yuan'17}$$

$\varphi$  - dilaton,  $\alpha(z) = e^{\varphi(z)}$  - running coupling in HQCD



# Holographic Equation of Motions

$$\varphi'' + \varphi' \left( \frac{g'}{g} + \frac{3b'}{2b} - \frac{3}{z} + c_B z \right) + \left( \frac{z}{L} \right)^2 \frac{\partial f_1}{\partial \varphi} \frac{(A'_t)^2}{2bg} - \left( \frac{z}{L} \right)^2 \frac{\partial f_B}{\partial \varphi} \frac{q_B^2}{2bg} = 0,$$

$$A_t'' + A_t' \left( \frac{b'}{2b} + \frac{f_1'}{f_1} - \frac{1}{z} + c_B z \right) = 0,$$

$$g'' + g' \left( \frac{3b'}{2b} - \frac{3}{z} + c_B z \right) - \left( \frac{z}{L} \right)^2 \frac{f_1 (A'_t)^2}{b} - \left( \frac{z}{L} \right)^2 \frac{q_B^2 f_B}{b} = 0,$$

$$b'' - \frac{3(b')^2}{2b} + \frac{2b'}{z} - \frac{4b}{3z^2} \left( -\frac{c_B z^2}{2} - \frac{c_B^2 z^4}{2} \right) + \frac{b(\varphi')^2}{3} = 0,$$

$$c_B z^2 (2g' + 3g) \left( \frac{b'}{b} - \frac{4}{3z} + \frac{2c_B z}{3} \right) - \left( \frac{z}{L} \right)^3 \frac{L q_B^2 f_B}{b} = 0,$$

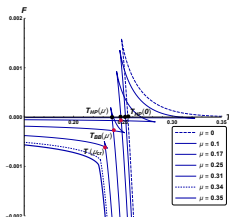
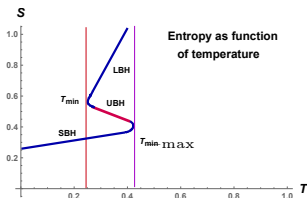
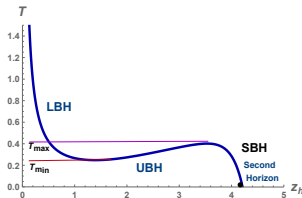
$$\begin{aligned} \frac{b''}{b} + \frac{(b')^2}{2b^2} + \frac{3b'}{b} \left( \frac{g'}{2g} - \frac{2}{z} + \frac{2c_B z}{3} \right) - \frac{g'}{3zg} (9 - 3c_B z^2) - \frac{2c_B}{3} (5 - c_B z^2) \\ + \frac{8}{z^2} + \frac{g''}{3g} + \frac{2}{3} \left( \frac{L}{z} \right)^2 \frac{bV}{g} = 0, \end{aligned}$$

We choose (LQ):  $f_1 = e^{-cz^2 - \mathcal{A}(z)} = (1 + bz^2)^a e^{-cz^2}$ ,

B.C.:  $A_t(0) = \mu$ ,  $A_t(z_h) = 0$ ;  $g(0) = 1$ ,  $g(z_h) = 0$ ;  $\varphi(z_0) = 0$  **We have to fix  $z_0$ .**

# Origin of 1-st order phase transition in HQCD

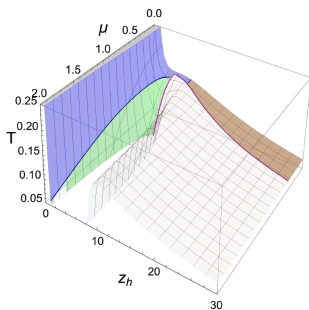
- $g(z)$  blackening function. The form of  $g(z)$  depends on  $\mathcal{A}(z)$ .
  - Due **non-monotonic** dependence of  $T = T(z_h) = g'(z)/4\pi \Big|_{z=z_h}$  on  $z_h$ , the entropy  $s = s(T)$  is **not monotonic**
  - As a consequence the free energy  $F = \int s dT$  undergoes the phase transition
- 1-st order phase transition describes transition from small black holes  $\rightarrow$  large black holes



