Formation of protons in a new cumulative region of central rapidities and large transverse momenta due to the interaction of fluctons

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Nuclear physics and elementary particle physics.
Nuclear physics technologies»**

**Saint-Petersburg State University,
St.Petersburg, July 1-6, 2025**

Study of inclusive production cross-sections
of pions and protons in a new cumulative region:
central rapidities, large transverse momenta

$d+d \rightarrow \pi+X$

$d+d \rightarrow p+X$

outside p+p kinematics:

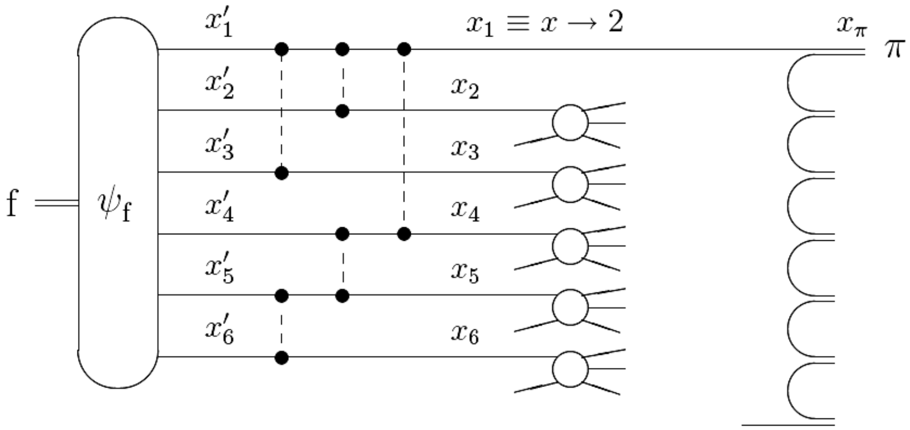
$p+p \rightarrow \pi+X$

$p+p \rightarrow p+X$

The mechanisms of pion and proton production are different!

Using the experience in describing the production of pions and protons in the traditional cumulative region of fragmentation of one of the colliding nuclei.

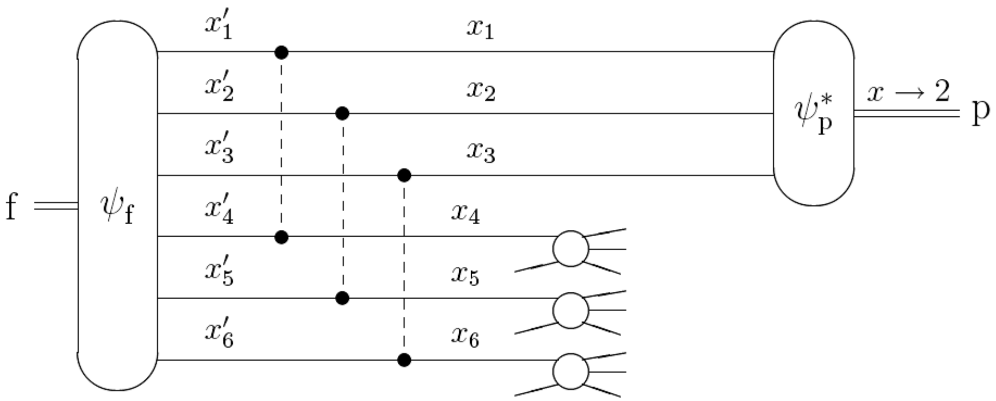
Coherent Quark Coalescence and Production of Cumulative Protons



- the cumulative pion production

by hadronization of one fast quark

*M.A. Braun, V.V. Vechernin, Nucl.Phys.**B 427**, 614 (1994); Phys.Atom.Nucl. **60**, 432 (1997); **63**, 1831 (2000)*



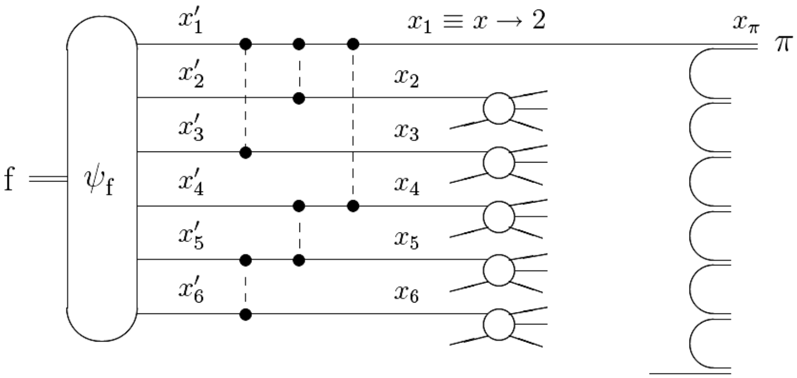
- the cumulative proton production

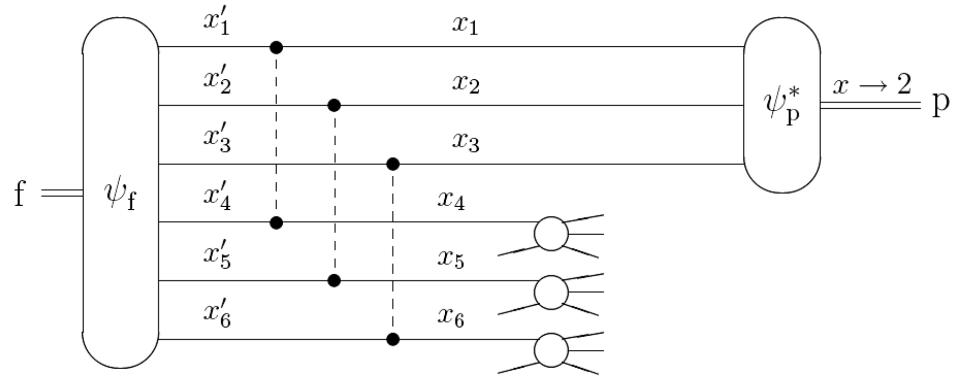
by **coherent** quark coalescence mechanism:

*M.A. Braun, V.V. Vechernin, Nucl.Phys.**B 92**, 156 (2001); Theor.Math.Phys **139**, 766 (2004); V.Vechernin, AIP Conf.Proc.1701 (2016) 060020.*

The last **recalls** the few nucleon **short-range correlations** in a nucleus
L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).
But instead of using the relativistic generalization of non-relativistic NN wave function
the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge
*Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl. Phys. **B369** (1992) 519.*
was developed and applied.

Comparison of the mechanisms of pion and proton production in dd collisions in traditional cumulative region of fragmentation of one of the colliding nuclei





$$f_\pi(x, k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C_\pi (2-x)^9\, \Phi_5\left(\frac{k_\perp}{m_q}\right) / \Phi_5(0)$$

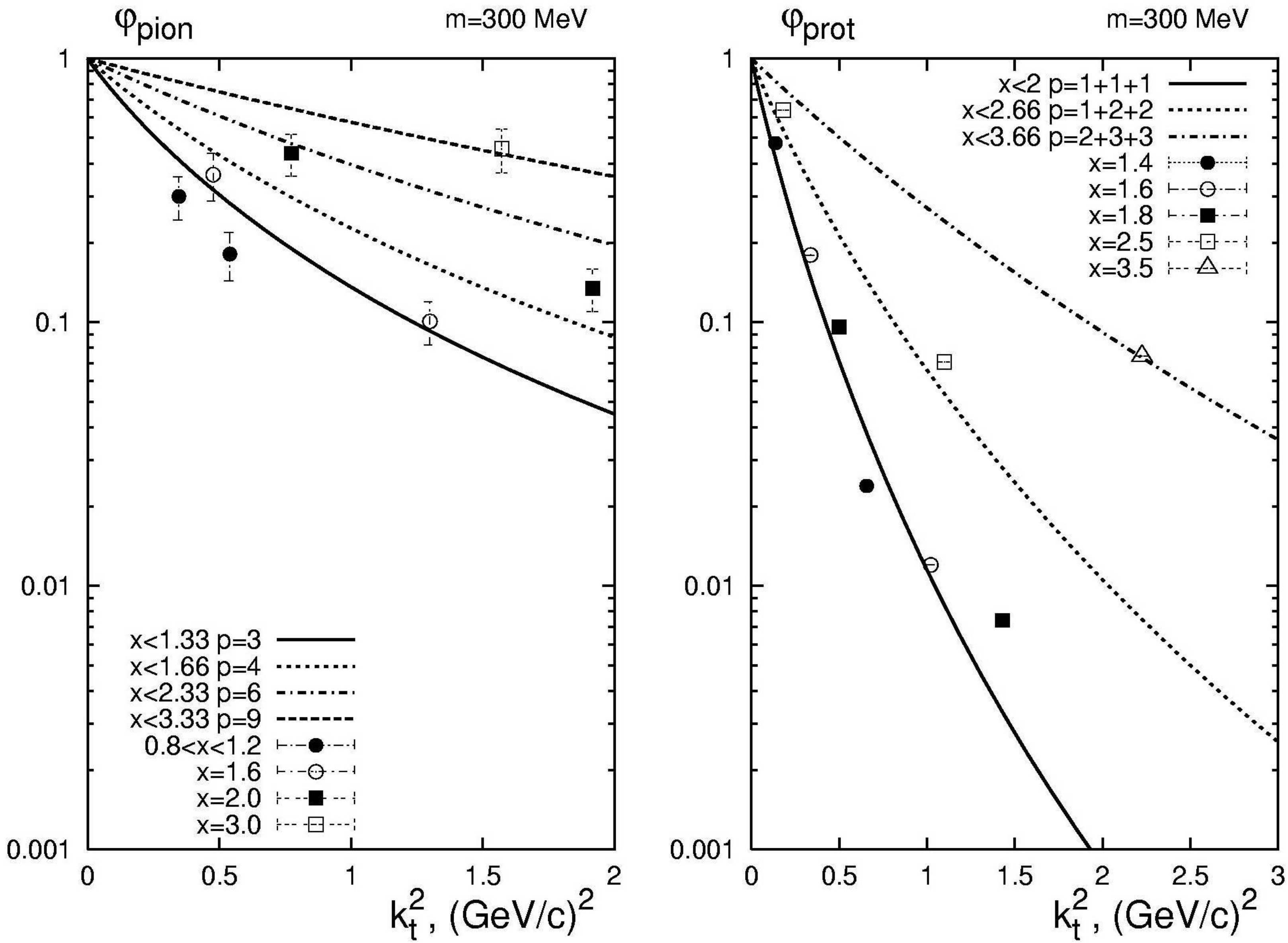
$$f_p(x, k_\perp) \equiv \frac{k_0\, d^3\sigma_p}{d^3\mathbf{k}} = C_p (2-x)^5\, \Phi_1^3\left(\frac{k_\perp}{3m_q}\right) / \Phi_1^3(0)$$

$$\Phi_p(t) = 2\pi \int_0^\infty dz\, z J_0(tz) [z K_1(z)]^p$$

$$\Phi_1(t) = \frac{4\pi}{(t^2+1)^2}$$

Energy scaling of cumulative inclusive cross section in the flucton fragmentation region:

$$f_\pi(x, k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C\, s^0\, (f - x)^{2p-1}\, \Phi_p\left(\frac{k_\perp}{m_q}\right)$$



V.Vechernin,
AIP Conference Proceedings
1701 (2016) 060020.

