Density dependent hyperonic interactions in neutron stars

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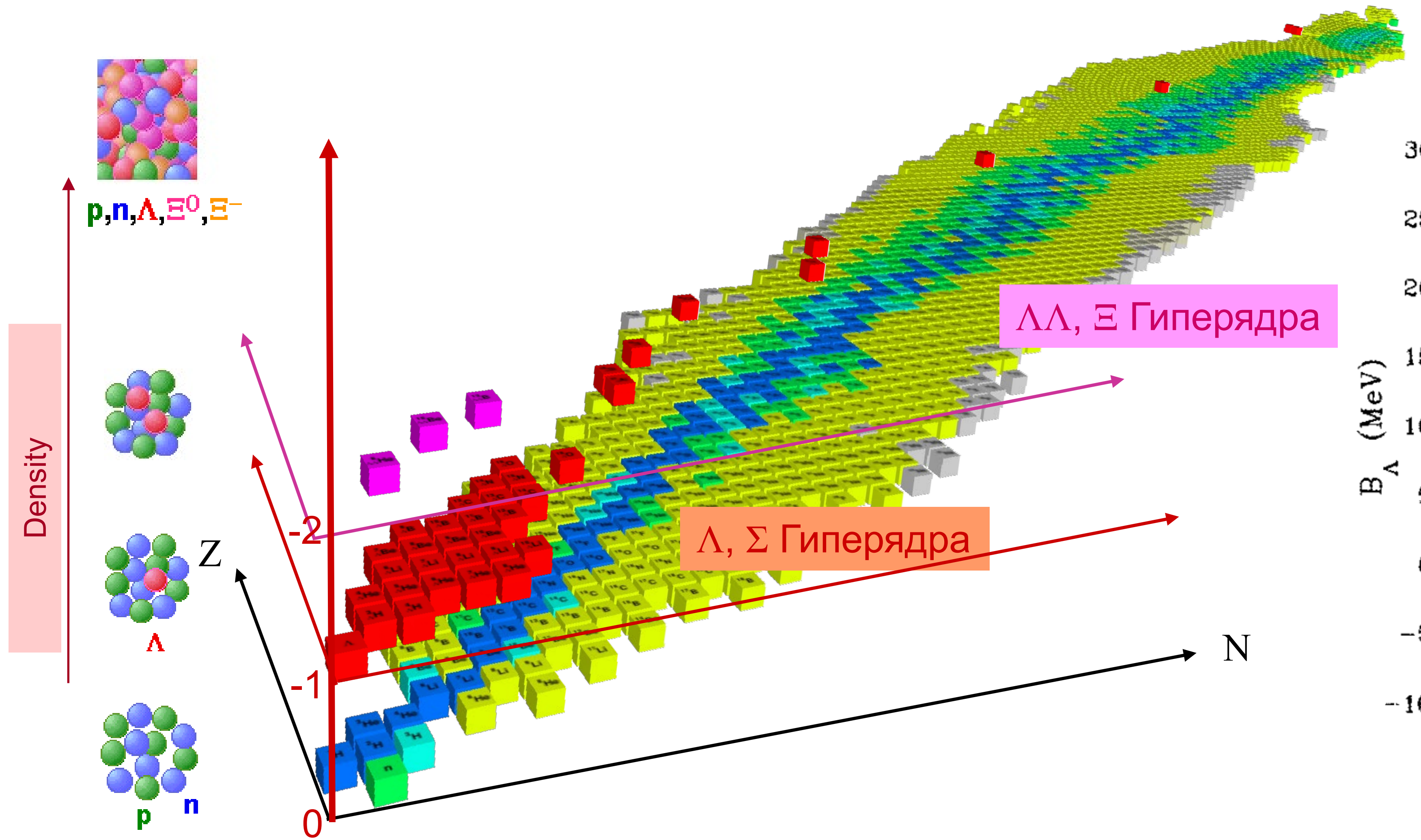
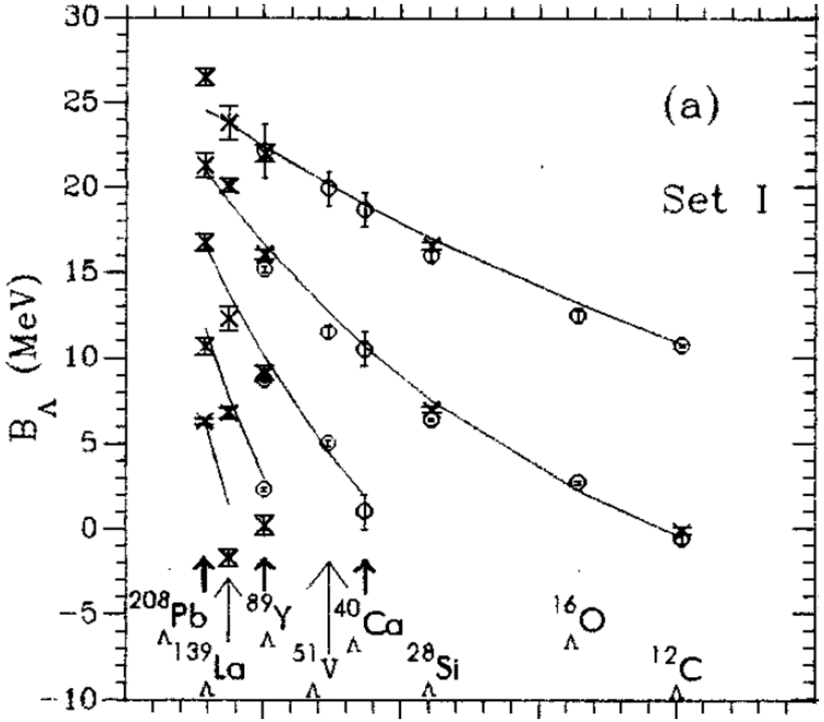
Hypernuclei and hyperonic interactions