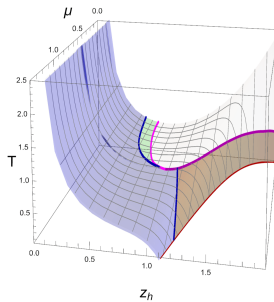
The swallow-tailed shape

- Physical quantities that probe backgrounds are smooth relative to  $z_h$   
 $\Rightarrow$  their dependence on  $T$  **should be taken from stable region**
- Non-monotonic dependence of  $T = T(z_h)$  gives the 1-st PT for corresponding characteristic of QCD

# 1-st order phase transition in HQCD. 1/3



LQ

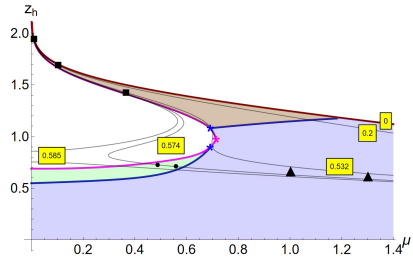
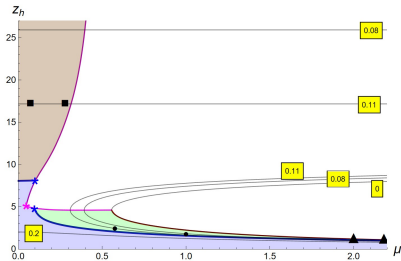


HQ

3D plot  $T = T(\mu, z_h)$ . The brown part of the surface corresponds to the hadronic phases, the blue one corresponds to the quark-gluon plasma and the green one to the quarkyonic phase. 3D plot  $T = T(\mu, z_h)$

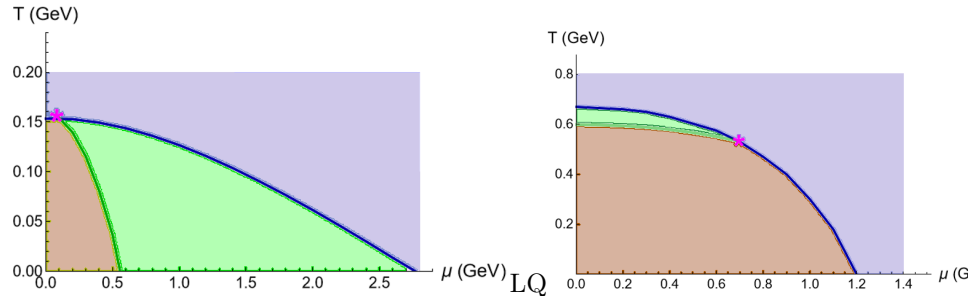
I.A., A.Hajilou, P.Slepov, M.Usova, PRD (2024) **110**, 126009;  
[arXiv:2402.14512]

# 1-st order phase transition in HQCD. 2/3



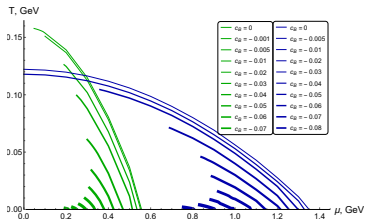
2D plots in  $(\mu, z_h)$ -plane for light quarks (A) and heavy quarks (B). Hadronic, quarkyonic and QGP phases are denoted by brown, green and blue, respectively. Solid gray lines show the temperature indicated in rectangles. The intersection of the confinement/deconfinement and 1-st order phase transition lines is denoted by the blue stars. The magenta star indicates CEP

# 1-st order phase transition in HQCD. 3/3

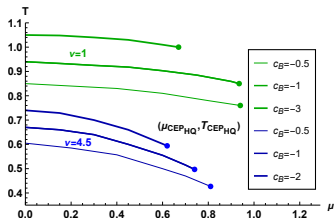


# 1-st order phase transition in HQCD, $B \neq 0$

## Light quarks



## Heavy quarks



I.A, Ermakov, Rannu, Slepov, EPJC'23

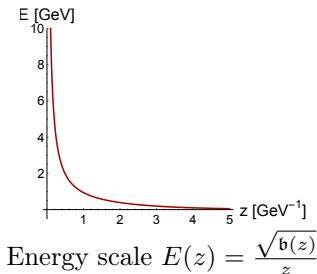
I.A, A. Hajilou, K.R., P.S. EPJC'23

# QCD Phase Diagram: Experiments

- Main experimental method to find 1-st order phase transition:  
Beam Energy Scan Method
- We propose **a new method** related with study of scattering amplitudes of particles created in colliding heavy ions beams.
- It is related with **special behaviour of scattering amplitudes near 1-st order phase transition**