S.V. Boyarinov et al., Sov.J.Nucl.Phys. ***46****, 871 (1987)*
S.V. Boyarinov et al., Physics of Atomic Nuclei ***57****, 1379 (1994)*
S.V. Boyarinov et al., Sov.J.Nucl.Phys. ***55****, 917 (1992)*

Pion and Proton Yields:

Estimation of Production Rates at Different Initial Energies

Problem with modeling using MC event generators:

Not done yet, as **this mechanism is absent in existing MC event generators**. It is necessary to introduce into the MC generator some **admixture of 6-quark flucton** into deuteron and **theoretically calculated inclusive cross-sections of production of various particles** in the new cumulative region **from flucton-flucton interaction.** **(in progress with Semyon Yurchenko)**.

Estimates based on extrapolation from the region of fragmentation of a nucleus:

Vechernin V.V., Belokurova S.N., Yurchenko S.V. Phys Part Nucl, 2024, Vol. 55, pp. 889–894.

$$\langle n \rangle_{\mathrm{dd}} \cdot \sigma_{\mathrm{dd}}^{tot} = 4\pi \int_{0.5}^{1} dy \int_{k_\perp^{min}(y)}^{k_\perp^{max}(y)} dk_\perp \, k_\perp \, f(\, x(y, k_\perp), \, k_\perp)$$

Table 1. The magnitude of the transverse momentum of cumulative pions and protons in dd scattering, corresponding to values of the variable $x = 1$ and 2 ($k_\perp^{min}$ and $k_\perp^{max}$) at a given value of the rapidity y, for two values of the initial energy.

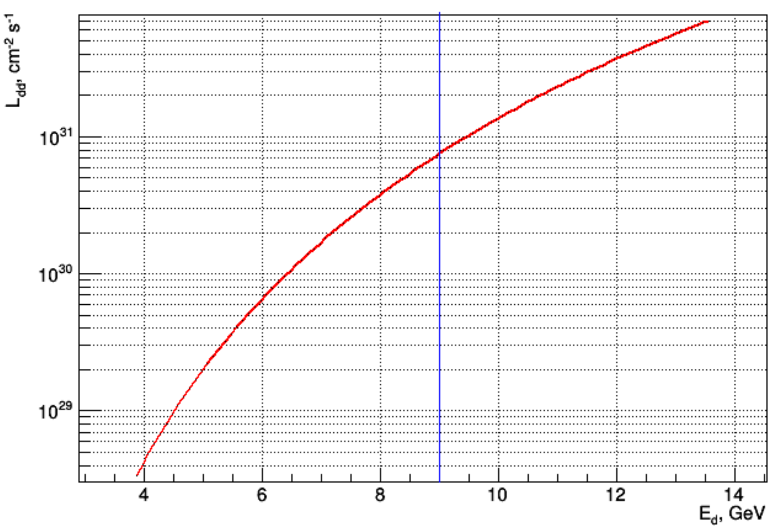
$\sqrt{s_{NN}}$	4 GeV			8 GeV		
	y	$k_\perp^{min}$	$k_\perp^{max}$	y	$k_\perp^{min}$	$k_\perp^{max}$
dd $\to \pi$	0.5	1.728	2.752	0.5	4.197	6.672
dd $\to \pi$	1.0	1.102	2.002	1.0	2.687	4.86
dd $\to$ p	0.5	1.741	2.999	0.5	4.218	6.803
dd $\to$ p	1.0	0.852	2.089	1.0	2.605	4.915

Pion and Proton Yields:

Estimation of Production Rates in dd collisions at $\sqrt{s_{NN}}$ = 4 and 8 GeV

Table 2. The results of calculations of the multiplicity of pions and protons in the cumulative region by the formula (9), using the inclusive cross sections (1) and (2), for rapidities in the interval $0.5 < |y| < 1$ formed in dd scattering due to the interaction of a nucleon with a 6-quark flucton.

	$\sqrt{s_{NN}}$	4 GeV	8 GeV
$\langle n_{\pi^-}\rangle_{\rm dd}$	$x > 1.0$	$9{\cdot}10^{-4}$	$1.9{\cdot}10^{-4}$
	$x > 1.2$	$6.6{\cdot}10^{-5}$	$1.2{\cdot}10^{-5}$
	$x > 1.5$	$3.6{\cdot}10^{-7}$	$5.8{\cdot}10^{-8}$
$\langle n_p\rangle_{\rm dd}$	$x > 1.0$	$2.3{\cdot}10^{-2}$	$9{\cdot}10^{-6}$
	$x > 1.2$	$1.2{\cdot}10^{-3}$	$4.6{\cdot}10^{-7}$
	$x > 1.5$	$1.04{\cdot}10^{-5}$	$4.2{\cdot}10^{-9}$



$$Y_{\rm dd} = 0.1 L_{\rm dd}\sigma^{\rm tot}_{\rm dd}\langle n\rangle_{\rm dd} t.$$

depends only on product $\langle n\rangle_{\rm dd}\sigma^{\rm tot}_{\rm dd}$
(see previous slide)

The SPD Collab. (V. Abazov et al.), Natural Science Review 1 1 (2024); arXiv:2404.08317 [hep-ex]

Pion and Proton Yields:

Estimation of Production Rates in dd collisions at $\sqrt{s_{NN}}$ = 4 and 8 GeV

Table 3. Estimates of the yields (Y_{dd}) of cumulative pions and protons in the rapidity interval $0.5 < |y| < 1$ in dd collisions in one hour of data acquisition at the SPD facility of the NICA collider, calculated by the formulas (9) and (11) using the inclusive cross sections (1) and (2) and taking into account the luminosity reduction at energy 4 GeV [21] (see text).

	$\sqrt{s_{NN}}$	4 GeV	8 GeV
$Y_{\mathrm{dd}} \to \pi^-$	$x > 1$	400	8 000
	$x > 1.2$	30	500
	$x > 1.5$	0.16	2.5
$Y_{\mathrm{dd}} \to \mathrm{p}$	$x > 1$	10 000	400
	$x > 1.2$	500	20
	$x > 1.5$	4.5	0.18

Inclusive cross sections for the production of pions and protons in dd-collisions, integrated over rapidity intervals 0.5 < |y| < 1

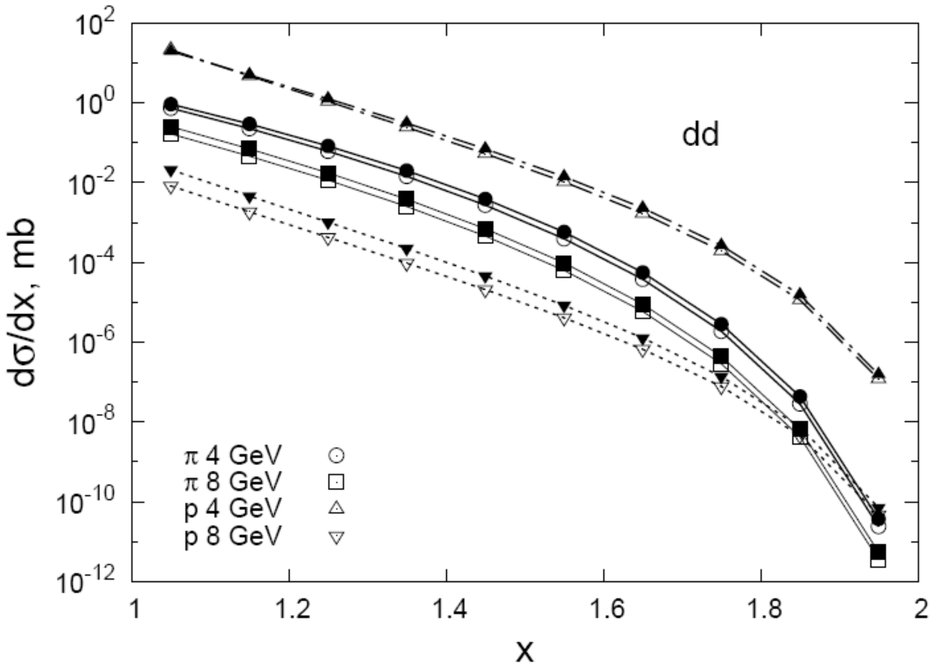


Figure 2. Inclusive cross sections for the production of pions (○, □) and protons (△,▽) in dd collisions, integrated over rapidity intervals $0.5 < |y| < 1$ and available for study with NICA SPD, respectively, for two initial energies $\sqrt{s_{NN}} = 4$ and 8 GeV, as a function of the light-cone cumulative variable $x = 2x_+$ (open simbols) and the cumulative number $x = x_M$ (solid symbols). Model calculations by (9) using (1) and (2). (Curves serve to guide the eye.)

$$\frac{d\sigma}{dx} = \frac{\langle n\rangle_{\rm dd}^{\Delta x}}{\Delta x}\sigma_{\rm dd}^{tot} = \frac{2\pi}{\Delta x}\int_{0.5}^{1} dy \int_{k_\perp^x(y)}^{k_\perp^{x+\Delta x}(y)} dk_\perp\, k_\perp \times$$

$$\times\, f(\,x(y,k_\perp),\, k_\perp)\ , \qquad (9)$$

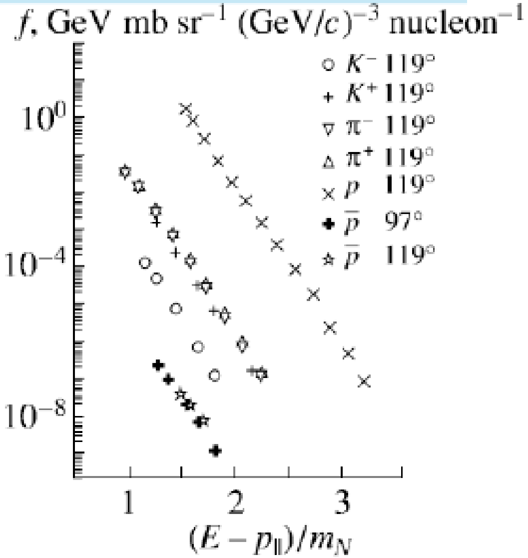
Vechernin V.V., Yurchenko S.V. Cumulative production at central rapidities and large transverse momenta in the quark model of flucton fragmentation, Moscow University Physics Bulletin, 2024, Vol. 79, Suppl. 1, pp. S174–S178.