Hyperon binding energy

$$B_\Lambda({}^{A+1}_{\ \ \Lambda}Z) = B_{tot}({}^{A+1}_{\ \ \Lambda}Z) - B_{tot}({}^{A}Z)$$



Hyperon binding energy in the pure nucleonic matter

$$\boxed{D_\Lambda(\rho_0) \approx 30 \text{ MeV}}$$

Skyrme interaction

ΛN-interaction

$$
\begin{aligned}
V_{\Lambda N}(\vec{r_\Lambda},\vec{r_N}) &= u_0(1+\xi_0 P_\sigma)\delta(\vec{r_\Lambda}-\vec{r_N}) \\
&+\frac{1}{2}u_1(1+\xi_1 P_\sigma)[\vec{P}'^2\delta(\vec{r_\Lambda}-\vec{r_N})+\delta(\vec{r_\Lambda}-\vec{r_N})\vec{P}^2] \\
&+u_2(1+\xi_2 P_\sigma)\vec{P}'\delta(\vec{r_\Lambda}-\vec{r_N})\vec{P} \\
&+iW_0^\Lambda\vec{P}'\delta(\vec{r_\Lambda}-\vec{r_N})[\vec{\sigma}\times\vec{P}]
\end{aligned}
$$

Parametrization of ΛN-interaction	γ
YBZ6	1
YBZ2	1
SLL4'	1
LYI	1/3
YMR	1/8

Three-body forces

$$V_3 = V_{\Lambda NN}(\vec{r_\Lambda},\vec{r_{N1}},\vec{r_{N2}}) = u_3\delta(\vec{r_\Lambda}-\vec{r_{N1}})\delta(\vec{r_\Lambda}-\vec{r_{N2}})$$

Density-dependent forces

$$V_3 = V_{\Lambda N}(\vec{r_\Lambda},\vec{r_N},\rho) = \frac{3}{8}u_3(1+\xi_3 P_\sigma)\delta(\vec{r_\Lambda}-\vec{r_N})\boxed{\rho_N^\gamma}\left(\frac{\vec{r_\Lambda}+\vec{r_N}}{2}\right)$$

ΛΛ-interaction

$$
\begin{aligned}
V_{\Lambda\Lambda}(\vec{r_1},\vec{r_2}) &= \lambda_0\delta(\vec{r_1}-\vec{r_2}) \\
&+\frac{1}{2}\lambda_1[\vec{P}'^2\delta(\vec{r_1}-\vec{r_2})+\delta(\vec{r_1}-\vec{r_2})\vec{P}^2]
\end{aligned}
$$

ΛΛ-interaction with density dependence

$$V_{\Lambda\Lambda} = \sum_1^3 (a_i + b_i k_F + c_i k_F^2)e^{-\frac{r^2}{\beta_i^2}}$$

Neutron stars

- **Chemical equilibrium**

$$\begin{cases} \mu_p + \mu_e = \mu_n \\ \mu_\mu = \mu_e \\ \mu_\Lambda + m_\Lambda = \mu_n + m_n \end{cases}$$