# Running coupling in QCD vs running coupling in HQCD

- $\alpha(z) = e^{\varphi(z)}$  running coupling in HQCD, here  $\varphi$  - dilaton,  
E. Kiritsis et al, 1401.0888, 1805.01769
- $\alpha = \alpha(E)$  - running coupling in QCD
- The energy scale as a function of the bulk coordinate  $z$ , corresponding to the warp factor for light quark model





# Holographic Running coupling

$$\alpha(z) = e^{\varphi(z)}$$

I.A. A.Hajilou, P.Slepov, M.Usova, 2402.14512

## Light Quark Model

$\varphi(z)$  - dilaton field       $\varphi(z)$  is defined up to a constant:  $\varphi(z)\Big|_{z=z_0} = 0$ .

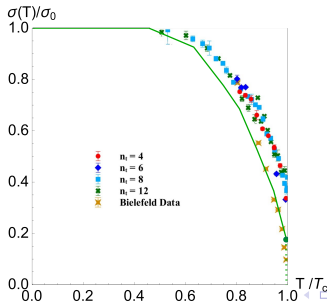
There are 3 choices:      a)  $z_0 = 0$       b)  $z_0 = f(z_h)$       c)  $z_0 = z_h$

$$z_0 = 10 \exp[-z_h/4] + 0.1$$

IA, K.Rannu, P.Slepov, JHEP'21

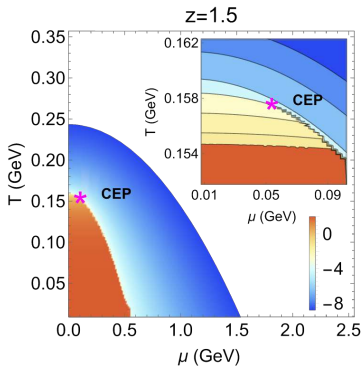
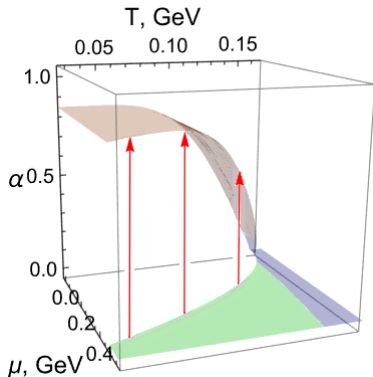
With this boundary condition the temperature dependence of  $\sigma_s$  fits the known lattice data

Cordaso, Bicudo 1111.1317



# Holographic Running coupling for $T \neq 0, \mu \neq 0$

I.A. Hajilou, Slepov, Usova, PRD, 2024

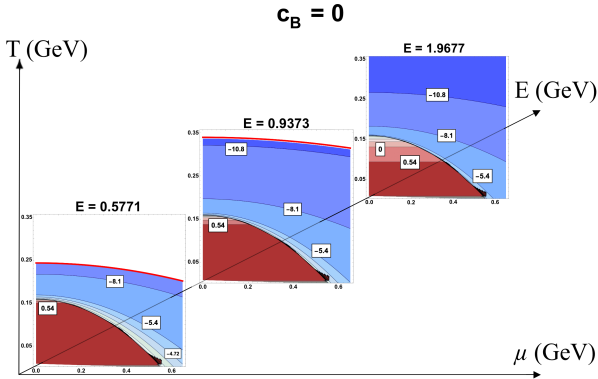


3D plot of  $\alpha(z; \mu, T)$

Density plot of  $\log \alpha(z; \mu, T)$

Light quark model,  $z = 0$

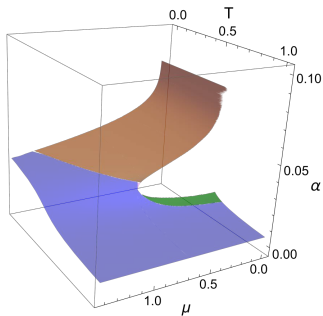
# Running Coupling for Light Quark Model



**Density plots** of  $\log \alpha(E; \mu, T)$  at different energy scales.

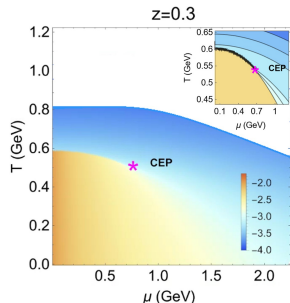
All values of  $E$  on the top of each panel show fixed value of energy  $E$ -coordinate.