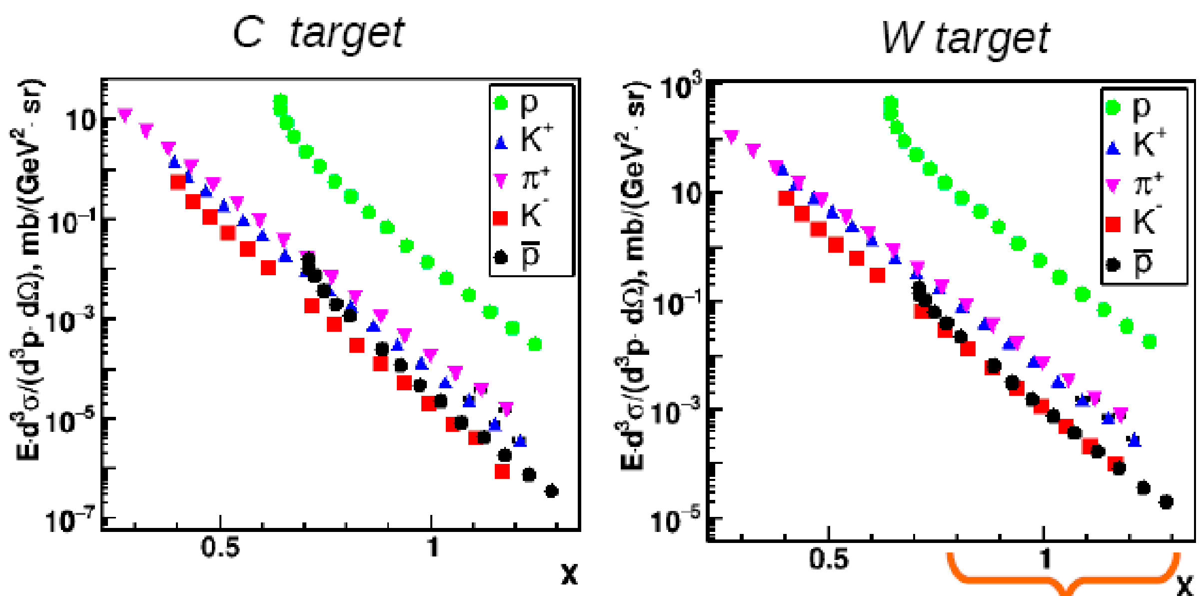
f + 2N

$x_1 = x = x_M$

$x_2 = 2$

Г.А. Лексин, ЯФ, т. 65, № 11, 2002, стр. 2042 — 2051





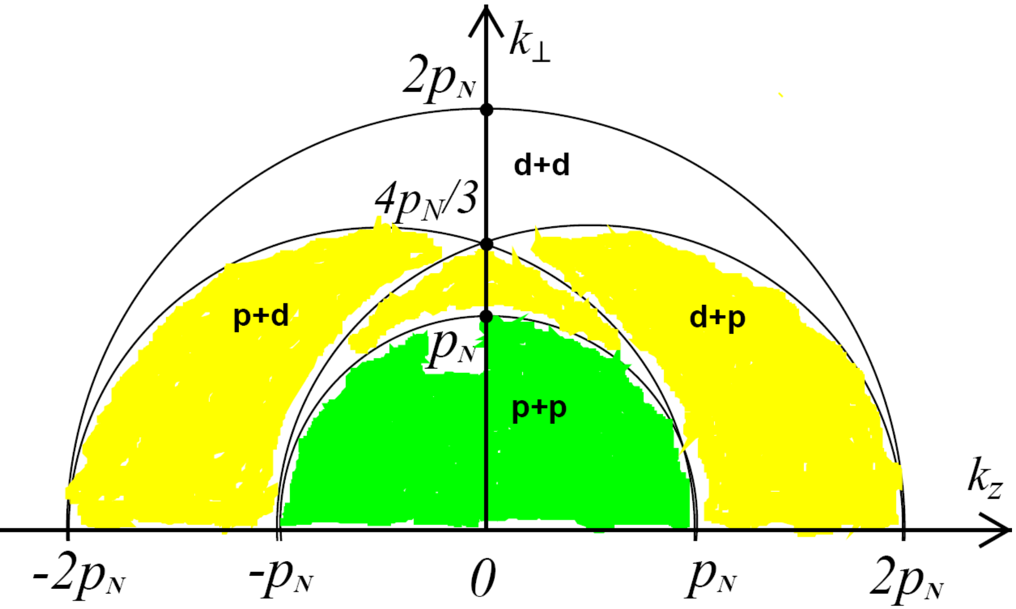
Large fraction of cumulative processes

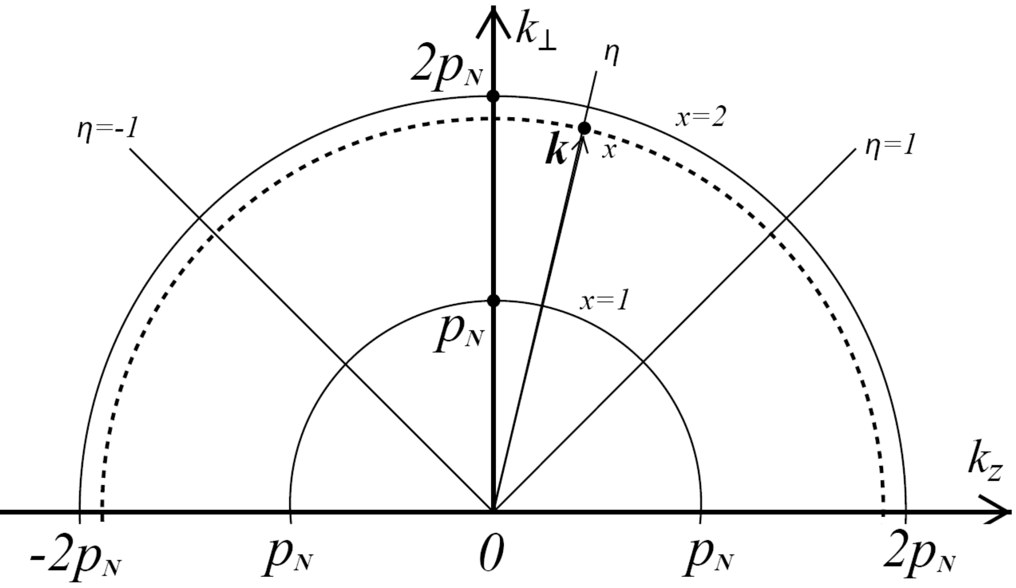
$\sqrt{s_{NN}}$ = 9.8 GeV

N. Antonov, V. Gapienko, G. Gapienko, M. Ilushin, A. Prudkoglyad, V. Romanovskiy, A. Semak, I. Solodovnikov, M. Ukhanov, V. Viktorov "High pt anti-proton and meson production in cumulative pA reaction at 50 GeV/c"
(National Research Center Kurchatov Institute - Institute for High Energy Physics, Protvino)
LXX International Conference "NUCLEUS – 2020. Nuclear physics and elementary particle physics. Nuclear physics technologies", St Petersburg, October 11-17, 2020.

The mechanism of pion production
in the new cumulative region of central rapidities and large transverse momenta due to flucton-flucton interaction.

Cumulative regions in dd and pd collisions





$\boldsymbol{p_N \gg m_N}$ $\qquad p_N \approx \sqrt{s}/2$

We expect
for the inclusive cross-section
of the production of particles with large transverse momenta at midrapidities:

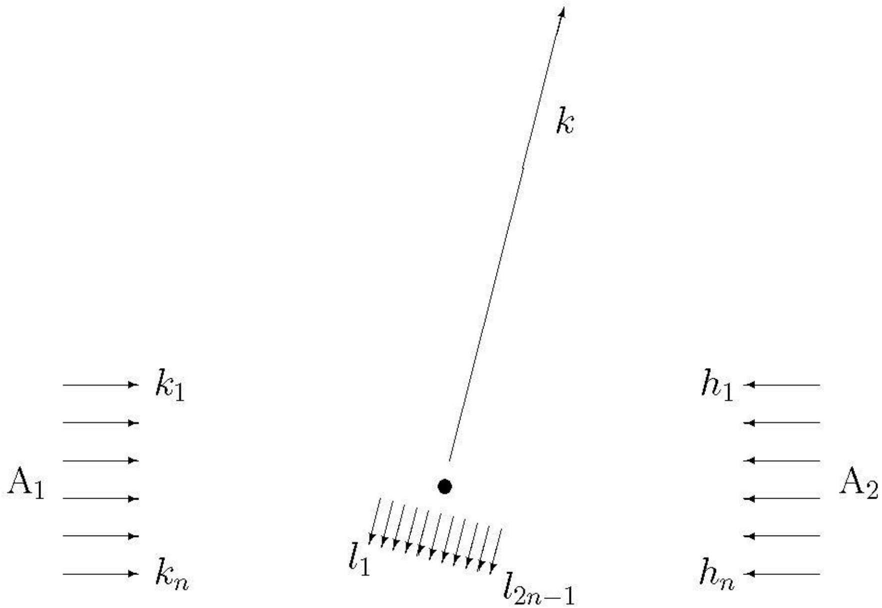
$$\frac{k_0\, d^3\sigma}{d^3\mathbf{k}} = f(x,\eta) = C\, s^{-q_1}\, (f-x)^{q_2}\, F(\eta)$$

Quark Counting Rules with
two asymptotic parameters:
s>>m² and (f-x)

q_1, q_2 are different for production of pions and protons!

x - cumulative number
$\eta = -\ln \operatorname{tg}\frac{\theta^*}{2}$ - pseudorapidity

Kinematics



d+d → π+X at quark level
($A_1 = A_2 = A = 2$, $n = 6$)

$$p_N = P_A/A \qquad p_N \gg m_N$$

Initial state:

$$k_i \sim P_A/n = p_N/3$$
$$h_i \sim -P_A/n = -p_N/3 \qquad n = 3A$$

Final state:

$$k \sim P_A = A\,p_N = n\,p_N/3$$
$$l_i \sim -P_A/(2n-1) = -\frac{n}{3(2n-1)}p_N$$