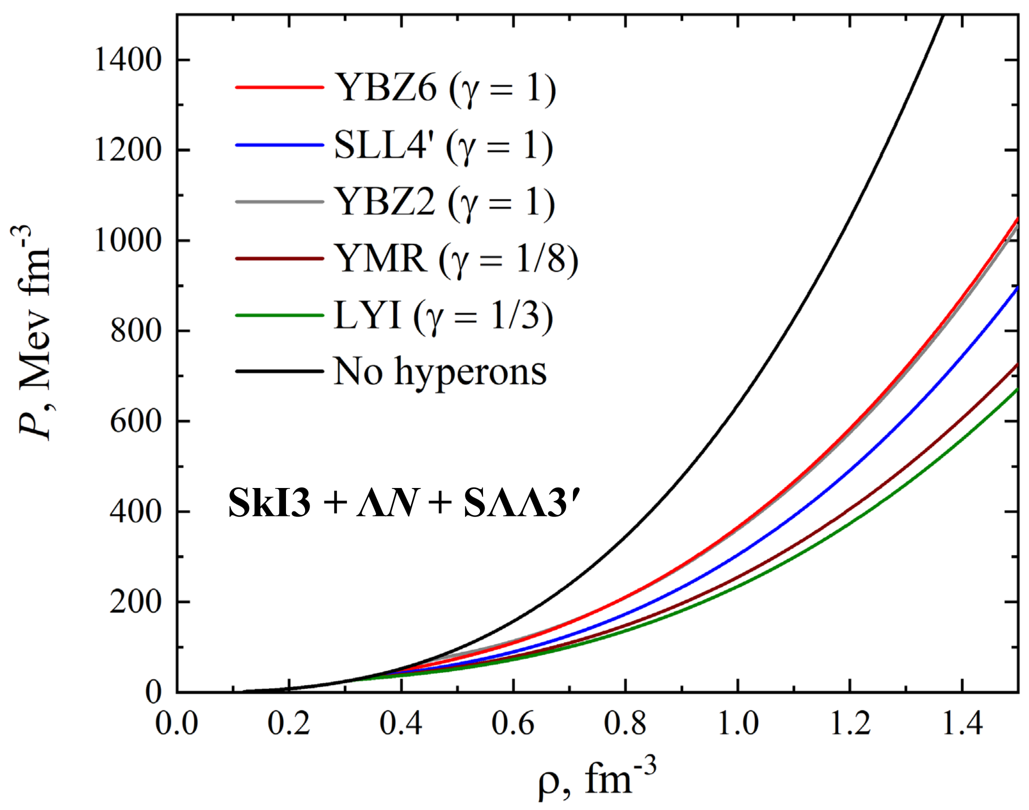
- **Tolman Oppenheimer Volkov equation**

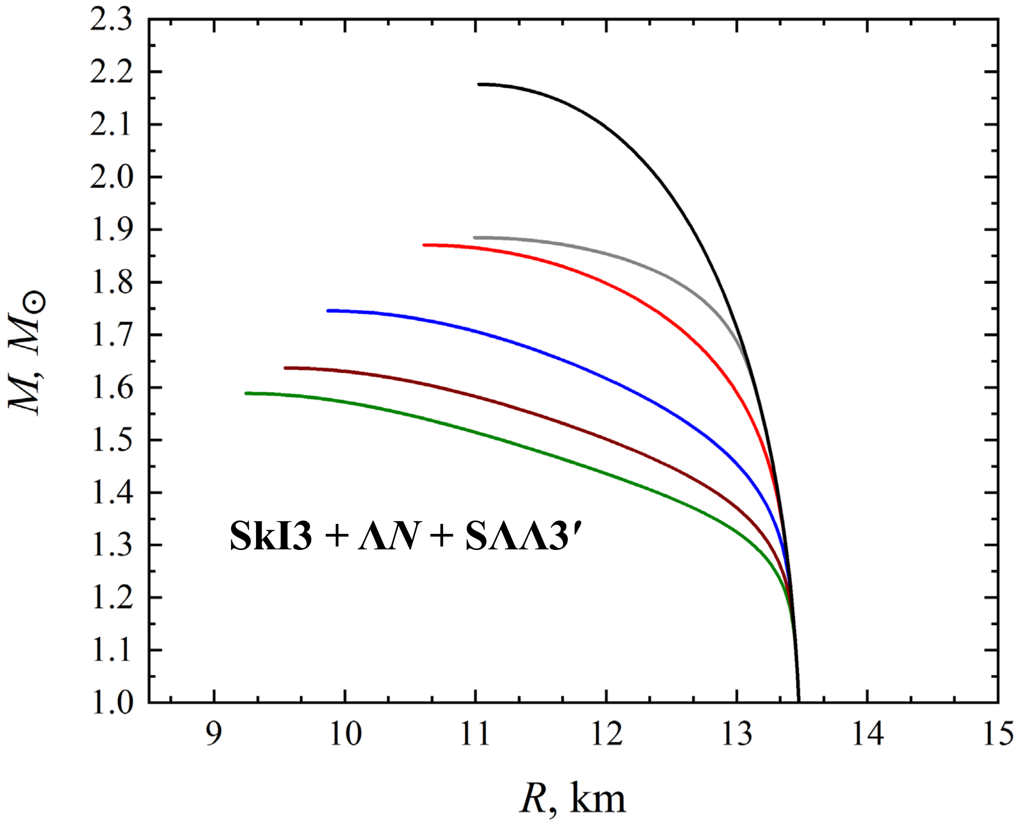
$$\frac{dP}{dr} = \frac{G}{r^2}\frac{[\rho(r) + P(r)/c^2][m(r) + (4\pi r^3 P(r)/c^2)]}{1 - (2Gm(r)/rc^2)}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