# Running coupling for Heavy Quark Model



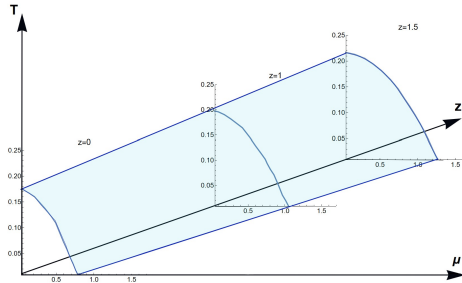
The 3D-plot for coupling constant  $\alpha = \alpha_{HQ}(z; \mu, T)$  for heavy quarks at  $z = 0$ .

Heavy quarks,  $z = 0$



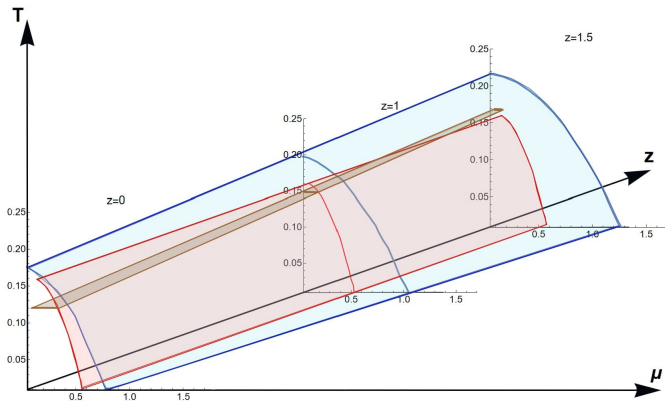
Density plots of  $\log \alpha(z; \mu, T)$  for

# Automodel behaviour of Running coupling



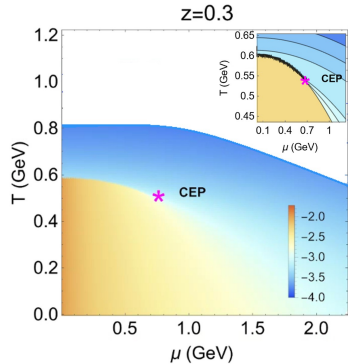
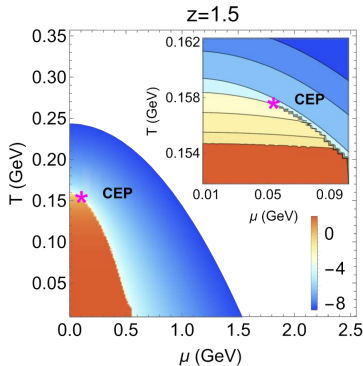
$$\alpha(T, \mu) = f_{up}(T^2 + c\mu^2)$$

# Automodel Behaviour of Running Coupling



$$\alpha(T, \mu) = f_{\text{below}}(T^2 + c\mu^2)$$

# Light vs Heavy Quarks Phase Diagrams from Scattering Amplitudes. $B=0$



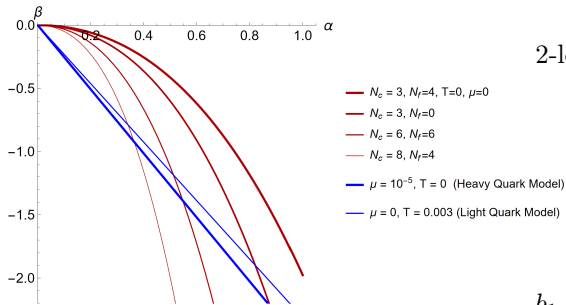
Density plots of  $\log \alpha(z; \mu, T)$

This is predicted by Holographic QCD

Dependence on magnetic field

I.A., A.Hajilow, A.Nikolaev, P.Slepov, PRD(2024)110

# $\beta$ function



Beta-function  $\beta(\alpha)$  for QCD at 2-loop level for  $T = 0, \mu = 0$  at different  $N_c$  and  $N_f$  in red lines, and holographic  $\beta$ -function for light quarks at  $\mu = 0, T = 0.003$  (light blue) and heavy quarks  $\mu = 10^{-5}, T = 0$  (dark blue);  $[\mu] = [T] = \text{GeV}$ .