First small parameter: $\frac{m_N}{p_N} = \frac{2m_N}{\sqrt{s_{NN}}} \ll 1$

Second small parameter: $1 - \frac{k}{k_{max}} \ll 1$

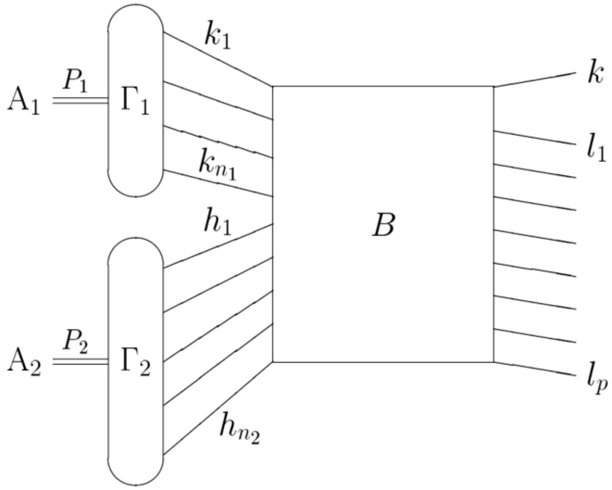
$k \to k_{max}$ => ${M_X}^2 = (\sum_{i=1}^{2n-1} l_i)^2 \to {M_{Xmin}}^2$

$$l_1 = l_2 = \dots = l_{2n-1} = -\frac{k_{max}}{2n-1}$$

V. Vechernin, S. Yurchenko
Int.J.Mod.Phys. E, 2441022,
DOI: 10.1142/S0218301324410222

$$s = s_{NN}$$

Quark counting rules for the inclusive cross sections of pion production in new cumulative region of large transverse momenta at mid-rapidities (the decrease with both the initial energy s and the cumulative number *x*)

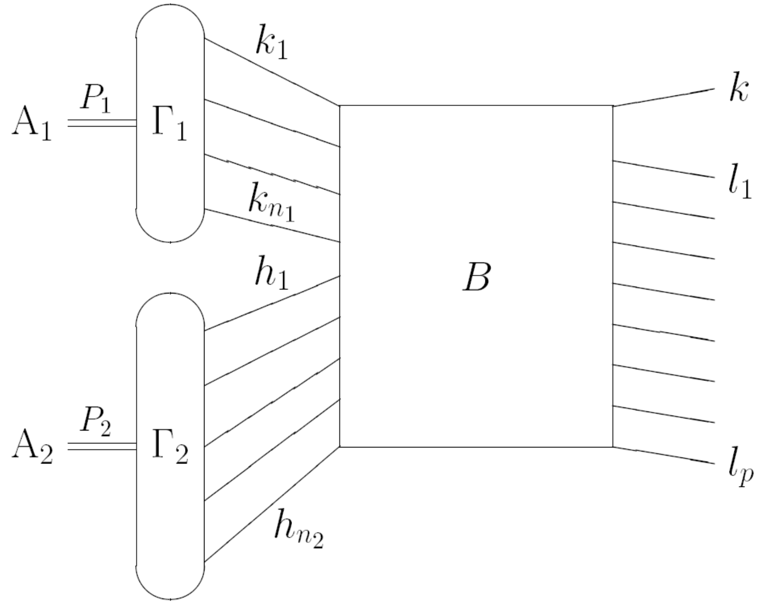


$$p = n_1 + n_2 - 1$$

To evaluate this behavior and find asymptotes at s>>m and (2-x)<<1
we need to generalize the quark counting rules, known now only for
a) the inclusive cross sections in the fragmentation region (|t|<<s) and
b) the elastic and quasielastic cross sections in the high pT region (|t|~s),
to the case of inclusive cross sections in the high pT region (|t|~s).

$$\boxed{I(x) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p-\frac{5}{2}}}{(m^2R^3)^{p-1} s^{(p+3)/2}}}$$

Amplitude (T)

$T =$ 

$p = n_1 + n_2 - 1$

$$I(\mathbf{k}) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{1}{J}\int |T|^2\, d\tau_p\ ,$$

$$J = 2A_1A_2\sqrt{s(s-4m_N^2)}$$

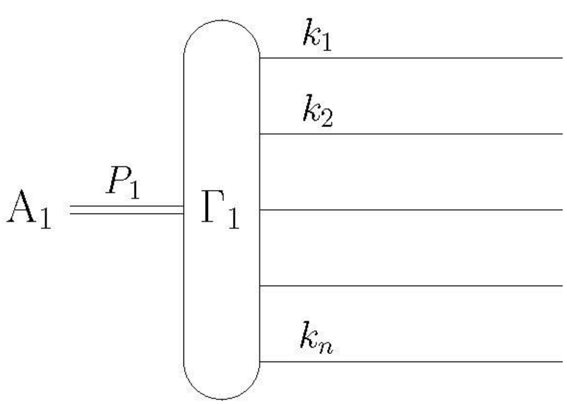
$$d\tau_p \equiv (2\pi)^4 \delta^4(P_1 + P_2 - k - \sum_{i=1}^{p} l_i) \prod_{i=1}^{p} \frac{d^{(3)}\mathbf{l}_i}{2l_{i0}(2\pi)^3}$$

$$s = s_{NN}$$

Light-cone partonic wave function

S.J. Brodsky, P. Hoyer, A. Mueller, W.-K. Tang, Nucl.Phys. B369 (1992) 519

$J_{n1} =$



$$x_i \equiv \frac{k_{i+}}{p_{1+}}\,, \qquad k_{i+} \equiv \frac{k_{i0}+k_{iz}}{\sqrt{2}}$$

$$p_1 = P_1/A_1$$

$$\psi_1(x_i, \mathbf{k}_{i\perp}) \equiv \frac{\Gamma_1(x_i, \mathbf{k}_{i\perp})}{\sum_{i=1}^{n_1} \frac{m^2+\mathbf{k}_{i\perp}^2}{x_i} - A_1 m_N^2}$$

$$J_{n_1} = \int \psi_1(x_i, \mathbf{k}_{i\perp})\, 2\delta(\sum_{i=1}^{n_1} x_i - A_1)\,(2\pi)^3 \delta^{(2)}(\sum_{i=1}^{n_1} \mathbf{k}_{i\perp}) \prod_{i=1}^{n_1} \frac{dx_i}{2x_i}\frac{d^2\mathbf{k}_{i\perp}}{(2\pi)^3}$$

M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614

$$J_{n_1} \sim \psi_1(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0)$$

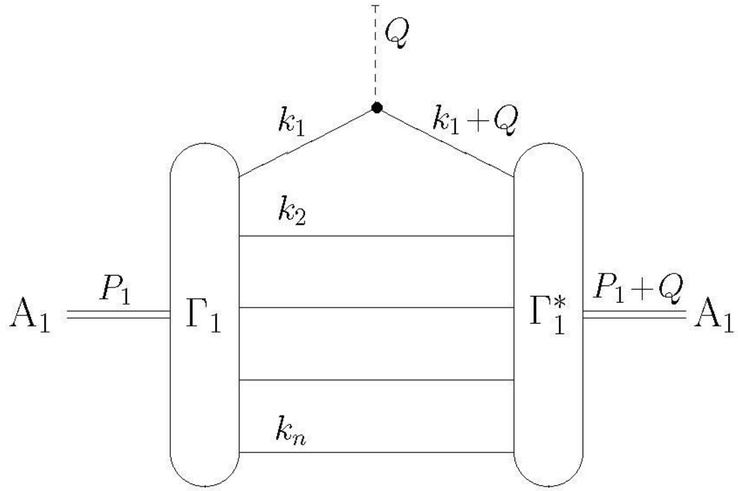
$$J_{n_1} = \frac{C_1}{m^{(n_1-1)/2} R_1^{3(n_1-1)/2}}$$

m - mass of the constituent quark
R_1- size of the system (R_1= R_A or R_N)
C_1- dimensionless constant, independent of the dimensional parameters of the model

$$J_n^2 \sim |\psi(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0)|^2 \sim \frac{1}{V^{n-1}} \sim \frac{1}{R^{3(n-1)/2}}$$

for a wave function with one dimensional parameter

Normalization of the wave function (Γ)

$$F(Q^2) =$$ 

$$Q^2 \to 0 \quad \Rightarrow \quad F(Q^2) \to 1$$

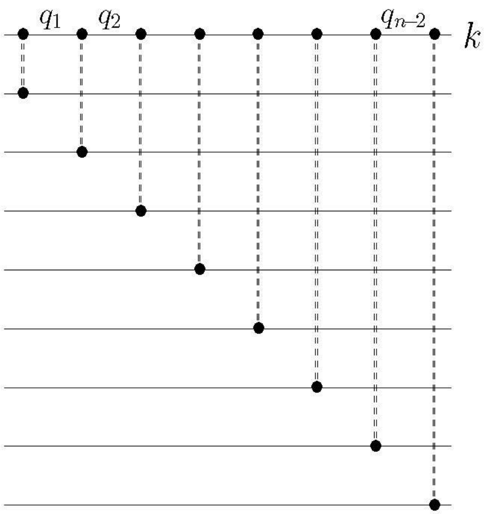
M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614

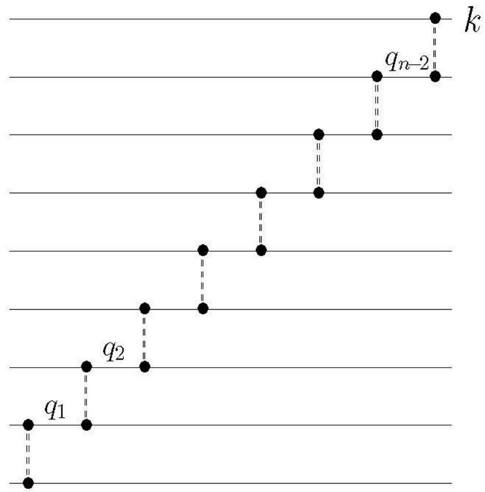
$$\int |\psi_1(x_i, \mathbf{k}_{i\perp})|^2\, 2\delta(\sum_{i=1}^{n_1} x_i - A_1)\,(2\pi)^3\delta^{(2)}(\sum_{i=1}^{n_1} \mathbf{k}_{i\perp}) \prod_{i=1}^{n_1} \frac{dx_i}{2x_i}\frac{d^2\mathbf{k}_{i\perp}}{(2\pi)^3} = A_1$$

$$\psi_1(x_i, \mathbf{k}_{i\perp}) \equiv \frac{\Gamma_1(x_i, \mathbf{k}_{i\perp})}{\sum_{i=1}^{n_1} \frac{m^2+\mathbf{k}_{i\perp}^2}{x_i} - A_1 m_N^2}$$

Similarly for $\psi_2(y_i, \mathbf{h}_{i\perp})$ $\qquad y_i \equiv \frac{h_{i-}}{p_{2-}}\,, \qquad h_{i-} \equiv \frac{h_{i0} - h_{iz}}{\sqrt{2}} \qquad p_2 = P_2/A_2$