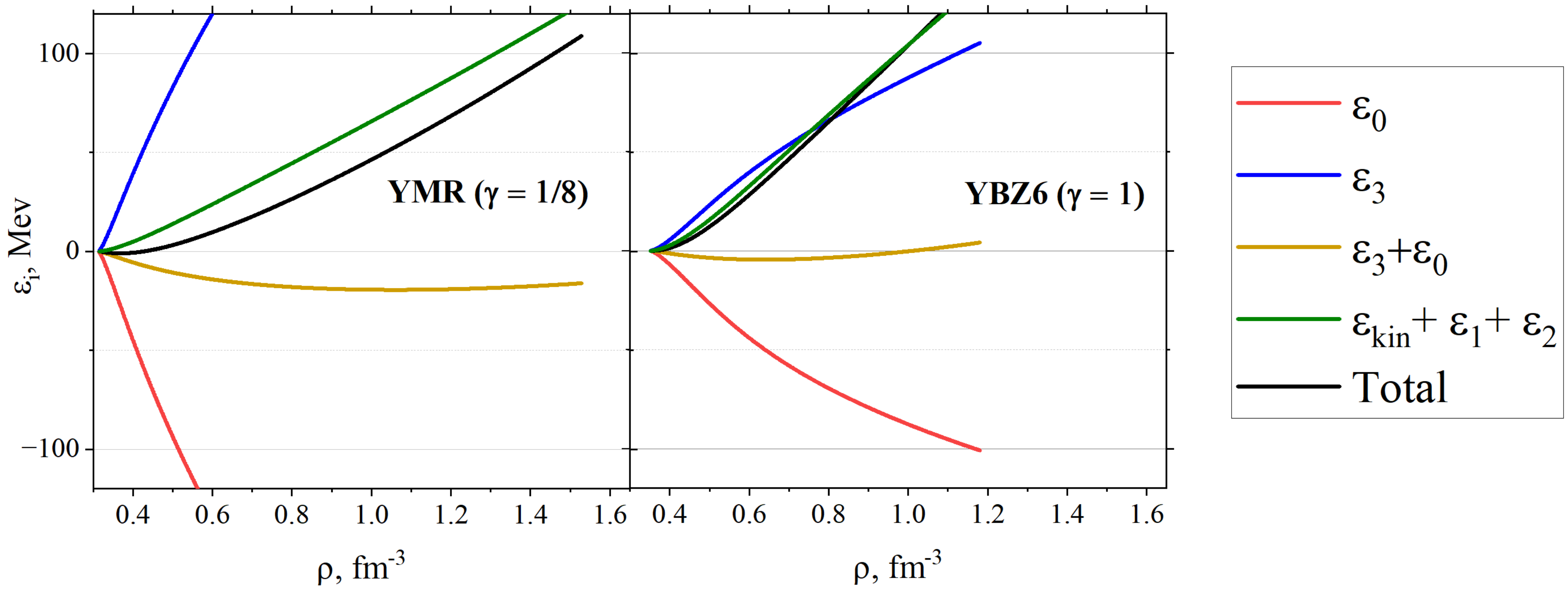
- **Hyperon puzzle**

PSR J0740+6620, M = 2.08 ± 0.07 M⊙
PSR J0952-0607, M = 2.35 ± 0.17 M⊙



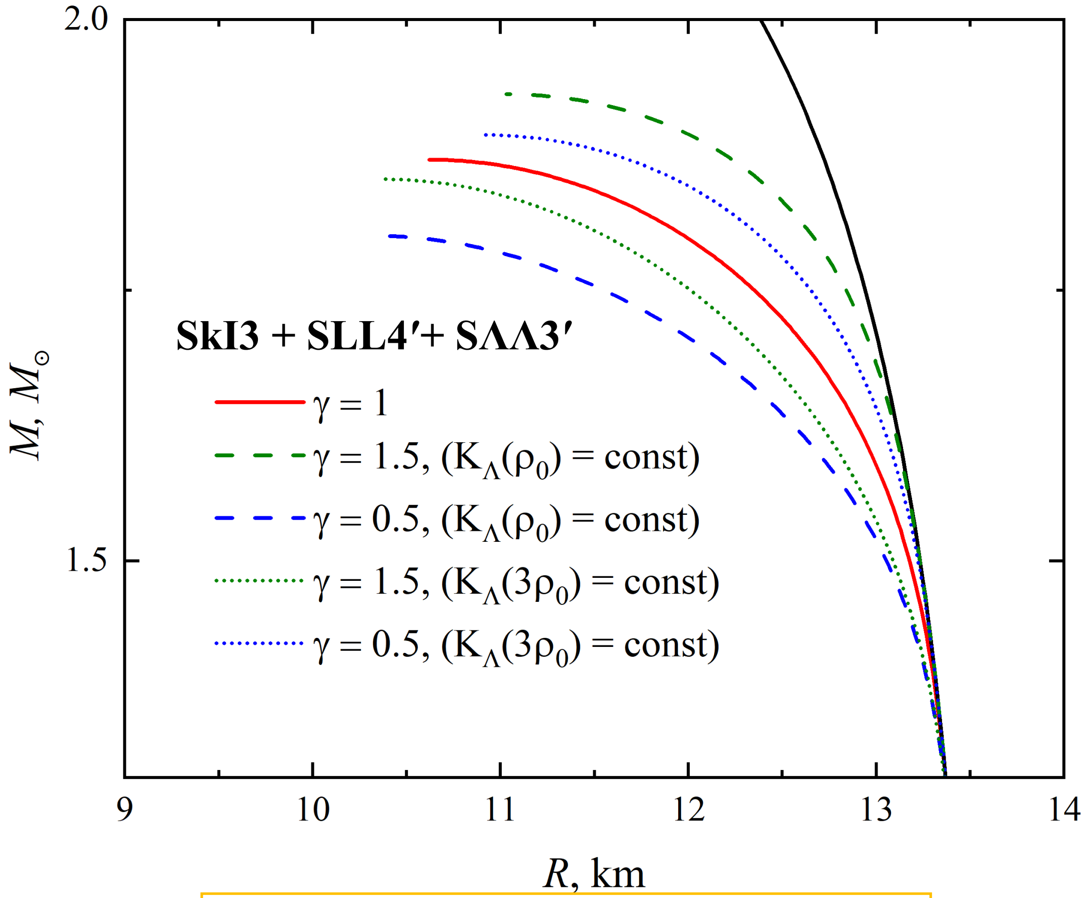


Contributions of various terms in energy per baryon



$$
\begin{aligned}
V_{\Lambda N}(\vec{r_\Lambda}, \vec{r_N}) = {} & u_0(1+\xi_0 P_\sigma)\delta(\vec{r_\Lambda} - \vec{r_N}) \\
& + \frac{1}{2}u_1(1+\xi_1 P_\sigma)[\vec{P}'^2\delta(\vec{r_\Lambda} - \vec{r_N}) + \delta(\vec{r_\Lambda} - \vec{r_N})\vec{P}^2] \\
& + u_2(1+\xi_2 P_\sigma)\vec{P}'\delta(\vec{r_\Lambda} - \vec{r_N})\vec{P} \\
& + \frac{3}{8}u_3(1+\xi_3 P_\sigma)\delta(\vec{r_\Lambda} - \vec{r_N})\rho_N^\gamma\left(\frac{\vec{r_\Lambda} + \vec{r_N}}{2}\right)
\end{aligned}
$$

