



Spin observables in $pd \rightarrow pd$ and $dd \rightarrow npd$ processes at SPD NICA energies

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CONTENT

- Motivation:
Spin-dependent pN-elastic scattering amplitudes are necessary for theoretical interpretation of nuclear spin observables, not yet derived from QCD theory .
- Phenomenological models (Regge, Regge- eikonal) for spin-dependent pN elastic scattering at SPD energies.
- Glauber spin-dependent theory of pd-elastic scattering and pN-amplitudes.
- Relations between spin observables for $dd \rightarrow n+p+d$ and $pd \rightarrow pd$.
- Results of MC simulations for $dd \rightarrow npd$ with separation of $pd \rightarrow pd$
- Summary

pN ELASTIC SCATTERING

NN forces is a basis of nuclear and hadronic physics.

NN-> NN data on spin dependent pN- amplitudes are noncomplete for pp at $T > 3$ GeV and scarce for pn at $T > 1.2$ GeV .

Important TASK: Measurement @ test of spin **amplitudes of NN elastic scattering in soft and hard NN- collisions**.

$$\phi_1(s, t) = \langle + + | M | + + \rangle,$$

$$\phi_2(s, t) = \langle + + | M | - - \rangle,$$

$$\phi_3(s, t) = \langle + - | M | + - \rangle,$$

$$\phi_4(s, t) = \langle + - | M | - + \rangle,$$

$$\phi_5(s, t) = \langle + + | M | + - \rangle.$$

$$\frac{d\sigma}{dt} = \frac{2\pi}{s^2} \{ |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2 \}.$$

$$A_N \frac{d\sigma}{dt} = -\frac{4\pi}{s^2} \text{Im} \{ \phi_5^* (\phi_1 + \phi_2 + \phi_3 - \phi_4) \},$$

$$A_{NN} \frac{d\sigma}{dt} = \frac{4\pi}{s^2} \{ 2|\phi_5|^2 + \text{Re}(\phi_1^* \phi_2 - \phi_3^* \phi_4) \},$$

>10 Observables are
required for complete
polarization experiment

01.07.2025 Amplitudes: 5 – pp, 6 – pn, TV gives additional terms

Phenomenological models of NN-elastic amplitudes

NN helicity amplitudes:

SAID data-base: Arndt R.A. et al. PRC 76 (2007) 025209; $\sqrt{s_{NN}} = 1.9 - 2.4 GeV$

Models:

- **A. Sibirtsev** et al., Eur. Phys. J. A 45 (2010) 357; arXiv:0911.4637 [hep-ph] (**Regge-parametrization for pp only**); $\sqrt{s_{NN}} = 2.5 - 15 GeV$

Isospin and G-parity:

$$\begin{aligned}\phi(pp) &= -\phi_{\omega} - \phi_{\rho} + \phi_{f_2} + \phi_{a_2} + \phi_P, \\ \phi(pn) &= -\phi_{\omega} + \phi_{\rho} + \phi_{f_2} - \phi_{a_2} + \phi_P.\end{aligned}$$

- **W.P. Ford, J. van Orden**, Phys. Rev. C 87 (2013) $\sqrt{s_{pN}} = 2.5 - 3.5 GeV$ (pp, pn; Regge);
- **O.V. Selyugin**, Symmetry., 13 N2 (2021) 164; (**Regge –eikonal**);
Phys.Rev.D 110 (2024) 11, 114028 ; e-Print: [2407.01311](https://arxiv.org/abs/2407.01311) [hep-ph] $\sqrt{s_{NN}} = 5 - 25 GeV$

Glauber spin-dependent theory of *pd-pd* and test of pN amplitudes

pd-pd: The simplest process with both **pp** and pn-amplitudes involved. **dd-dd** elastic is also suitable, but much more complicated, spin-dependent Glauber formalism is not yet developed.

M.N. Platonova, V.I. Kukulin, Phys.Rev.C 81 (2010) 014004, Phys.Rev.C 94 (2016) 6, 069902; (erratum) e-Print: 1612.08694[nucl-th].

M.N. Platonova, V.I.Kukulin, Eur.Phys.J.A56 (2020) 5, 132; e-Print: 1910.05722[nucl-th]

A.A. Temerbayev, Yu.N. Uzikov, Yad. Fiz. 78(2015)38; Bull. Rus. Ac. Sci. v.80 №3 (2016) 242. [Madison ref. frame](#)

Elastic $pd \rightarrow pd$ transitions

$$\begin{aligned}\hat{M}(\mathbf{q}, \mathbf{s}) = & \exp\left(\frac{1}{2}i\mathbf{q} \cdot \mathbf{s}\right)M_{pp}(\mathbf{q}) + \exp\left(-\frac{1}{2}i\mathbf{q} \cdot \mathbf{s}\right)M_{pn}(\mathbf{q}) + && \text{Single pN-scattering} \\ & + \frac{i}{2\pi^{3/2}} \int \exp(i\mathbf{q}' \cdot \mathbf{s}) \left[M_{pp}(\mathbf{q}_1)M_{pn}(\mathbf{q}_2) + p \leftrightarrow n \right] d^2\mathbf{q}'. && \text{Double scattering}\end{aligned}$$

On-shell elastic pN scattering amplitude (**T-even, P-even**)

$$\begin{aligned}M_{pN} = & A_N + (C_N\boldsymbol{\sigma}_1 + C'_N\boldsymbol{\sigma}_2) \cdot \hat{\mathbf{n}} + B_N(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{k}})(\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{k}}) + \\ & + (G_N - H_N)(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{n}})(\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{n}}) + (G_N + H_N)(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{q}})(\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{q}})\end{aligned}$$

GENERAL SPIN STRUCTURE OF THE pd-pd AMPLITUDES AND SPIN OBSERVABLES

A.A. Temerbayev, Yu. N. U. Yad. Fiz. 78 (2015) 38; Bull. Rus, Ac. Sci. v.80 №3 (2016) 242.