Block of hard exchanges (*B*)





S.J. Brodsky, B.T. Chertok, Phys.Rev. D 14 (1976) 3003

$$B = \frac{C_B}{s^{n-2}} = \frac{C_B}{s^{n_1+n_2-2}}$$

$$n = n_1 + n_2$$

$$p = n - 1$$

$$r_B \sim \frac{1}{\sqrt{s}}$$

$$B(k_i,\, h_i;\, \mathbf{k},\, \mathbf{l}_i) \approx B(P_1/n_1,\, P_2/n_2;\, \mathbf{k}_{max},\, -\mathbf{k}_{max}/p)$$

$$T = \frac{C_1\, C_2\, C_B}{m^{(n-2)/2} R_1^{3(n_1-1)/2} R_2^{3(n_2-1)/2} s^{n-2}} = \frac{C_1\, C_2\, C_B}{m^{(n-2)/2} R^{3(n-2)/2} s^{n-2}}$$

$$I(\mathbf{k}) = \frac{1}{J} |T|^2\, \tau_p$$

$$\tau_p = (2\pi)^4 \int \delta^4(P_1 + P_2 - k - \sum_{i=1}^{p} l_i) \prod_{i=1}^{p} \frac{d^{(3)}\mathbf{l}_i}{2 l_{i0} (2\pi)^3}$$

After Calculation of Phase Volume and Relation with Cumulative Number:

$$x\sqrt{s} = \sqrt{k^2 + m^2} + \sqrt{k^2 + [p(x)\, m]^2}\ .$$

$$p(x) = n_1 + n_2 - 1 = 3A_1 + 3A_2 - 1 = 6A - 1 = 6x - 1.$$

$$\boxed{I(x) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p-\frac{5}{2}}}{(m^2R^3)^{p-1} s^{(p+3)/2}}}$$

$$p = p(A)$$

two (!)

small parameters:

$m/\sqrt{s} \ll 1$

$A - x \ll 1$

A=2 d+d → π+X $I_{dd\to\pi}(x) \sim s^{-7}\,(2-x)^{14}$

A=1 p+p → π+X $I_{pp\to\pi}(x) \sim s^{-4}\,(1-x)^{5}$

$$r_B \sim \frac{1}{\sqrt{s}}$$

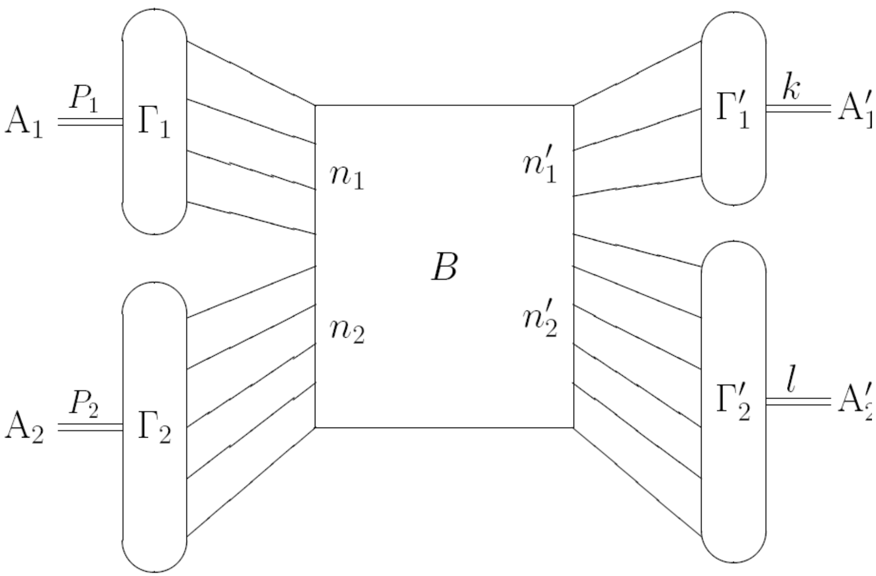
Changes may occur at moderate NICA energy:

$$M_X \sim (A - x)\sqrt{s}$$

so at rather small NICA energies: partonic phase volume => hadronic phase volume only **d+d** → **π+NNNN,** neglecting **d+d** → **π+ πNNNN** and so on.

Comparison with Quark Counting Rules for Quasi-Elastic Processes

one
small parameters:
$m/\sqrt{s} \ll 1$



$$\boxed{\frac{d\sigma}{dt} \sim \frac{1}{s^{n_1+n_2+n'_1+n'_2-2}}}$$

Matveev V.A., Muradyan R.M., Tavkhelidze A.N., Lett. Nuovo Cimento 7 (1973) 719
Brodsky S., Farrar G., Phys.Rev.Lett. 31 (1973) 1153
Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003
Uzikov Yu. N., JETP Lett. 81 (2005) 303

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\, s^2_{A_1A_2}}|T_{2\to 2}|^2 = \frac{1}{16\pi\, A_1^2 A_2^2 s^2}|T_{2\to 2}|^2$$

$$T_{2\to 2} = J_{n_1}\, J_{n_2}\, B\, J_{n'_1}\, J_{n'_2}$$

$$\boxed{\frac{d\sigma}{dt} = \frac{C'}{s^{2n-2}\, m^{2n-4} R_1^{3(n_1-1)} R_2^{3(n_2-1)} R_1'^{3(n'_1-1)} R_2'^{3(n'_2-1)}}}$$

$$n = n_1 + n_2 = n'_1 + n'_2$$

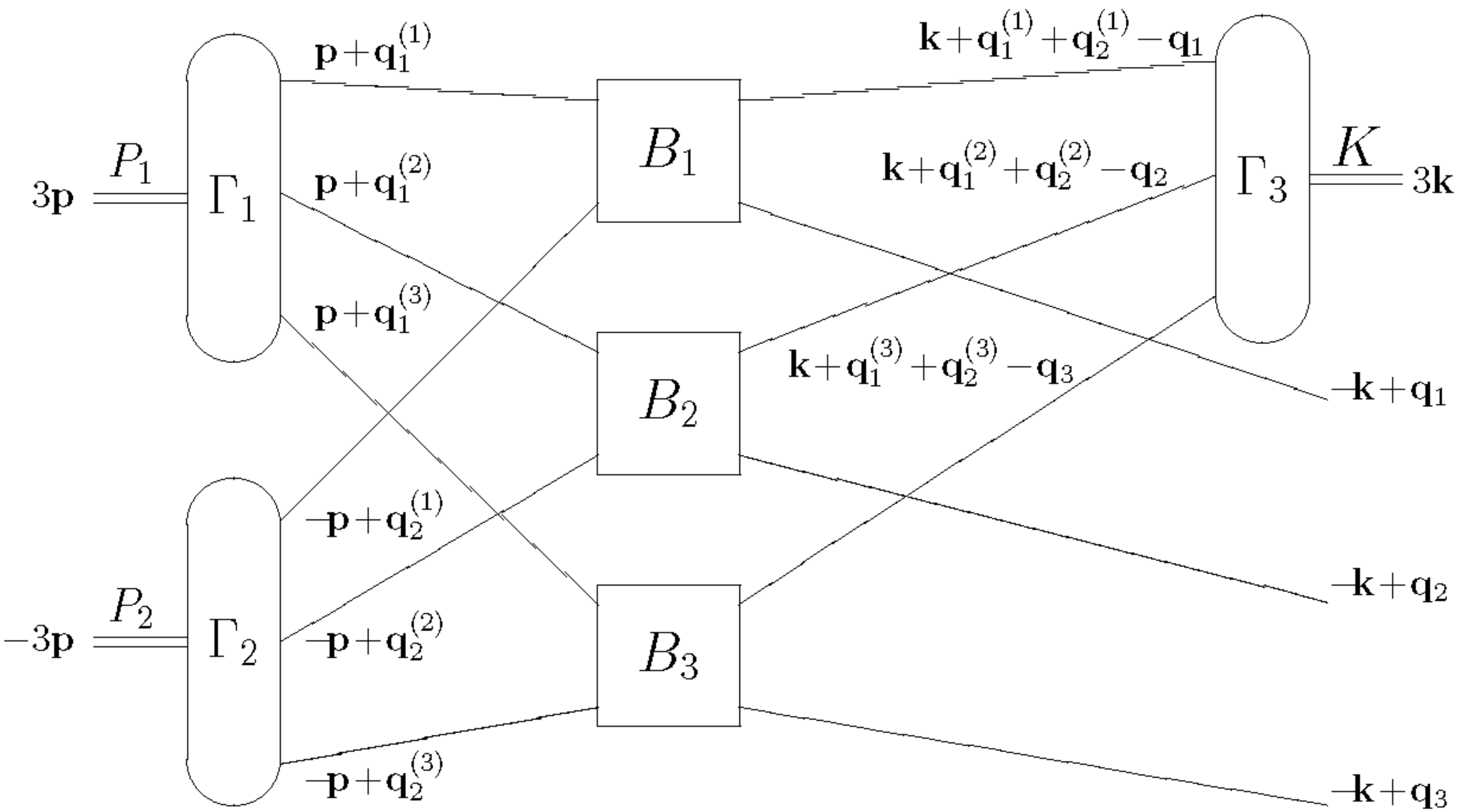
Yu.L. Dokshitzer, QCD Phenomenology, Lectures at the CERN–Dubna School, Pylos, August 2002

Early validity of QCR in
the deuteron break-up by a photon, $\gamma + D \to p + n$

The mechanism of proton production
at central rapidities and large transverse momenta due to flucton-flucton interaction.