Masses and radii of neutron star for different values of γ



Changed parameters: γ, u_0, u_3

The binding energy of Λ-hyperon in the pure nucleonic matter

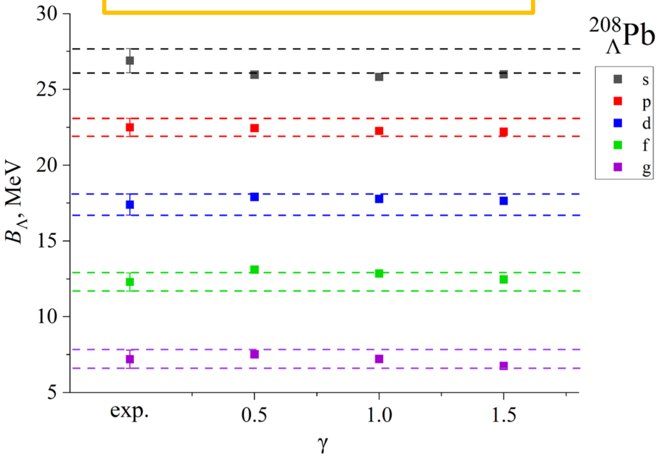
$$D_\Lambda = -\mu_\Lambda$$

$$\boxed{D_\Lambda(\rho_0) \approx 30 \text{ MeV}}$$

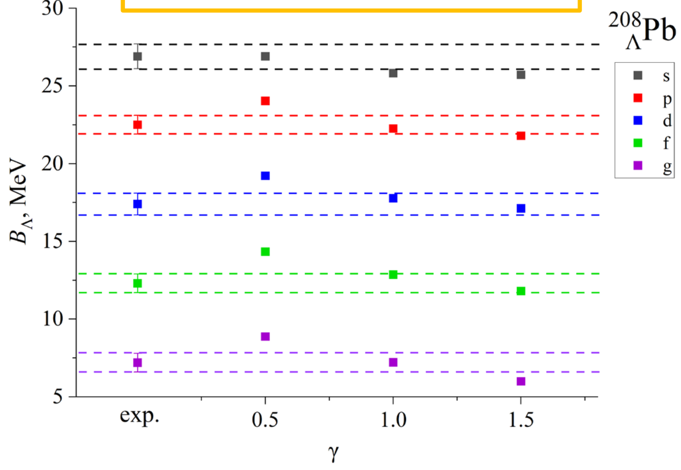
Compression power of ΛN-interaction

$$K_\Lambda = 3\rho \frac{dD_\Lambda(\rho)}{d\rho}$$

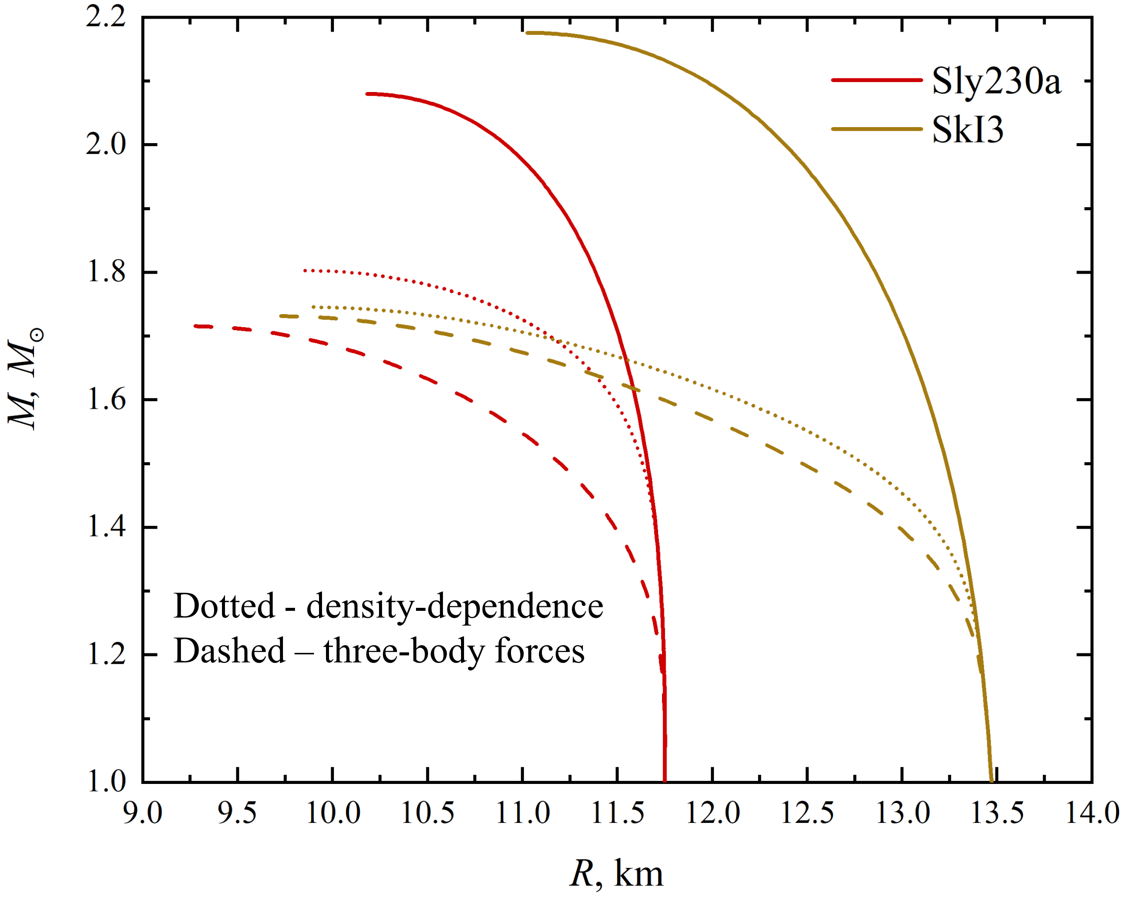
$$\boxed{K_\Lambda(\rho_0) = const}$$

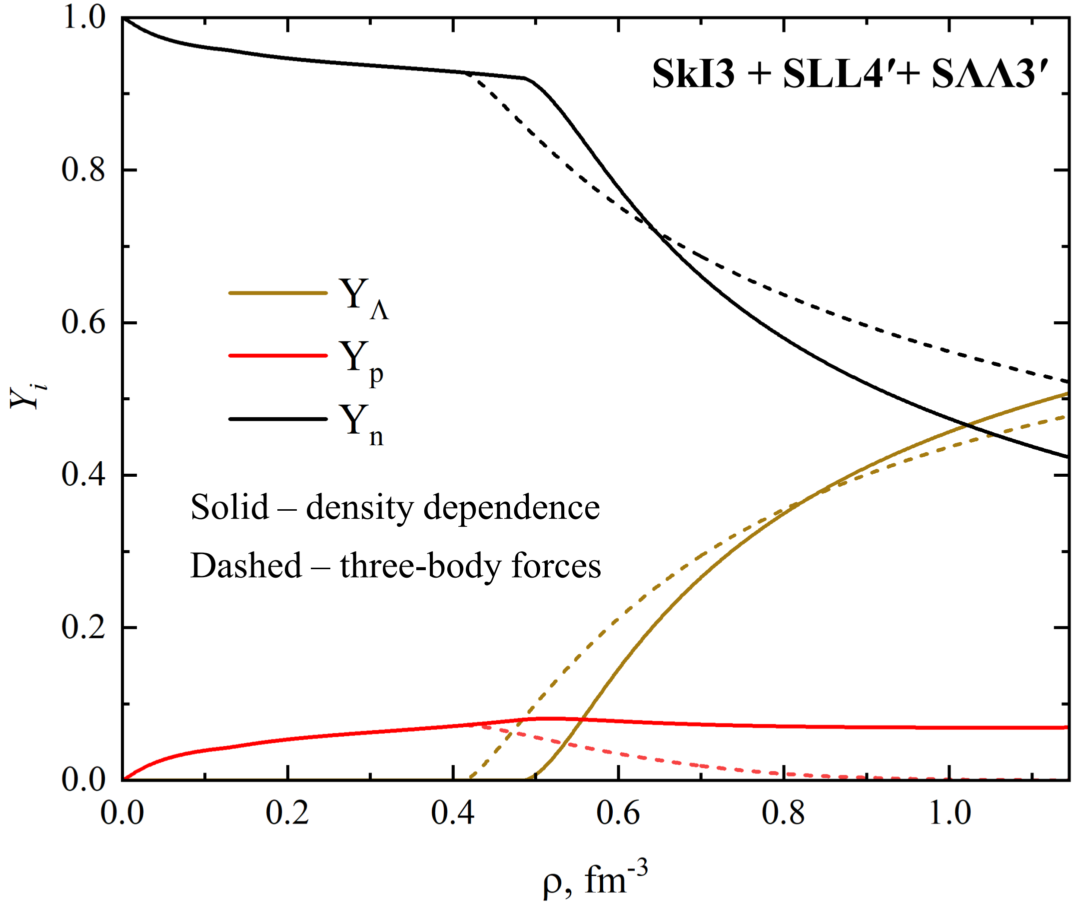


$$\boxed{K_\Lambda(3\rho_0) = const}$$



Three-body and density-dependent forces





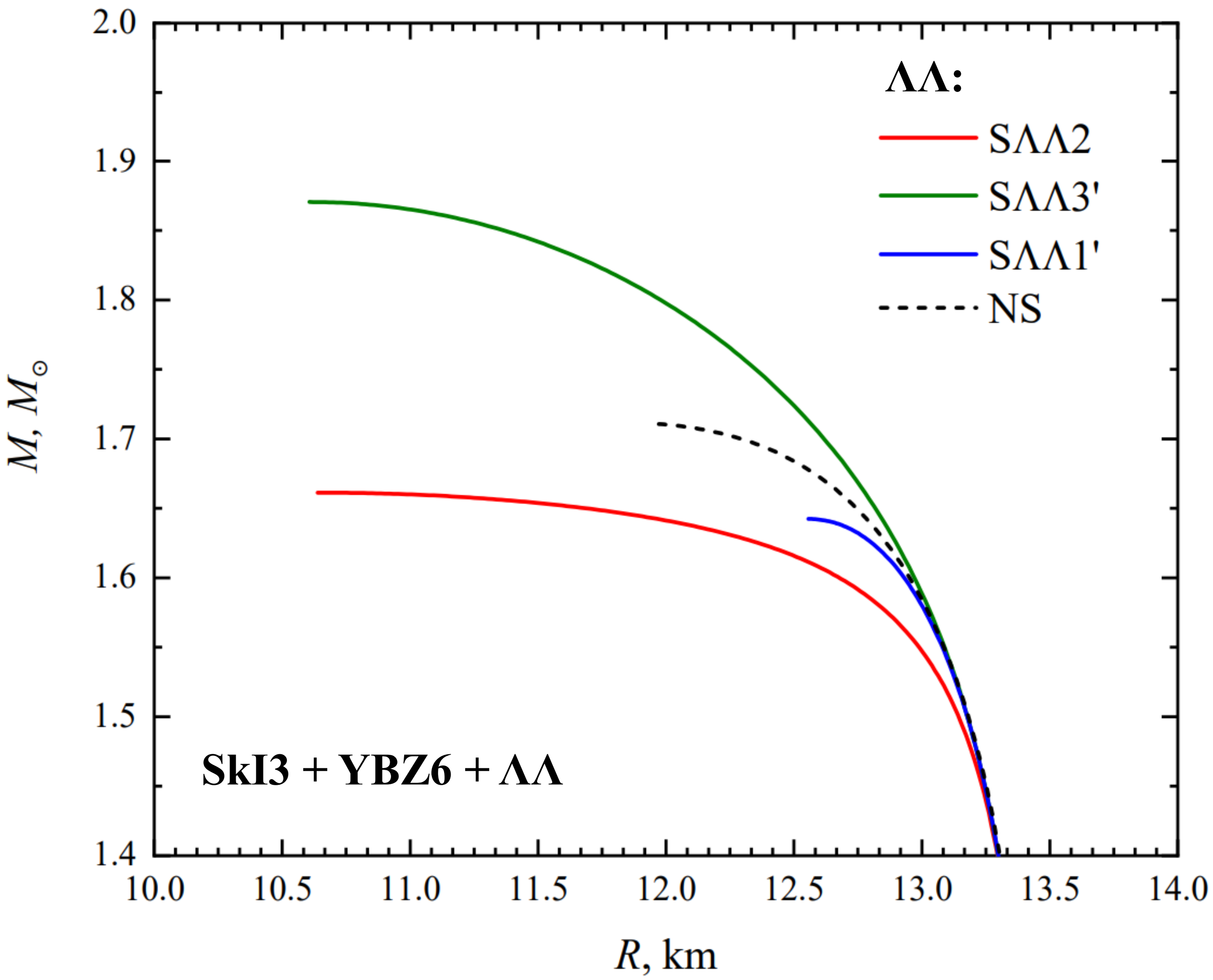
Three-body forces

$$V_3 = V_{\Lambda NN}(\overrightarrow{r_\Lambda}, \overrightarrow{r_{N1}}, \overrightarrow{r_{N2}}) = u_3 \delta(\overrightarrow{r_\Lambda} - \overrightarrow{r_{N1}}) \delta(\overrightarrow{r_\Lambda} - \overrightarrow{r_{N2}})$$

Density-dependent forces

