Calculation of the contribution of loop diagrams in the Glauber model for dd interaction

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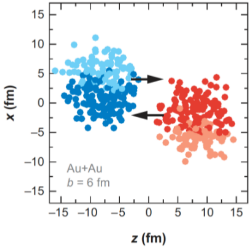
Glauber model

Glauber, R. Lectures in theoretical physics, ed. WE Brittin and LG Dunham / R. Glauber // Interscience, New York. — 1959. — T. 1. — C. 315.

- A nucleus-nucleus collision is considered as a sequence of independent nucleon-nucleon collisions;
- The cross-section of nucleon interaction is constant throughout the entire time of the passage of nucleons from one nucleus to another;
- The nuclei move in a straight line along the collision axis.



We will characterize nucleon-nucleon collisions by the quantity $\sigma(a)$ — the probability of inelastic interaction of two nucleons when the value of the impact parameter is a:

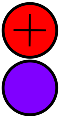
$$\int da\ \sigma(a) = \sigma_{NN}. \quad (1)$$

Difference from the case of heavy nuclei

The widely used assumption in the case of heavy nuclei that the profile function has a factorization:

$$T_A(a_1, \ldots, a_A) = \prod_{j=1}^{A} T_A(a_j)$$

cannot be used, since the positions of the nucleons in the deuteron are not independent.



Let us assume that the profile function of the deuteron has the following form:

$$T_d(a_1, a_2) = T_{n_1}(a_1) T_{n_1}(a_2) \delta(a_1 - a_2). \quad (2)$$

Calculation of the mean value of some random variable X

$$\langle X\rangle = \left\langle \langle \overline{X} \rangle_{n_2} \right\rangle_{n_1} = c^{-1} \int \overline{X}(a_1, a_2, b_1, b_2, \beta)\times$$
$$\times T_{n_1}(a_1) T_{n_1}(a_2) \delta(a_1 - a_2)\, da_1\, da_2\; T_{n_2}(b_1) T_{n_2}(b_2) \delta(b_1 - b_2)\, db_1\, db_2 =$$
$$= c^{-1} \int \overline{X}(a, -a, b, -b, \beta)\; T_{n_1}(a) T_{n_1}(-a)\, da\; T_{n_2}(b) T_{n_2}(-b)\, db. \tag{3}$$

Here $\overline{X}$ is the average value of some random variable X for a fixed position of all nucleons in deuterons d_1 and d_2 in the plane of the impact parameter. (All integrations are two-dimensional — over the entire plane of the impact parameter)

$$c^{-1} \int T_{n_1}(a) T_{n_1}(-a)\; da\;\; T_{n_2}(b) T_{n_2}(-b)\; db = 1. \tag{4}$$

$$\langle X\rangle = \left\langle \langle \overline{X} \rangle_{n_2} \right\rangle_{n_1} = c^{-1} \int \overline{X}(a, -a, b, -b, \beta)\times$$
$$\times\, (T_{n_1}(a))^2\;\; da\;\; (T_{n_2}(b))^2\;\; db. \tag{5}$$

Variance of the total number of participants

$$\left\langle N_w^{d_1}(\beta) \right\rangle = 2P(\beta), \tag{6}$$

$$V[N_w^{d_1}(\beta)] = 2Q(\beta)P(\beta) + 2\left(Q^{(12)}(\beta) - Q^2(\beta)\right), \tag{7}$$

where

$$Q(\beta) \equiv c^{-1} \int [1 - \sigma(a - b + \beta)]\,[1 - \sigma(a + b + \beta)] \times \\ \times \left(T_{n_1}(a)\right)^2 \; da \; \left(T_{n_2}(b)\right)^2 \; db, \tag{8}$$

$$Q^{(12)}(\beta) \equiv c^{-1} \int [1 - \sigma(a - b + \beta)]\,[1 - \sigma(a + b + \beta)] \times \\ \times [1 - \sigma(-a - b + \beta)]\,[1 - \sigma(-a + b + \beta)]\,\left(T_{n_1}(a)\right)^2 da\,\left(T_{n_2}(b)\right)^2 db, \tag{9}$$

$$P(\beta) = 1 - Q(\beta). \tag{10}$$

Let's consider

$$Q^{(12)}(\beta) \equiv c^{-1} \int [1-\sigma(a-b+\beta)]\,[1-\sigma(a+b+\beta)] \times$$
$$\times [1-\sigma(-a-b+\beta)]\,[1-\sigma(-a+b+\beta)]\,(T_{n_1}(a))^2\,da\,(T_{n_2}(b))^2\,db, \tag{11}$$

Similar to the case of heavy nuclei V.V.Vechernin and H.S.Nguyen, Phys.Rev.C 84, 054909 (2011)., it includes contribution

$$\sigma(a-b+\beta)\sigma(a+b+\beta)\sigma(-a-b+\beta)\sigma(-a+b+\beta) \tag{12}$$

which depends on the details of the NN interaction at nucleon distances, which are much smaller than typical nuclear distances.