Consider the technique using the simplest example p+p → p+X   at

$$x \to 1, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}/2, \quad \theta^* \sim 90^\circ$$



$$\sum_{i=1}^{3} \mathbf{q}_1^{(i)} = 0$$

$$\sum_{i=1}^{3} \mathbf{q}_2^{(i)} = 0$$

$$\sum_{i=1}^{3} \mathbf{q}_i = 0$$

Here will be no proportionality $\quad J_{n_1} \sim \psi_1(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0)$

Used approximations

$$l^2 - m^2 + \mathrm{i}\epsilon = [l_0 + E(\mathbf{l})][l_0 - E(\mathbf{l}) + \mathrm{i}\epsilon] \approx 2E(\mathbf{l})[l_0 - E(\mathbf{l}) + \mathrm{i}\epsilon]$$

$$E(\mathbf{p}+\mathbf{q}) \equiv \sqrt{(\mathbf{p}+\mathbf{q})^2 + m^2} = E(|\mathbf{p}+\mathbf{q}|)$$

$$E(\mathbf{p}+\mathbf{q}) \approx E_p + \frac{(\mathbf{q}\mathbf{p})}{E_p} + \frac{1}{2E_p}\left[\mathbf{q}^2 - \frac{(\mathbf{q}\mathbf{p})^2}{E_p^2}\right]$$

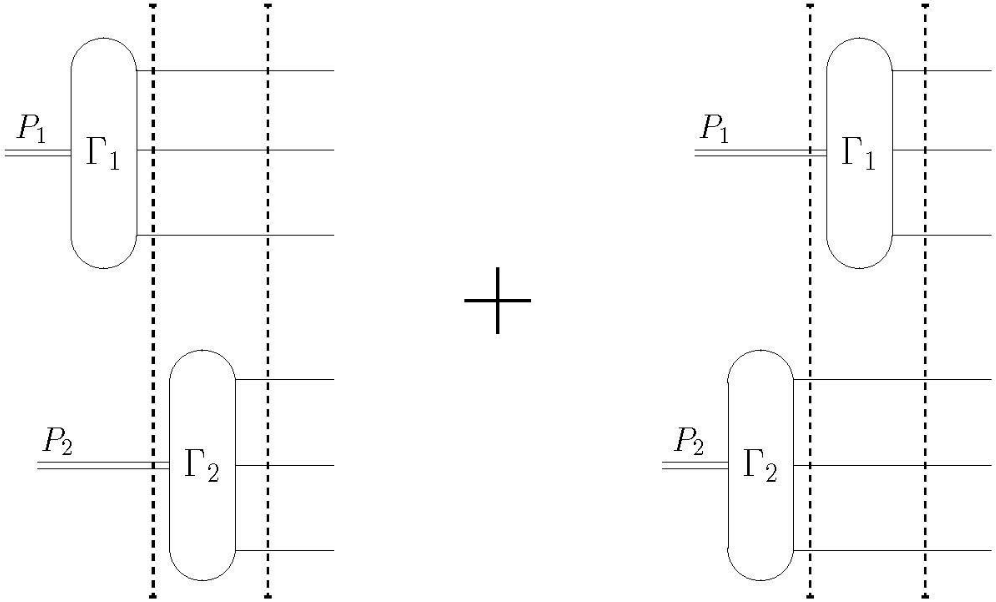
$$|\mathbf{q}| \ll E_p \equiv \sqrt{\mathbf{p}^2 + m^2}$$

$$E_p \equiv E(\mathbf{p}) \ , \ \ E_k \equiv E(\mathbf{k})$$

Heitler's (“old fashioned”) perturbation theory:

$$\boxed{\frac{1}{[\sum_i E(\mathbf{l}_i) - E_{init} - \mathrm{i}\epsilon]}}$$

Correspondence to wave functions



$$\varphi_{3\mathbf{p}}(\mathbf{q}_1^{(i)}) \equiv \frac{\Gamma_1}{[E_1 - E_1^{init}]2E_p\sqrt{3}} ,$$

$$\varphi_{-3\mathbf{p}}(\mathbf{q}_2^{(i)}) \equiv \frac{\Gamma_2}{[E_2 - E_2^{init}]2E_p\sqrt{3}}$$

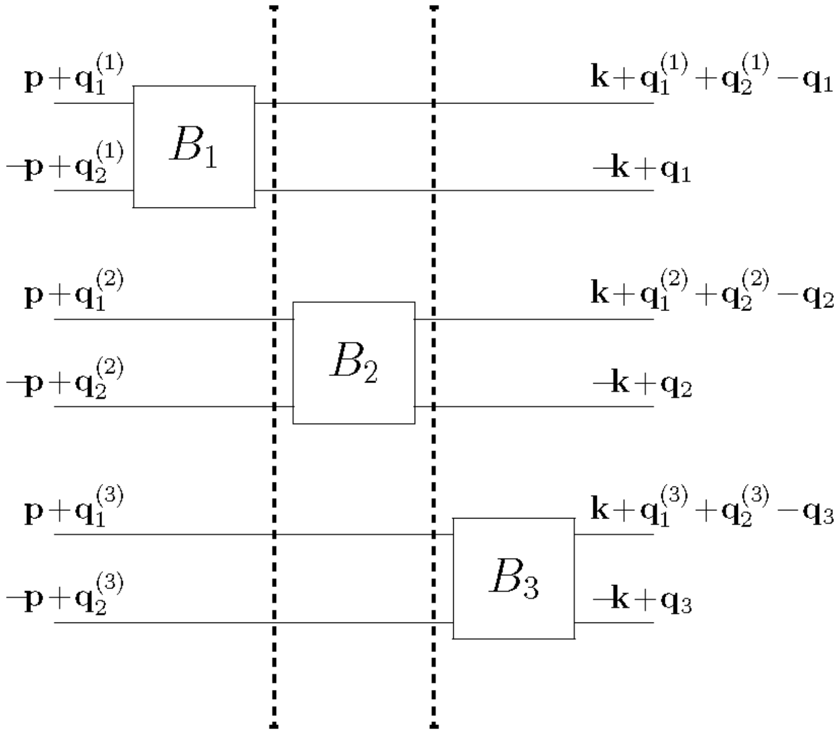
$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=2}^{n} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

$$\left\{\frac{1}{[E_1 - E_1^{init} - \mathrm{i}\epsilon]} + \frac{1}{[E_2 - E_2^{init} - \mathrm{i}\epsilon]}\right\} \frac{1}{[E_1 + E_2 - E_1^{init} + E_2^{init} - \mathrm{i}\epsilon]} =$$

$$= \frac{1}{[E_1 - E_1^{init}][E_2 - E_2^{init}]}$$

$$E_1 \equiv \sum_{i=1}^{3} E(\mathbf{p} + \mathbf{q}_1^{(i)}) \ , \quad E_2 \equiv \sum_{i=1}^{3} E(-\mathbf{p} + \mathbf{q}_2^{(i)}) = \sum_{i=1}^{3} E(\mathbf{p} - \mathbf{q}_2^{(i)})$$

Permutations of B blocks



$$\frac{E_p^2}{[-a_1-\mathrm{i}\epsilon][a_3-\mathrm{i}\epsilon]}$$

$$\boxed{a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)}-\mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-2\mathbf{q}_i)}$$

$$\sum_{i=1}^{3} a_i = 0$$

$$\frac{1}{b-\mathrm{i}\epsilon} = \frac{1}{b} + i\pi\delta(b)$$

$$\frac{-E_p^2}{[a_1+\mathrm{i}\epsilon][a_3-\mathrm{i}\epsilon]} + \frac{-E_p^2}{[a_3+\mathrm{i}\epsilon][a_1-\mathrm{i}\epsilon]} = -\frac{2E_p^2}{a_1a_3} - 2\pi^2E_p^2\delta(a_1)\delta(a_3)$$

$$\frac{1}{a_1a_3} + \frac{1}{a_1a_2} + \frac{1}{a_2a_3} = \frac{a_1+a_2+a_3}{a_1a_2a_3} = 0$$

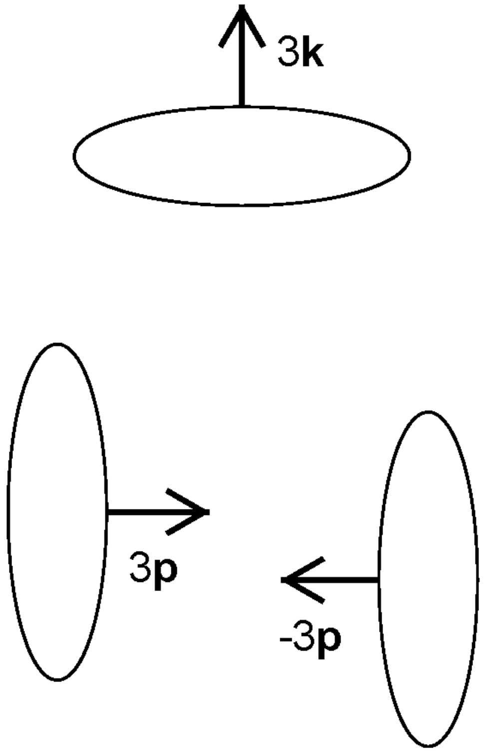
$$\boxed{D(a_1,a_2,a_3) = -2\pi^2E_p^2[\delta(a_1)\delta(a_2)+\delta(a_2)\delta(a_3)+\delta(a_1)\delta(a_3)]}$$

Amplitude of proton production in p+p → p+X at

$$x \to 1, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}/2, \quad \theta^* \sim 90^\circ$$

$$T(\mathbf{p};\mathbf{k},\mathbf{q}_i) = J(\mathbf{p};\mathbf{k},\mathbf{q}_i)\prod_{j=1}^{3} B_j(\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_i) = \int \prod_{i=1}^{2} \frac{d^3\mathbf{q}_1^{(i)}}{2E_p(2\pi)^3}\frac{d^3\mathbf{q}_2^{(i)}}{2E_p(2\pi)^3}\times$$
$$\times\varphi_{3\mathbf{p}}(\mathbf{q}_1^{(i)})\varphi_{-3\mathbf{p}}(\mathbf{q}_2^{(i)})D(a_1,a_2,a_3)\varphi^*_{3\mathbf{k}}(\mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-\mathbf{q}_i)$$



$$a_i \equiv (\mathbf{p},\ \mathbf{q}_1^{(i)}-\mathbf{q}_2^{(i)}) - (\mathbf{k},\ \mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-2\mathbf{q}_i)$$

$$D(a_1,a_2,a_3) = -2\pi^2E_p^2[\delta(a_1)\delta(a_2)+\delta(a_2)\delta(a_3)+\delta(a_1)\delta(a_3)]$$

$$\int|\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2\prod_{i=1}^{n-1}\frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

Characteristic features of the amplitude in mechanism of Coherent Quark Coalescence

- **no proportionality to the wave function value at zero**
- **smaller number of hard exchanges compared to fragmentation mechanism**
- **the convolution of wave functions (not probabilities!)**
- **MC simulations cannot be applied (just like for Gribov screening)**
- **usual factorization assumption is invalid**
- **calculation of Feynman graphs automatically leads to**

  the correct space-time picture of the process

(see papers on **Coherent Coalescence Mechanism** at nucleon level:

Braun M.A., Vechernin V.V., Yad.Fiz. 36 (1982) 614; 44 (1986) 784.