Yu.N. Uzikov, A. Bazarova, A.A. Temerbayev,

Physics of Particles and Nuclei, 2022, Vol. 53, No. 2, pp. 419–425.

$$\langle p' \mu', d' \lambda' | T | p \mu, d \lambda \rangle = \varphi_{\mu}^+ \cdot e_{\beta}^{(\lambda')*} T_{\beta\alpha}(\mathbf{p}, \mathbf{p}', \boldsymbol{\sigma}) e_{\alpha}^{(\lambda)} \varphi_{\mu}, \quad (1)$$

$$\begin{aligned} T_{xx} &= M_1 + M_2 \sigma_y & T_{xy} &= M_7 \sigma_z + M_8 \sigma_x & T_{xz} &= M_9 + M_{10} \sigma_y \\ T_{yx} &= M_{13} \sigma_z + M_{14} \sigma_x & T_{yy} &= M_3 + M_4 \sigma_y & T_{yz} &= M_{11} \sigma_x + M_{12} \sigma_z \\ T_{zx} &= M_{15} + M_{16} \sigma_y & T_{zy} &= M_{17} \sigma_x + M_{18} \sigma_z & T_{zz} &= M_5 + M_6 \sigma_y, \end{aligned}$$

12 independent spin amplitudes (i=1,...,12) M_i for P-and T-invariance included.

Any spin observables can be calculated for pd-pd using full set of M_i

Some spin observables for pd-pd

$$\frac{d\sigma}{dt} = \frac{1}{6} \text{Tr}MM^+, \quad \text{Tr}MM^+ = 2 \sum_{i=1}^{18} |M_i|^2, \quad (3)$$

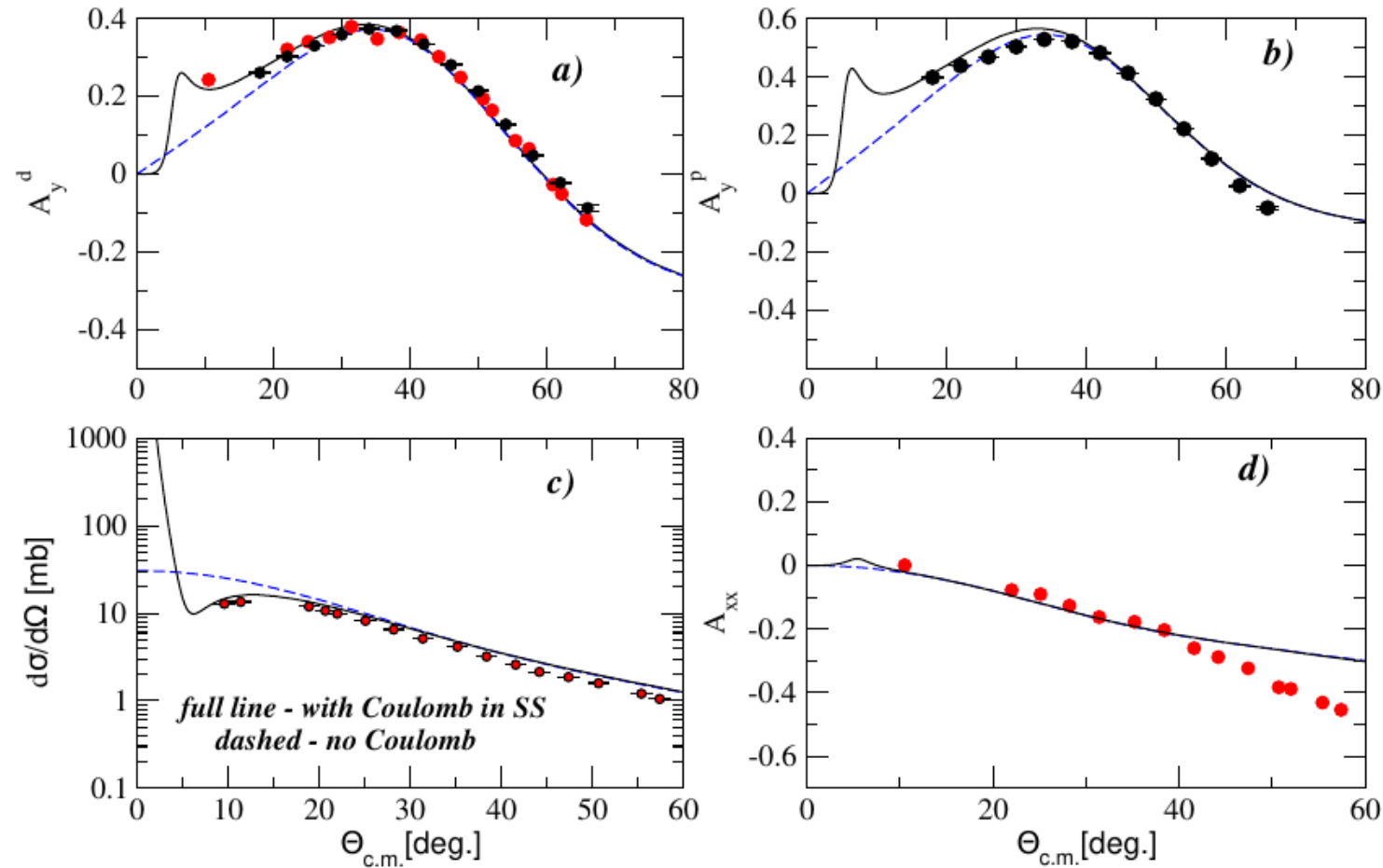
$$\begin{aligned} A_y^d &= \text{Tr}MS_y M^+ / \text{Tr}MM^+ = -\frac{2}{\sum_{i=1}^{18} |M_i|^2} \text{Im}(M_1 M_9^* \\ &+ M_2 M_{10}^* + M_{13} M_{12}^* + M_{14} M_{11}^* + M_{15} M_5^* + M_{16} M_6^*), \\ A_y^p &= \text{Tr}M\sigma_y M^+ / \text{Tr}MM^+ = \frac{2}{\sum_{i=1}^{18} |M_i|^2} [\text{Re}(M_1 M_2^* \\ &+ M_9 M_{10}^* + M_3 M_4^* + M_{15} M_{16}^* + M_5 M_6^*) \\ &- \text{Im}(M_8 M_7^* + M_{14} M_{13}^* + M_{11} M_{12}^* + M_{17} M_{18}^*)], \\ A_{yy} &= \text{Tr}MP_{yy} M^+ / \text{Tr}MM^+ = 1 - \frac{3}{\sum_{i=1}^{18} |M_i|^2} \\ &\times (|M_3|^2 + |M_4|^2 + |M_7|^2 + |M_8|^2 + |M_{17}|^2 + |M_{18}|^2), \end{aligned}$$