$$V_3 = V_{\Lambda N}(\overrightarrow{r_\Lambda}, \overrightarrow{r_N}, \rho) = \frac{3}{8} u_3 (1 + \xi_3 P_\sigma) \delta(\overrightarrow{r_\Lambda} - \overrightarrow{r_N}) \rho_N^\gamma \left(\frac{\overrightarrow{r_\Lambda} + \overrightarrow{r_N}}{2}\right)$$

ΛΛ-interaction



D. E. Lanskoy, 1998, Minato F., 2011

ΛΛ-interaction	Radius of interaction
SΛΛ1'	Small
SΛΛ2	Medium
SΛΛ3'	Large

NS – density dependent ΛΛ-interaction

NSC89 (Nijmegen) → **NS** (Gauss) → **NS** (Skyrme)

Conclusion

- The equation of state of the neutron star matter is determined by a complex interplay of the contributions of the different terms of the potential of ΛN-interaction and various values of $\boldsymbol{\gamma}$ in the density dependence can be acceptable.

- The three-body ΛNN forces in neutron stars lead to a softer equation of state than density-dependent ΛN forces and can even considerably affect the chemical composition of the star leading to disappearance of protons at a certain density.

- Density dependence in $\Lambda\Lambda$-interaction does not have such a large effect on the characteristics of neutron stars as in ΛN-interaction interactions and this issue requires further study.

THANK YOU FOR ATTENTION

Back up

NS approximation

$$V_{\Lambda\Lambda} = \sum_{1}^{3} (a_i + b_i k_F + c_i k_F^2) e^{-\frac{r^2}{\beta_i^2}}$$

$$\begin{aligned} V_{\Lambda\Lambda}(\mathbf{r}_1, \mathbf{r}_2) &= \lambda_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \\ &+ \frac{1}{2}\lambda_1 [\mathbf{P}'^2 \delta(\mathbf{r}_1 - \mathbf{r}_2) + \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{P}^2] \\ &+ \lambda_2 \mathbf{P}' \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{P} \\ &+ \sum_i \lambda_3^i \delta(\mathbf{r}_1 - \mathbf{r}_2) \rho_N^{\delta_i} \\ &+ \frac{1}{2} \sum_i \lambda_4^i [\mathbf{P}'^2 \delta(\mathbf{r}_1 - \mathbf{r}_2) + \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{P}^2] \rho_N^{\delta_i}. \end{aligned}$$