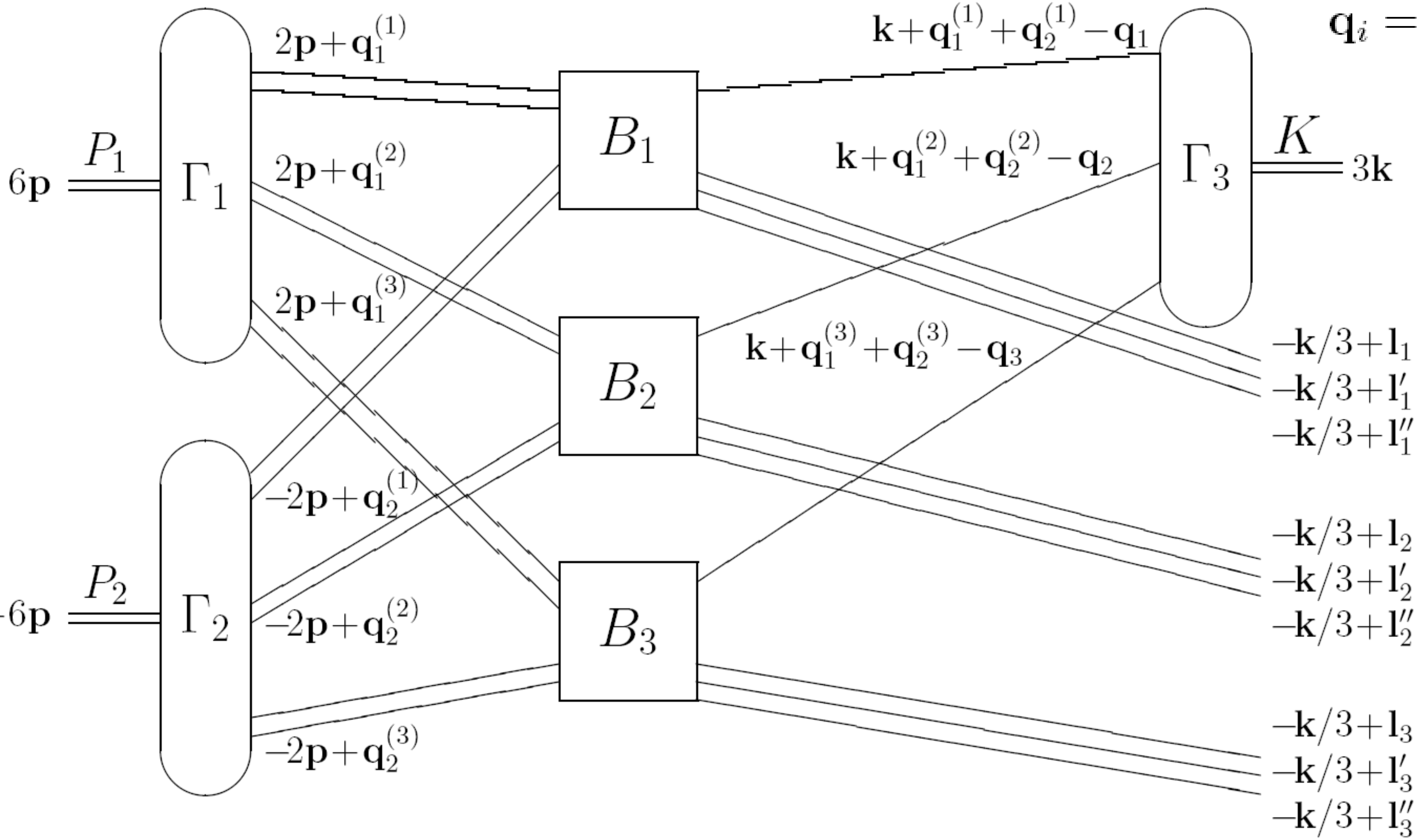
Braun M.A., Vechernin V.V., Production of fast fragments in high-energy hadron collisions with nuclei. J. Phys. G 16 (1990) 1615-1626.)

$$r_d \gg r_N \sim r_f \gg r_B \sim \frac{1}{\sqrt{s}}$$

Amplitude of proton production in d+d → p+X at

$$x \to 2, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}, \quad \theta^* \sim 90^\circ$$

$$\mathbf{q}_i = \mathbf{l}_i + \mathbf{l}'_i + \mathbf{l}''_i$$



$$\mathbf{q}_1^{(i)} = \mathbf{k}_1^{(i)} + \mathbf{k}_1'^{(i)}$$

$$\overline{\mathbf{q}}_1^{(i)} = (\mathbf{k}_1^{(i)} - \mathbf{k}_1'^{(i)})/2$$

$$\sum_{i=1}^{3} \mathbf{q}_i = \sum_{i=1}^{3} [\mathbf{l}_i + \mathbf{l}'_i + \mathbf{l}''_i] = 0$$

Amplitude of proton production in d+d → p+X at

$$x \to 2, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}, \quad \theta^* \sim 90^\circ$$

$$T(\mathbf{p};\mathbf{k},\mathbf{l}_i,\mathbf{l}'_i,\mathbf{l}''_i) = J(\mathbf{p};\mathbf{k},\mathbf{l}_i,\mathbf{l}'_i,\mathbf{l}''_i)\prod_{j=1}^{3} B_j(2\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{l}_i,\mathbf{l}'_i,\mathbf{l}''_i) = \int \prod_{i=1}^{2} \frac{d^3\mathbf{q}_1^{(i)}}{2E_p(2\pi)^3}\frac{d^3\mathbf{q}_2^{(i)}}{2E_p(2\pi)^3} \times$$

$$\times \left[\int \varphi_{6\mathbf{p}}(\mathbf{q}_1^{(i)},\overline{\mathbf{q}}_1^{(i)})\prod_{i=1}^{3}\frac{d^3\overline{\mathbf{q}}_1^{(i)}}{2E_p(2\pi)^3}\right]\left[\int \varphi_{-6\mathbf{p}}(\mathbf{q}_2^{(i)},\overline{\mathbf{q}}_2^{(i)})\prod_{i=1}^{3}\frac{d^3\overline{\mathbf{q}}_2^{(i)}}{2E_p(2\pi)^3}\right]$$

$$\times D(a_1,a_2,a_3)\,\varphi^*_{3\mathbf{k}}(\mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-\mathbf{q}_i)$$

$$D(a_1,a_2,a_3) = -2\pi^2 E_p^2[\delta(a_1)\delta(a_2)+\delta(a_2)\delta(a_3)+\delta(a_1)\delta(a_3)]$$

$$a_i \equiv (2\mathbf{p},\,\mathbf{q}_1^{(i)}-\mathbf{q}_2^{(i)}) - (\mathbf{k},\,\mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-2\mathbf{q}_i) \qquad \mathbf{q}_i = \mathbf{l}_i+\mathbf{l}'_i+\mathbf{l}''_i$$

$$\mathbf{q}_1^{(i)} = \mathbf{k}_1^{(i)} + \mathbf{k}_1^{\prime(i)}$$

$$\overline{\mathbf{q}}_1^{(i)} = (\mathbf{k}_1^{(i)} - \mathbf{k}_1^{\prime(i)})/2$$

$$\sum_{i=1}^{3}\mathbf{q}_i = \sum_{i=1}^{3}[\mathbf{l}_i+\mathbf{l}'_i+\mathbf{l}''_i] = 0$$

Summary

- Mechanism of Coherent Quark Coalescence (recombination, fusion) leads to the increase of proton production in considered region, compared to the mechanism of fragmentation of one quark into proton, which was used for description of pion production, due to fewer number of hard exchanges.

- it is difficult to calculate, as the usual factorization assumption is invalid, it's expressed through the superposition of amplitudes (not probabilities!), MC simulations cannot be applied (just like for Gribov screening).

Future studies

Inclusion of diquarks in the scheme.

V.T. Kim, Modern Phys. Lett. A 3 (1988) 909

Two options:

- diquark fragmentation or
- coherent coalescence of quark and diquark

M.A. Braun, V.V. Vechernin, Nucl.Phys.B 427 (1994) 614

The work was supported by the Russian Science Foundation grant 23-12-00042.

Backup slides

Flucton-flucton interaction in dd collisions

- **It can be studied only in new cumulative region of large transverse momenta in mid-rapidity region at NICA**
  (not in the traditional cumulative region of fragmentation of one of the nuclei).
- **There are no additional interactions in dd collision, compared with collisions of heavier nuclei, if both deuterons are in flucton configuration at the moment of collision.**
- **Higher frequency of dd collisions that can be recorded by the SPD, compared to the slower MPD (important for a registration of rare cumulative events).**
- **The studies in new cumulative region becomes possible due to the moderate energy of the NICA collider and is completely impossible at ultrahigh energies of the RHIC and LHC.**

Calculation of Phase Volume

$$\tau_p = (2\pi)^{4-3p} \int \prod_{i=1}^{p} \frac{d^3 \mathbf{l}'_i}{2 l_{i0}} \delta^{(3)}\left(\sum_{i=1}^{p} \mathbf{l}'_i\right) \times$$
$$\times \delta\left(\sum_{i=1}^{p}\left[\sqrt{(\mathbf{k}/p + \mathbf{l}'_i)^2 + m^2} - \sqrt{(\mathbf{k}/p)^2 + m^2}\right] - \Delta\right)$$

$$\mathbf{l}'_i = -\mathbf{k}/p + \mathbf{l}_i$$

$$l_{i0} = \sqrt{(\mathbf{k}/p + \mathbf{l}'_i)^2 + m^2}$$

$$p = n_1 + n_2 - 1$$

$$\Delta = A\sqrt{s} - \sqrt{k^2 + m^2} - \sqrt{k^2 + p^2 m^2}$$

$k \to k_{max}$ => $\Delta \to 0$

$$\sqrt{k_{max}^2 + m^2} + \sqrt{k_{max}^2 + (p\,m)^2} = A\sqrt{s}$$

$$\tau_p = \frac{1}{2^p m^{p-1} p^{\frac{3}{2}}} \frac{\left(\frac{E_p}{2\pi}\Delta\right)^{\frac{3}{2}p - \frac{5}{2}}}{\left(\frac{3}{2}p - \frac{5}{2}\right)!}$$