$$\begin{aligned} A_{xx} &= \text{Tr}MP_{xx} M^+ / \text{Tr}MM^+ = 1 - \frac{3}{\sum_{i=1}^{18} |M_i|^2} \\ &\times (|M_1|^2 + |M_2|^2 + |M_{13}|^2 + |M_{14}|^2 + |M_{15}|^2 + |M_{16}|^2), \\ C_{y,y} &= \text{Tr}MS_y \sigma_y M^+ / \text{Tr}MM^+ = -\frac{2}{\sum_{i=1}^{18} |M_i|^2} \\ &\times [\text{Im}(M_2 M_9^* + M_1 M_{10}^* + M_{16} M_5^* + M_{15} M_6^*) \\ &+ \text{Re}(M_{14} M_{12}^* - M_{13} M_{11}^*)], \\ C_{x,x} &= \text{Tr}MS_x \sigma_x M^+ / \text{Tr}MM^+ \\ &= -\frac{2}{\sum_{i=1}^{18} |M_i|^2} [\text{Im}(M_8 M_9^* + M_3 M_{11}^* + M_{17} M_5^*) \\ &+ \text{Re}(M_7 M_{10}^* - M_4 M_{12}^* + M_{18} M_6^*)], \\ C_{xx,y} &= \text{Tr}M_{xx} \sigma_y M^+ / \text{Tr}MM^+ = A_y^p - \frac{6}{\sum_{i=1}^{18} |M_i|^2} \\ &\times [\text{Re}(M_2 M_1^* + M_{16} M_{15}^*) - \text{Im}(M_{14} M_{13}^*), \end{aligned}$$

Test calculations: *pd* elastic scattering at 135 MeV

A.A. Temerbavev, Yu.N.Uzikov, *Yad. Fiz.* **78** (2015) 38

Glauber is comparable with results of Faddeev calculations



Data: K. Sekiguchi et al. PRC (2002); B. von Przewoski et al. PRC (2006)

See also Faddeev calculations: A.Deltuva, A.C. Fonseca, P.U. Sauer, PRC 71 (2005) 054005.

A.A. Temerbayev, Yu.N. Uzikov, Yad. Fiz, 78 (2015) 38

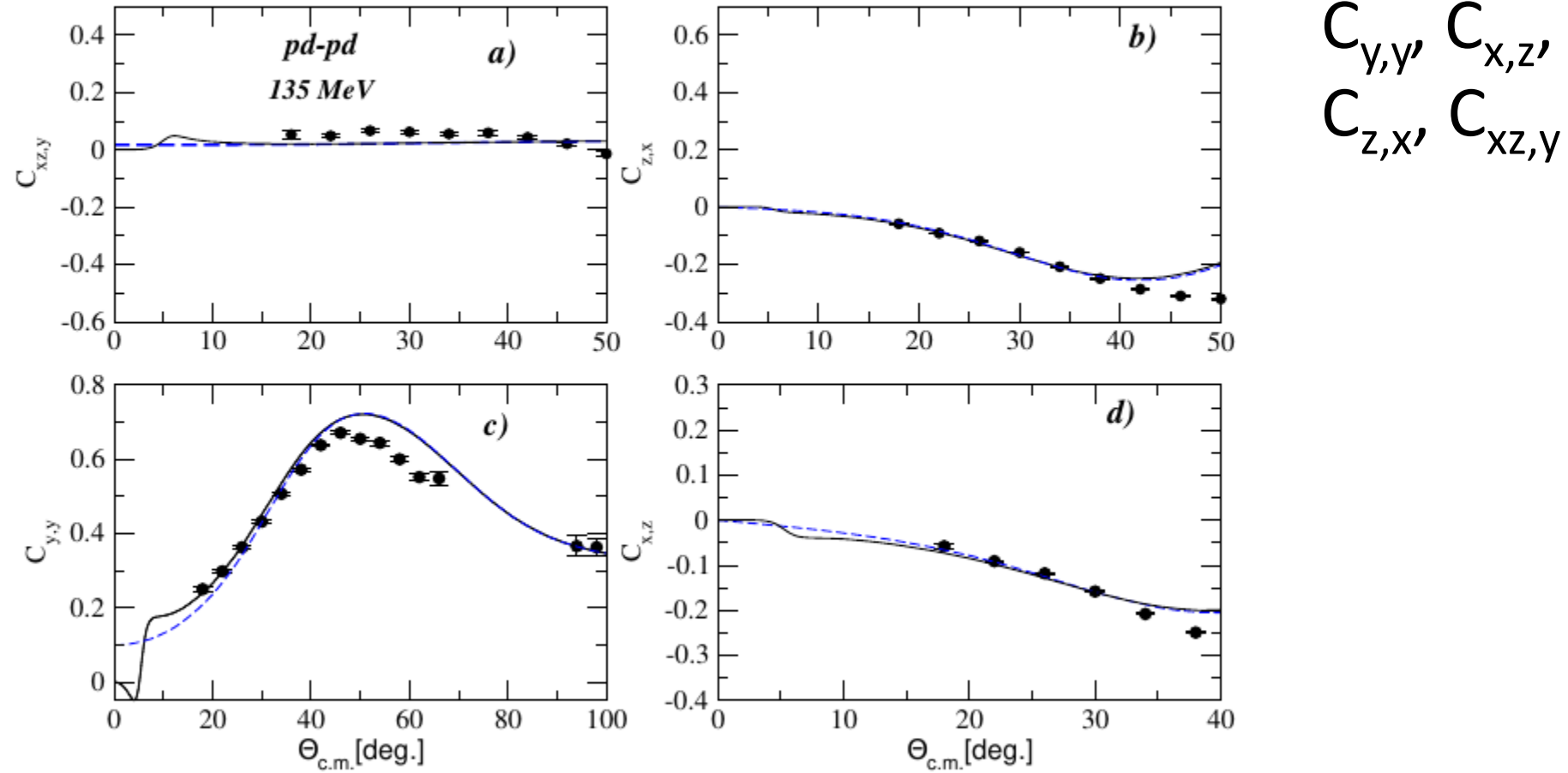
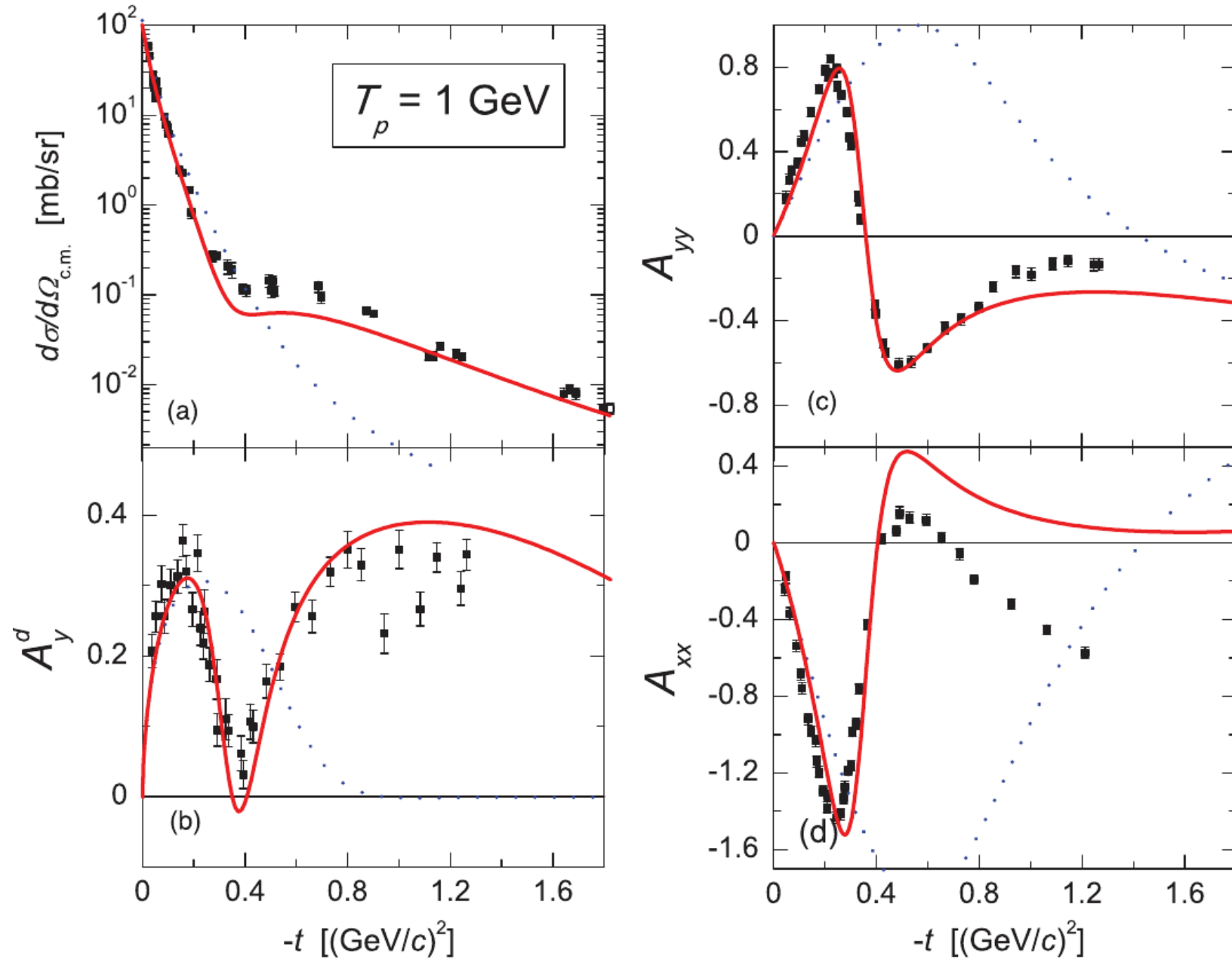