NS parameters

$$\lambda_0 = \pi^{3/2} \sum_{1}^{3} a_i \beta_i^3,$$

$$\lambda_1 = -\frac{1}{2}\pi^{3/2} \sum_{1}^{3} a_i \beta_i^5,$$

$$\lambda_3^1 = \left(\frac{3\pi^2}{2}\right)^{1/3} \pi^{3/2} \sum_{1}^{3} b_i \beta_i^3,$$

$$\lambda_3^2 = \left(\frac{3\pi^2}{2}\right)^{2/3} \pi^{3/2} \sum_{1}^{3} c_i \beta_i^3,$$

$$\lambda_4^1 = -\frac{1}{2}\left(\frac{3\pi^2}{2}\right)^{1/3} \pi^{3/2} \sum_{1}^{3} b_i \beta_i^5,$$

$$\lambda_4^2 = -\frac{1}{2}\left(\frac{3\pi^2}{2}\right)^{2/3} \pi^{3/2} \sum_{1}^{3} c_i \beta_i^5.$$

Table 1. Parameters λ_0, λ_1, λ_3^i and λ_4^i (9) of NS parametrization in Skyrme form

λ_0, MeV fm^3	λ_1, MeV fm^5	λ_3^1, MeV fm^4	λ_3^2, MeV fm^5	λ_4^1, MeV fm^6	λ_4^2, MeV fm^7
−833.1	646.4	1268.8	−960.5	−735.4	625.0

CSB in three-body forces

$$\varepsilon_3 = \frac{1}{4} u_3 Y_\Lambda (\rho_N^2 + 2\rho_p \rho_n) = \frac{1}{8} u_3 Y_\Lambda (3\rho_N^2 - \rho_-^2).$$