2-loops QCD  $\beta$ -function:

$$\beta(\alpha) = -b_0 \alpha^2 - b_1 \alpha^3,$$

$$b_0 = \frac{1}{2\pi} \left( \frac{11}{3} N_c - \frac{2}{3} N_f \right)$$

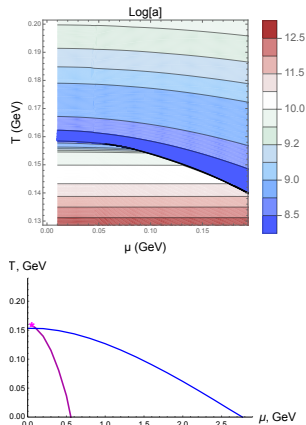
$$b_1 = \frac{1}{8\pi^2} \left( \frac{34}{3} N_c^2 - \left( \frac{13}{3} N_c - \frac{1}{N_f} \right) \right)$$

$N_c$  # of colors,  $N_f$  #of flavors.

I.A, A. Hajilou, P. Slepov and M. Usova, Phys.Rev.D (2025) 111



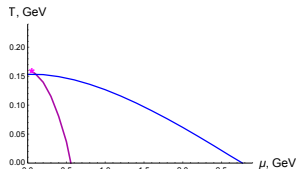
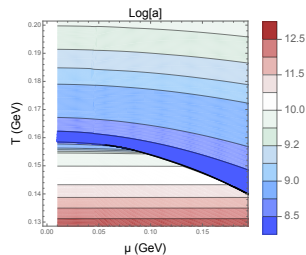
# Jet Quenching as function of $T$ and $\mu$ for LQ



1st order - magenta

2nd order - blue

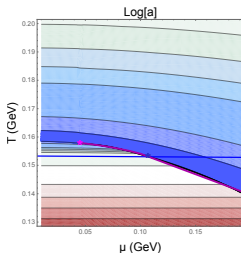
# Jet Quenching as function of $T$ and $\mu$ for LQ



1st order - magenta

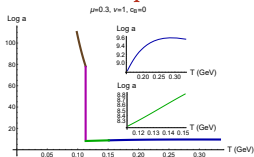
2nd order - blue

IA, A.Hajilow, A.Nikolaev, P.Slepov

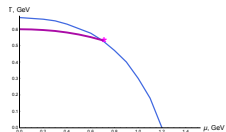
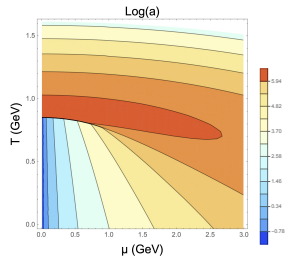


We see that junction in JQ is exactly on the line of the first order transition (as should be)