Figure 1: Spin correlation coefficients $C_{xz,y}$ (a), $C_{z,x}$ (b), $C_{y,y}$ (c), $C_{x,z}$ (d) at 135 MeV versus the c.m.s. scattering angle calculated within the modified Glauber model [15] without (dashed lines) and with (full) Coulomb included in comparison with the data from [22].



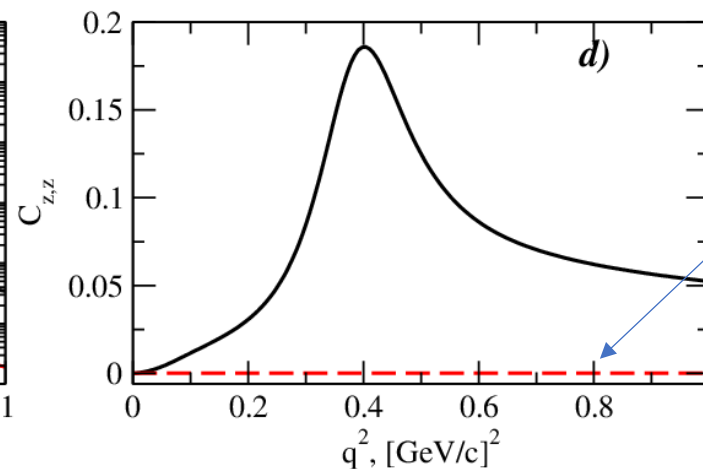
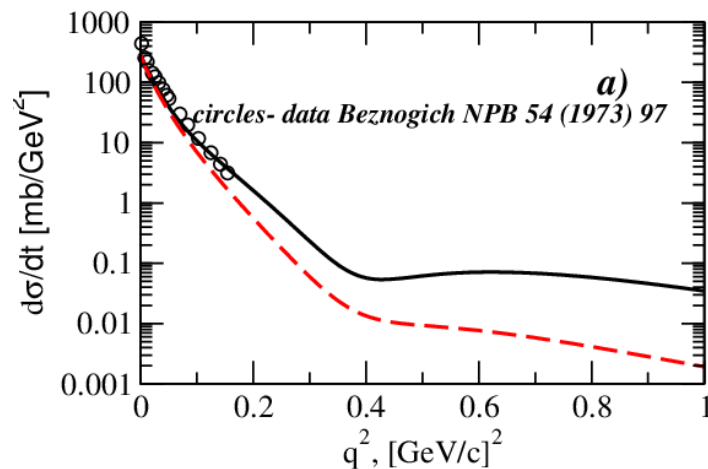
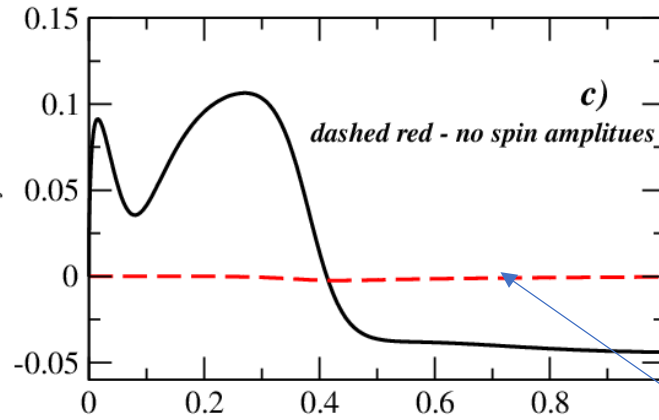
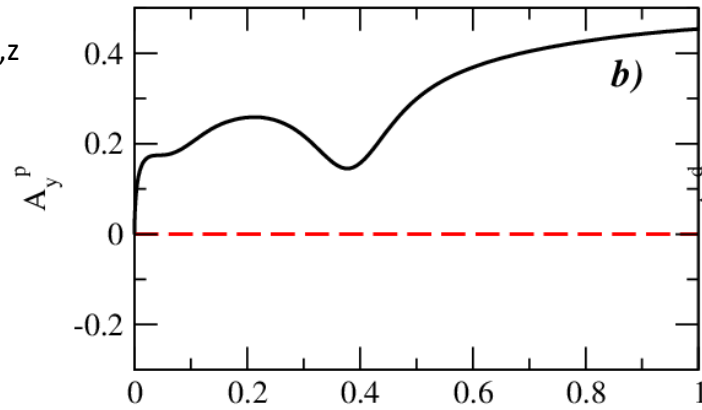
pd elastic scattering at SPD energies in Glauber model