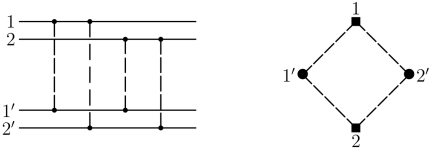


Figure: An example of the loop diagram in AA collisions. 1 and 2 are nucleons of the nucleus A; 1' and 2' are nucleons of the nucleus B

The resulting expression for the average value of the number of wounded nucleons
$\left\langle N_w^{d_1}(\beta)\right\rangle = 2(1 - c^{-1}\int [1-\sigma(a-b+\beta)]\,[1-\sigma(a+b+\beta)] \times$
$\times \left(T_{n_1}(a)\right)^2 \; da \; \left(T_{n_2}(b)\right)^2 \; db),$
includes contributions corresponding not only to the average interaction between one nucleon in one deuteron and another nucleon in the second deuteron,

$$c^{-1}\int \sigma(a-b+\beta) \; \left(T_{n_1}(a)\right)^2 \; da \; \left(T_{n_2}(b)\right)^2 \; db,$$

but also to the average interaction between one nucleon in one deuteron and two nucleons of the second deuteron

$$c^{-1}\int \sigma(a-b+\beta)\sigma(a+b+\beta) \; \left(T_{n_1}(a)\right)^2 \; da \; \left(T_{n_2}(b)\right)^2 \; db.$$

In the limit $r_N \ll R_{d_1}, R_{d_2}$, in contradistinction to the case of heavy nuclei, $\left\langle N_w^{d_1}(\beta)\right\rangle$ depends not only on the integral inelastic NN cross section σ_{NN} but also on the shape of the function $\sigma(b)$, i.e., on details of the NN interaction at nucleon distances.

Variance of the total number of participants

$$V[N_w^{d_1}(\beta) + N_w^{d_2}(\beta)] = V[N_w^{d_1}(\beta)] + V[N_w^{d_2}(\beta)] + \\ + 2\left[\left\langle N_w^{d_1}(\beta) N_w^{d_2}(\beta)\right\rangle - \left\langle N_w^{d_1}(\beta)\right\rangle \left\langle N_w^{d_2}(\beta)\right\rangle\right], \quad (13)$$

$$\left\langle N_w^{d_1}(\beta) N_w^{d_2}(\beta)\right\rangle - \left\langle N_w^{d_1}(\beta)\right\rangle \left\langle N_w^{d_2}(\beta)\right\rangle = 4\left[Q^{(11)}(\beta) - Q(\beta)\widetilde{Q}(\beta)\right]. \quad (14)$$

$$\widetilde{Q}(\beta) \equiv c^{-1} \int [1 - \sigma(a - b + \beta)]\,[1 - \sigma(-a - b + \beta)] \times \\ \times (T_{d_1}(a))^2\, da\, (T_{d_2}(b))^2\, db. \quad (15)$$

$$Q^{(11)}(\beta) \equiv c^{-1} \int [1 - \sigma(a - b + \beta)]\,[1 - \sigma(-a - b + \beta)] \times \\ \times [1 - \sigma(a + b + \beta)]\,(T_{d_1}(a))^2\, da\, (T_{d_2}(b))^2\, db. \quad (16)$$

Fluctuations of the number of nucleon collisions

$$\langle N_{coll}(\beta)\rangle = 4\chi(\beta). \quad (17)$$

$$V\left[N_{coll}(\beta)\right] = 4\left[\chi(\beta) + \chi_1(\beta) + \widetilde{\chi_1}(\beta) + \chi_2(\beta) - 4\chi^2(\beta)\right]. \quad (18)$$

where

$$\chi(\beta) \equiv c^{-1}\int \sigma(a-b+\beta)\ \left(T_{n_1}(a)\right)^2\ da\ \left(T_{n_2}(b)\right)^2\ db,$$

$\chi(\beta)$ represents the averaged probability of NN interaction.

$$\chi_1(\beta) \equiv c^{-1}\int \sigma(a-b+\beta)\sigma(a+b+\beta)\ \left(T_{n_1}(a)\right)^2\ da\ \left(T_{n_2}(b)\right)^2\ db,$$

$$\widetilde{\chi_1}(\beta) \equiv c^{-1}\int \sigma(a-b+\beta)\sigma(-a-b+\beta)\ \left(T_{n_1}(a)\right)^2\ da\ \left(T_{n_2}(b)\right)^2\ db,$$

$$\chi_2(\beta) \equiv c^{-1}\int \sigma(a-b+\beta)\sigma(-a+b+\beta)\ \left(T_{n_1}(a)\right)^2\ da\ \left(T_{n_2}(b)\right)^2\ db.$$

In contradistinction to the case of heavy nuclei, $\chi_1(\beta)$, $\widetilde{\chi_1}(\beta)$, $\chi_2(\beta)$ depends on the shape of the function $\sigma(b)$, i.e., on details of the NN interaction at nucleon distances.

Conclusions

Within the framework of the Glauber approach for deuteron-deuteron scattering, exact analytical expressions were obtained for the mean value and variance of the number of wounded nucleons, the variance of the total number of participating nucleons, the mean value and variance of the number of nucleon collisions for dd interaction at a fixed value of the impact parameter.

Similar to the case of heavy nuclei, there is the additional contact-term contribution which depends on details of the NN interaction at nucleon distances. This contact contribution arises from loop diagram corresponding to the interactions between two pairs of nucleons in colliding nuclei.

But in contradistinction to the case of heavy nuclei, this contribution also appears for the case of "tree" interactions (the interaction of one nucleon from the first deuteron with a pair of nucleons from the second one) due to the fact that the positions of the nucleons are not independent