## 2-nd order phase transition

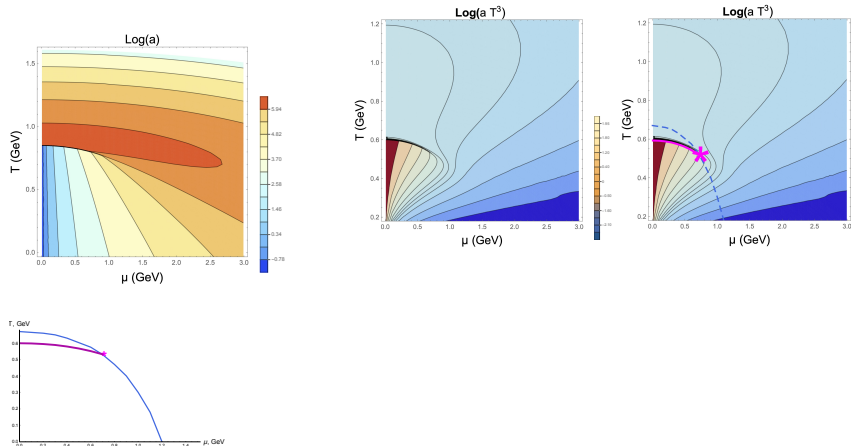


# Jet Quenching as function of $T$ and $\mu$ for heavy quarks



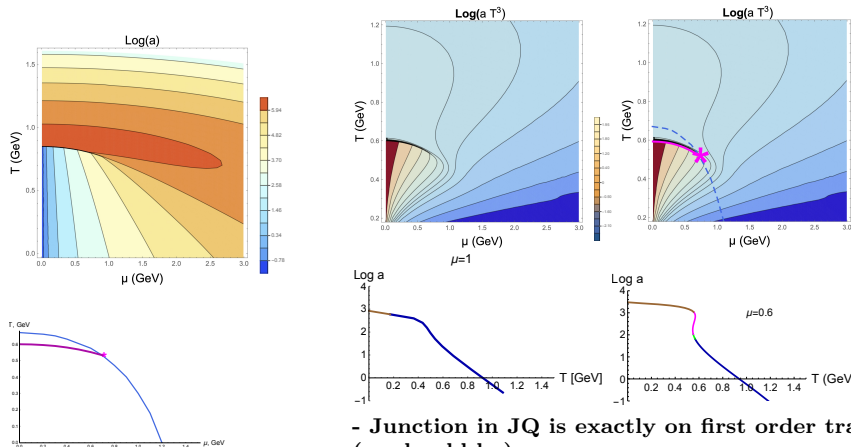
Heavy quarks

# Jet Quenching as function of $T$ and $\mu$ for heavy quarks



Heavy quarks

# Jet Quenching as function of $T$ and $\mu$ for heavy quarks



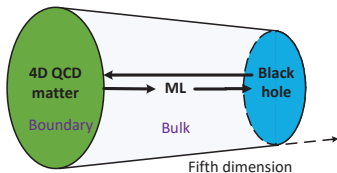
Heavy quarks

- Junction in JQ is exactly on first order trans.  
(as should be).

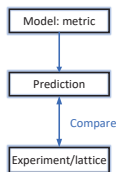
- After conf./deconf. line - change of the slop

# Few comments on applications of machine learning for fitting parameters in HQCD

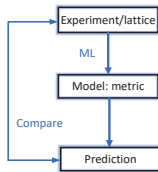
- The parameter space is high-dimensional and non-linear, making traditional fitting methods less effective.
- Goal: an effectively automatic way through machine learning to determine the parameters of HQCD
- The sketch of holographic QCD and machine learning.  
From X. Chen and M. Huang, JHEP 02 (2025), 123



Conventional Holographic model:



ML Holographic model:



# Conclusion

- Phase structure in  $(T, \mu, B, \nu)$ -space;  
     $B$  - characterizes the magnetic field,  
     $\nu$  -anisotropy
- Dependence of the phase structure in  $(T, \mu, B, \nu)$  - space on quark mass
- **Jumps** in physical quantities (jet quenching, energy loss, etc.) at the first-order phase transition and their dependence on  $B$  and anisotropy parameter  $\nu$ .
- **Smooth** variation of physical quantities (running couplings, jet quenching parameter, etc.) across the confinement-deconfinement transition.

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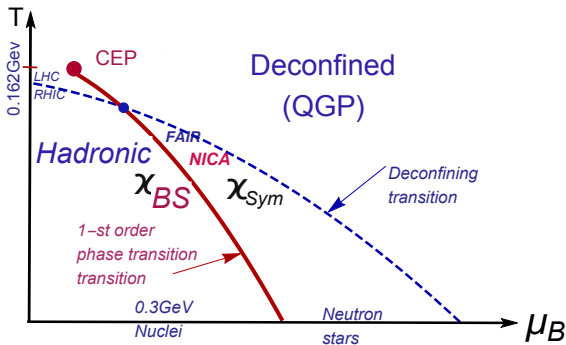
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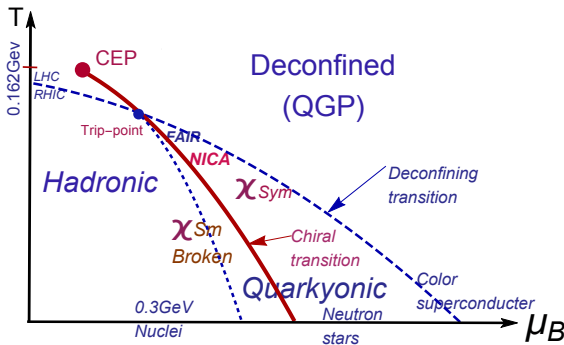
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# Backup. The expected more detailed QCD phase diagram



- Parameter of the chiral symmetry breaking  $\langle \bar{\psi}\psi \rangle$ 
  - $\langle \bar{\psi}\psi \rangle = 0 \iff \chi\text{-symmetry}$
  - $\langle \bar{\psi}\psi \rangle \neq 0 \iff \text{broken } \chi\text{-symmetry}$

**Backup. The expected more detailed QCD phase diagram**



- Quarkyonic phase: baryon free  $\Rightarrow$  dense baryons *McLerran, Pisarski 0706.2191*
  - Baryon density jumps