pd- elastic

High sensitivity to spins:

full black - $P_L=20.4$ GeV/c with Sibirtsev amplitudes

A_y^p A_y^d $C_{z,z}$

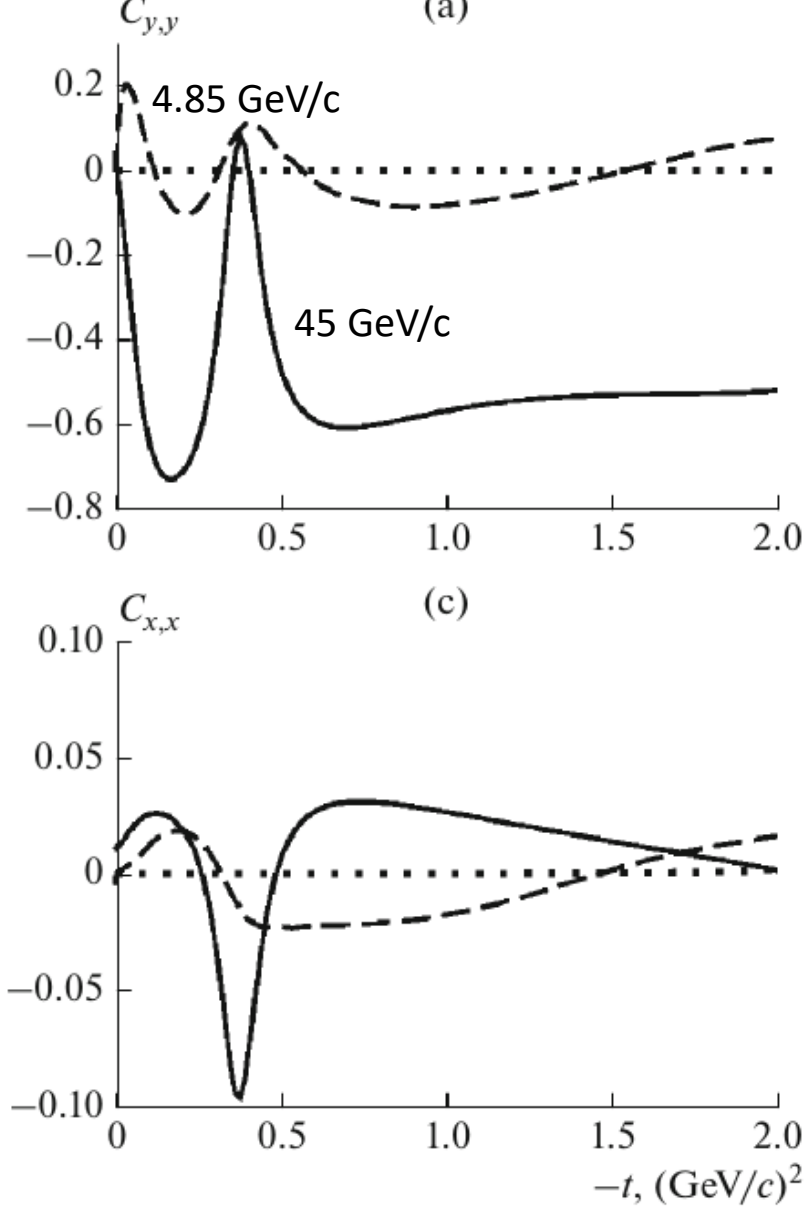


Yu.N.U., A.Bazarova, A. Temerbayev, Phys.Part. Nucl.53 (2022)419;