$$E_p \equiv \sqrt{k^2/p^2 + m^2}$$

Relation with Cumulative Number

$$x\sqrt{s} = \sqrt{k^2 + m^2} + \sqrt{k^2 + [p(x)\, m]^2}\ .$$

$$p(x) = n_1 + n_2 - 1 = 3A_1 + 3A_2 - 1 = 6A - 1 = 6x - 1.$$

$$\Delta = (A - x)[\sqrt{s} + O(1/\sqrt{s})]$$

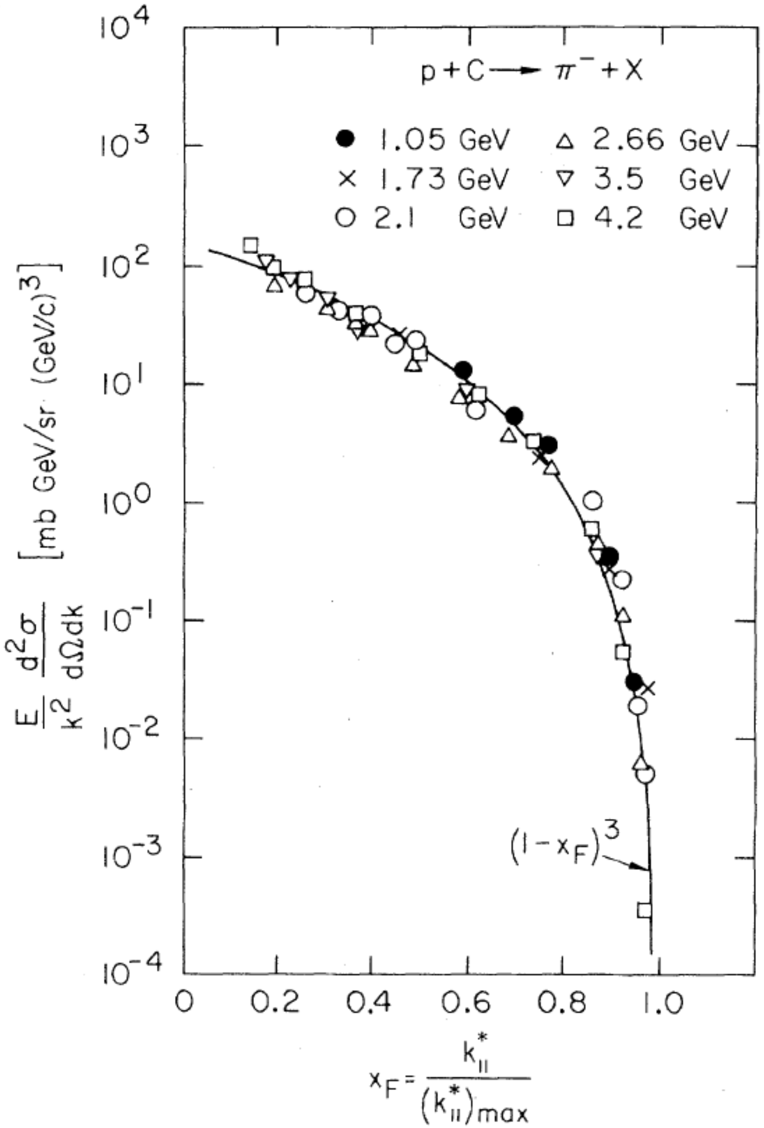
$$\tau_p = \frac{1}{2^{4p-5} p^{3p/2-1} m^{p-1}} \frac{\left[\frac{A}{\pi} s(A-x)\right]^{\frac{3}{2}p-\frac{5}{2}}}{\left(\frac{3}{2}p - \frac{5}{2}\right)!}$$

$$p = p(A)$$

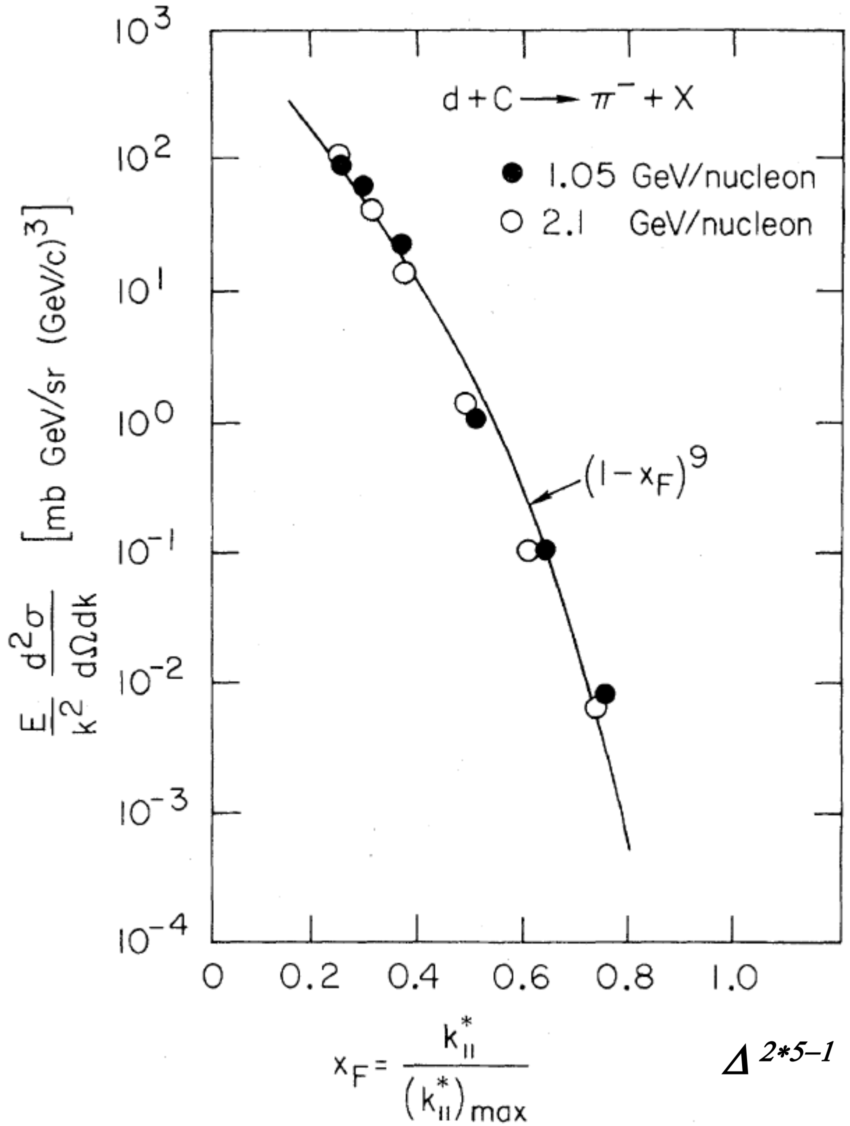
$$\boxed{I(x) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p-\frac{5}{2}}}{(m^2 R^3)^{p-1} s^{(p+3)/2}}}$$

two (!)
small parameters:
$m/\sqrt{s} \ll 1$
$A - x \ll 1$

Schmidt I.A., Blankenbecler R. Phys.Rev. ***D15*** *(1977) 3321*



Δ^{2*2-1}



Δ^{2*5-1}

Threshold behaviour of ***inclusive cross sections*** (quark counting rules) at |t|<<s.

The experimental points from J. Papp et al., Phys.Rev.Lett. 34, 601 (1975).

Quark counting rules for *elastic and quasi elastic* reactions with nuclei

Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett. Nuovo Cimento 7 (1973) 719
Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153; Phys.Rev. D11 (1975) 1309
Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269

$s \to \infty, \quad t/s$ fixed

$$(d\sigma/dt)_{\pi p \to \pi p} \sim s^{-8}, \ (d\sigma/dt)_{pp \to pp} \sim s^{-10}, \ (d\sigma/dt)_{\gamma p \to \pi p} \sim s^{-7}, \ (d\sigma/dt)_{\gamma p \to \gamma p} \sim s^{-6}$$

$\sim s^{-n}$ $\qquad$ $A+B\text{->}C+D$ $\qquad$ $n=n_A+n_B+n_C+n_D-2$ $\qquad$ $n_p=3 \ \ n_\pi=2 \ \ n_\gamma=1$

$$\frac{d\sigma}{dt}(A+B \to C+D) \to \frac{1}{t^{N-2}} f(t/s) \qquad N=n_A+n_B+n_C+n_D$$

Yu.L. Dokshitzer, QCD Phenomenology,
Lectures at the CERN–Dubna School, Pylos, August 2002

the deuteron break-up by a photon, $\gamma + D \to p + n$

$$\frac{d\sigma}{dt} = \frac{f(\Theta)}{s^{K-2}}; \qquad \frac{t}{s} = \text{const}, \qquad K-2=1+6+3+3-2=11$$

For light nuclei:

Yu.N. Uzikov, Indication of Asymptotic Scaling in the Reactions
$dd\text{->}p^3H$, $dd\text{->}n^3He$ and $pd\text{->}pd$, JETP Letters 81 (2005) 303.

$\sim s^{-22}$ (6+6+3+9-2=22) and $\sim s^{-16}$ (3+6+3+6-2=16)

Incorporating diquarks

V.T. Kim, Diquarks and Dynamics of Large P(T) Baryon Production, Mod.Phys.Lett.A 3 (1988) 909.

p/π^+ - ratio explanation, using that the diquark distribution function is harder: $(1-x)^1$ vs $(1-x)^3$ for quarks [$(1-x)^{2p-1}$].

Yu.L. Dokshitzer, QCD Phenomenology,
Lectures at the CERN–Dubna School, Pylos, August 2002

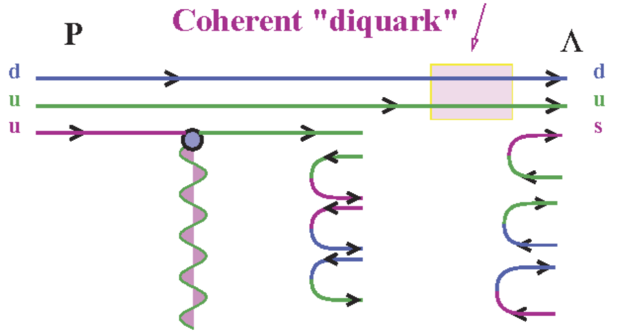


Fig. 4a: Gluon exchange produces a leading baryon.

M.A. Braun, V.V. Vechernin, Nuclear Structure Functions and Particle Production in the Cumulative Region in the Parton Model, Nucl.Phys. B427 (1994) 614

Can string junction carries the baryon number?

L. Montanet, G. C. Rossi, and G. Veneziano, “Baryonium Physics,”
Phys. Rept. 63, 149–222 (1980).

D. Kharzeev, “Can gluons trace baryon number?”
Phys.Lett. B 378, 238–246 (1996), arXiv:nucl-th/9602027.
Can be verified experimentally by studing of
baryon stopping in central pp and AA collisions.

Yu.M. Shabelski,
String Junction and Diffusion of Baryon Charge in Multiparticle Production Processes,
arXiv: 0705.0947 [hep-ph], (2007).
F. Bopp, Yu.M. Shabelski,
String junction effects for forward and central baryon production in hadron-nucleus collisions
Eur.Phys.J.A 28 (2006) 237-243

G.Pihan, A.Monnai, B.Schenke, Chun Shen,
Unveiling baryon charge carriers through charge stopping in isobar collisions
arXiv:2405.19439v1 [nucl-th] (2024).

Connection with diquarks:
Now B=1 corresponds to diquark