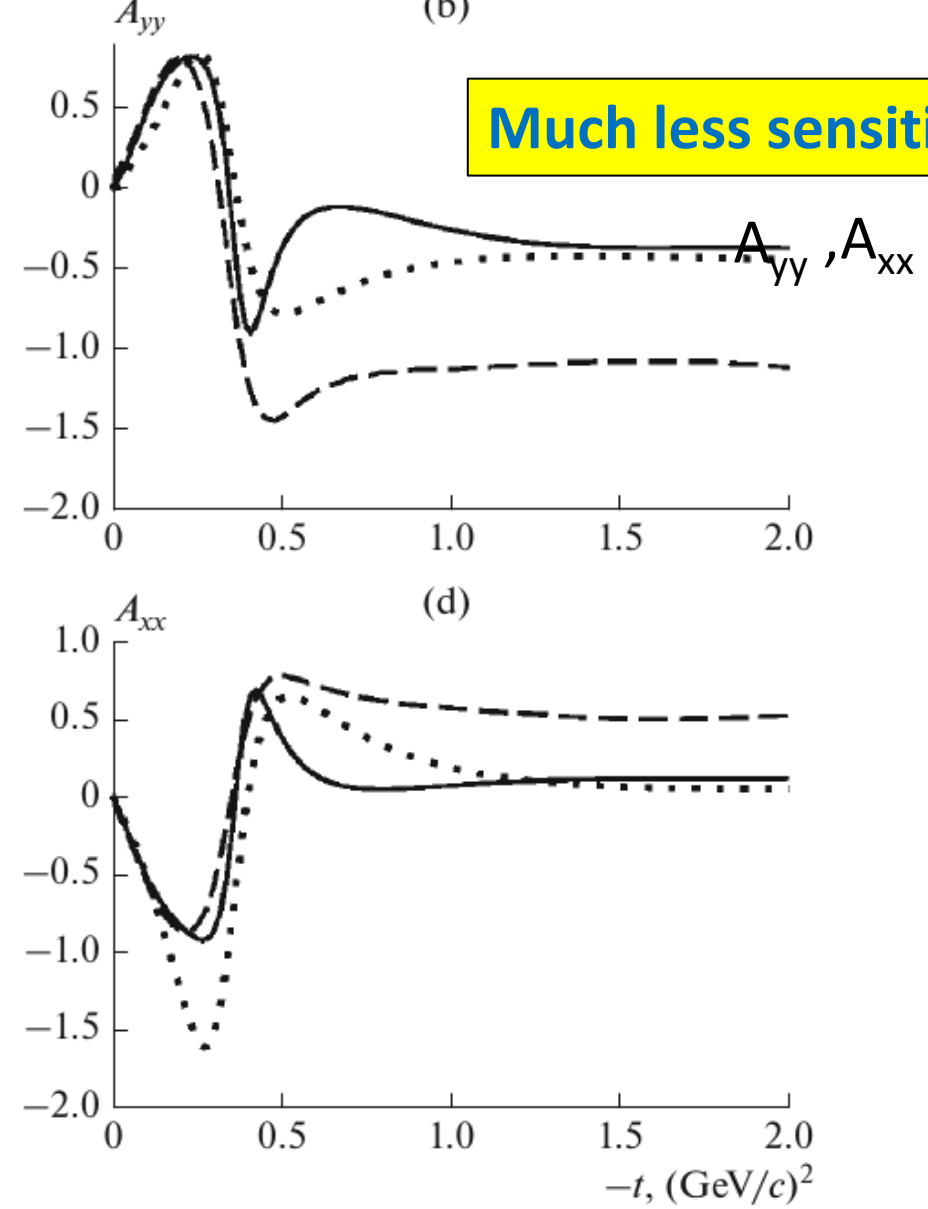
pN- amplitudes from A. Sibirtsev et al. EPJ A (2010)

Highly sensitive:

$C_{y,y}$, $C_{x,x}$



Much less sensitive:



Dotted : spin-independent pN-amp.

Fig. 4. Results for spin-dependent pd observables. Same description of curves as in Fig. 3. The dotted lines are results where the spin-dependent pN amplitudes have been omitted in the calculation.

Search for T-invariance violation

in double polarized $pd, {}^3\text{He}d$, dd - collisions and pN -elastic amplitudes

BAU problem: $R_{\text{exp}} = 6.7 \cdot 10^{-10}$, $R_{\text{SM}} \sim 10^{-19}$

CP violation beyond the SM (or T-violation under CPT-inv.)

L. B. Okun, “Note concerning CP parity,” *Sov. J. Nucl. Phys.* **1**, 670 (1965).

TVPC NN forces:

$n^{165}\text{Ho}$, P.R. Huffman et al. *PRC* **55** (1997)

No data on TVPC effect at SPD NICA energies

T-even P-even pN single spin-flip amplitude ϕ_5 and also ϕ_1 , ϕ_3 are necessary to extract the null-test signal of T-violation

Y. Uzikov, M. Platonova, A. Kornev *et al.*, *Int. Jour. Mod. Phys. E* <https://doi.org/10.1142/S0218301324410039> (2024)

Y. N. Uzikov and A. Temerbayev, *Phys. Rev. C* **92**, 014002 (2015), arXiv: 1506.08303

Y. N. Uzikov and J. Haidenbauer, *Phys. Rev. C* **94**, 035501 (2016), arXiv: 1607.04409

Y. N. Uzikov and M. N. Platonova, *JETP Lett.* **118**, 785 (2023), arXiv: 2311.10841

M.N. Platonova, Yu. N. Uzikov, *Chin. Phys. C* **49**, N3 (2025) 034108

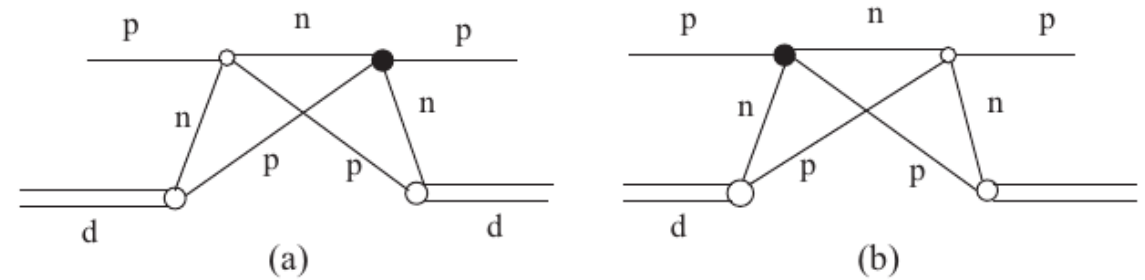
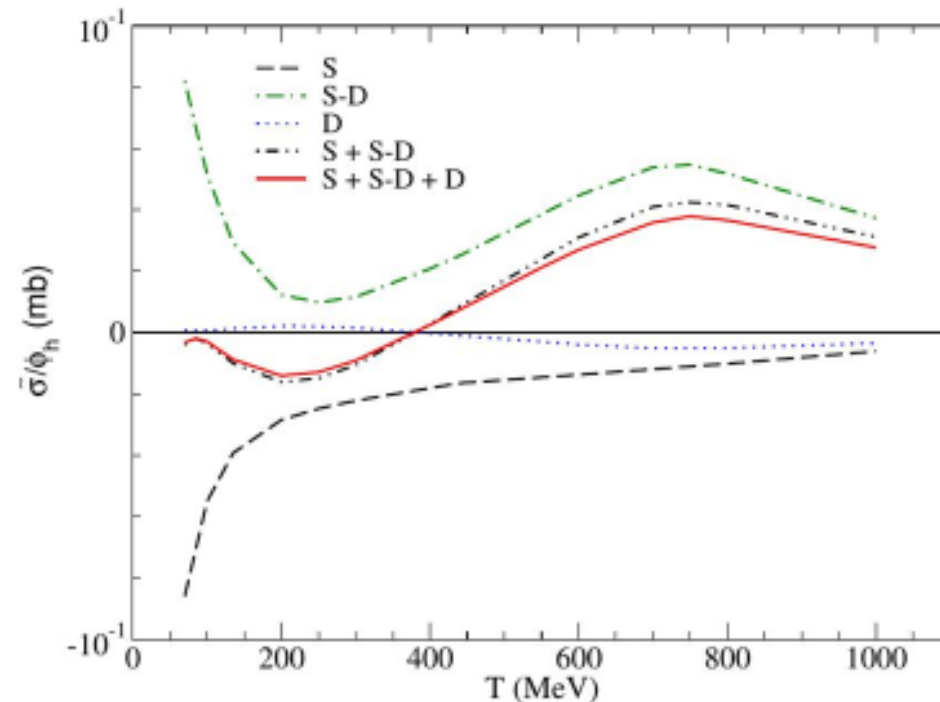
$p^\uparrow d^\uparrow$

$$\sigma_{tot} = \underbrace{\sigma_0 + \sigma_1 \mathbf{p}^p \cdot \mathbf{P}^d + \sigma_2 (\mathbf{p}^p \cdot \hat{\mathbf{k}})(\mathbf{P}^d \cdot \hat{\mathbf{k}}) + \sigma_3 P_{zz}}_{T\text{-even}, P\text{-even}} + \underbrace{\tilde{\sigma}_{tvpc} p_y^p P_{xz}^d}_{T\text{-odd}, P\text{-even}}$$

$$C' \approx i\phi_5 + iq/2m(\phi_1 + \phi_3)/2$$

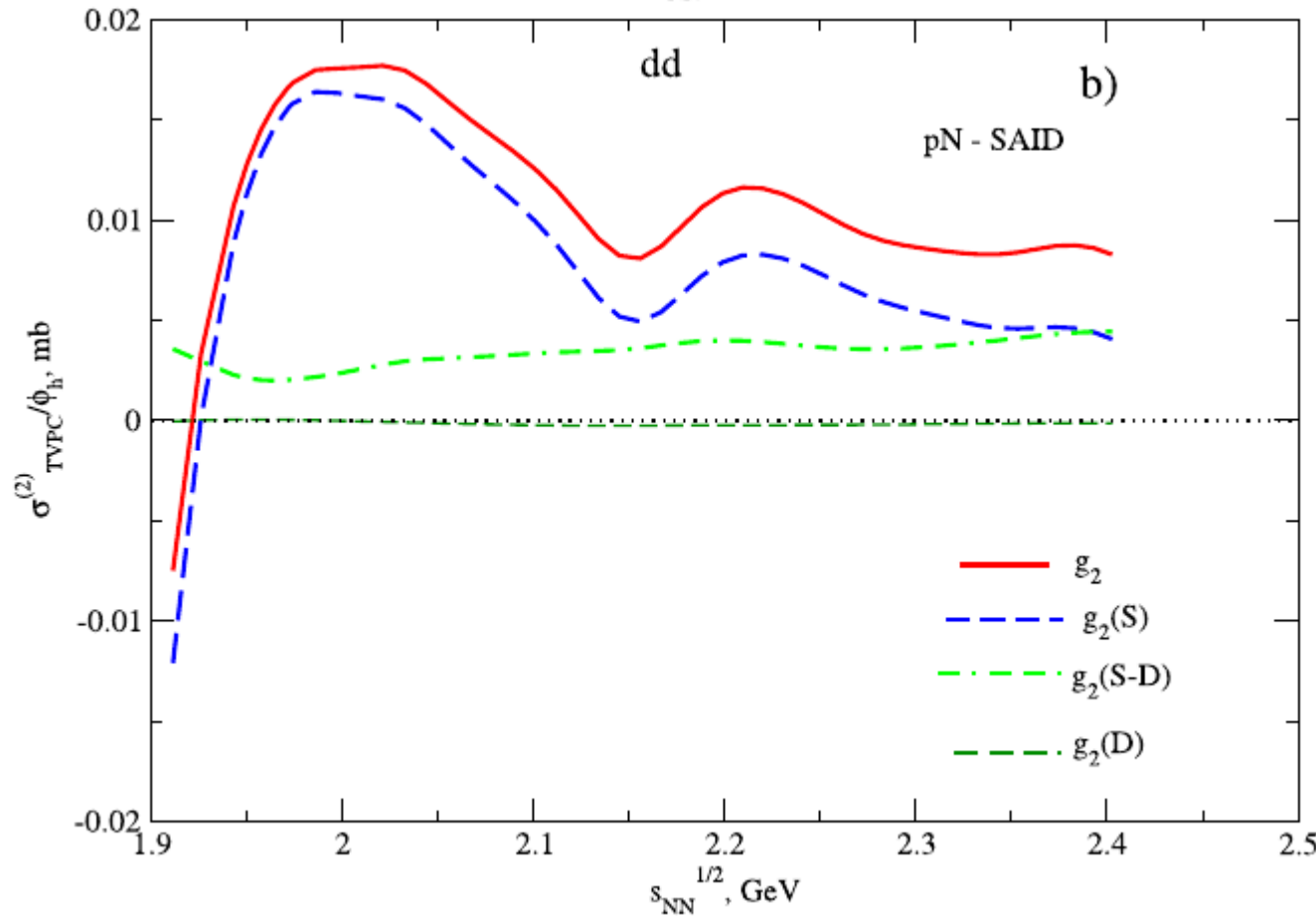
Yu.N.U., A.A. Temerbayev, PRC 92 (2015) 014002;
Yu.N.U., J. Haidenabuer, PRC 94 (2016) 035501.

TVPC. The S- and D- wave contributions-I



Null-test signal:

$$\begin{aligned} \tilde{g} = & \frac{i}{4\pi m_p} \int_0^\infty dq q^2 \left[S_0^{(0)}(q) - \sqrt{8} S_2^{(1)}(q) - 4 S_0^{(2)}(q) \right. \\ & \left. + \sqrt{2} \frac{4}{3} S_2^{(2)}(q) + 9 S_1^{(2)}(q) \right] [-C'_n(q) h_p + C'_p(q) (g_n - h_n)] \end{aligned}$$



Null-test signal in dd:

$$g_2 = \frac{i}{2\pi m} \int_0^\infty dq q^2 [Z_0 + Z(q)] \zeta(q) h_N(q) (C'_n(q) + C'_p(q))$$

$$M_N = A_N + C_N \boldsymbol{\sigma}_p \cdot \hat{\mathbf{n}} + C'_N \boldsymbol{\sigma}_N \cdot \hat{\mathbf{n}}, \\ + B_N (\boldsymbol{\sigma}_p \cdot \hat{\mathbf{k}}) (\boldsymbol{\sigma}_N \cdot \hat{\mathbf{k}}), \\ + (G_N + H_N) (\boldsymbol{\sigma}_p \cdot \hat{\mathbf{q}}) (\boldsymbol{\sigma}_N \cdot \hat{\mathbf{q}}), \\ + (G_N - H_N) (\boldsymbol{\sigma}_p \cdot \hat{\mathbf{n}}) (\boldsymbol{\sigma}_N \cdot \hat{\mathbf{n}});$$

$$A_N = (\phi_1 + \phi_3)/2, \quad B_N = (\phi_3 - \phi_1)/2,$$

$$C_N = i\phi_5, \quad G_N = \phi_2/2, \quad H_N = \phi_4/2;$$

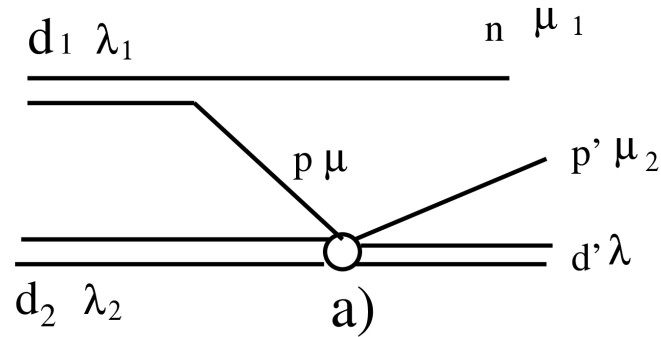
$$C'_N = C_N + \frac{iq}{2m} A_N.$$

T-odd P-even (TVPC)

$$t_N = h_N [(\boldsymbol{\sigma}_p \cdot \hat{\mathbf{k}}) (\boldsymbol{\sigma}_N \cdot \hat{\mathbf{q}}) \\ + (\boldsymbol{\sigma}_p \cdot \hat{\mathbf{q}}) (\boldsymbol{\sigma}_N \cdot \hat{\mathbf{k}}) - \frac{2}{3} (\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma}_p) (\mathbf{q} \cdot \hat{\mathbf{k}})] \\ + g_N [\boldsymbol{\sigma}_p \times \boldsymbol{\sigma}_N] \cdot [\hat{\mathbf{q}} \times \hat{\mathbf{k}}] (\boldsymbol{\tau}_p - \boldsymbol{\tau}_N)_z \\ + g'_N (\boldsymbol{\sigma}_p - \boldsymbol{\sigma}_N) \cdot i[\hat{\mathbf{q}} \times \hat{\mathbf{k}}] [\boldsymbol{\tau}_p \times \boldsymbol{\tau}_N]_z;$$

FACTORIZATION of the SQUARED MATRIX ELEMENT $dd\text{-}npd$ in IA

IA =



If final neutron takes one half of the deuteron momentum (S-wave dominance, suppressed p_T momenta), then the $pd \rightarrow pd$ amplitude can be extracted with minimum distortions.

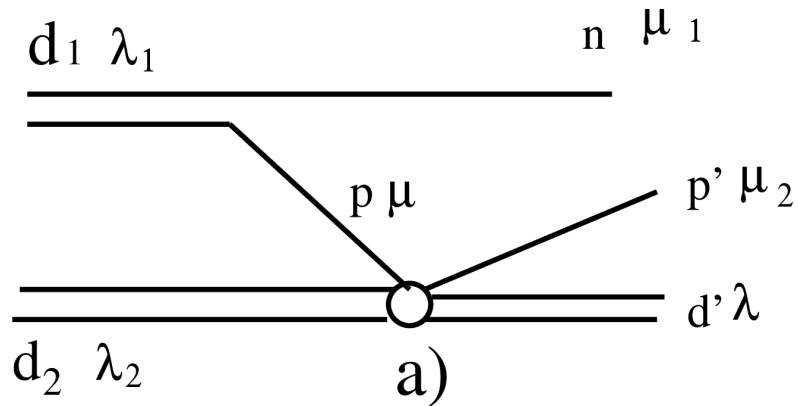
Transition matrix element for the $dd \rightarrow npd$ in IA, S-wave of d.w.f. :

$$M_{\lambda_1 \lambda_2}^{\mu_1 \mu_2 \lambda'} = K \sum_{\mu} \left(\frac{1}{2} \mu_1 \frac{1}{2} \mu |1 \lambda_1) u(q) M_{\lambda_2 \mu}^{\lambda' \mu_2} (pd \rightarrow pd) \right). \quad (1)$$

$$|M_{\lambda_1 \lambda_2}^{\mu_1 \mu_2 \lambda'}|^2 = K^2 u^2(q) |M_{\lambda_2 \mu}^{\lambda' \mu_2} (pd \rightarrow pd)|^2. \quad (2)$$

$$d\sigma_{\lambda_2} = \frac{1}{3} \sum_{\lambda_1} \sum_{\mu_1 \mu_2 \lambda'} |M_{\lambda_1 \lambda_2}^{\mu_1 \mu_2 \lambda'} (dd \rightarrow npd)|^2 = K^2 u^2(q) \frac{1}{2} \sum_{\mu \lambda' \mu_2} |M_{\lambda_2 \mu}^{\lambda' \mu_2} (pd \rightarrow pd)|^2.$$

Relations between $dd \rightarrow npd$ and $pd \rightarrow pd$



Rescatterings have to be taken into account

Ratio $R=A_d^y/A_p^y$ is of special interest at small t :
Yu.N.Uzikov, C. Wilkin, PLB 793 (2019) 224

$$|M(dd \rightarrow npd)|^2 = K[u^2(q) + w^2(q)] |M(pd \rightarrow pd)|^2$$

$d_2^\uparrow$: Vector or tensor Polarized

$$A_Y^d(dd_2^\uparrow \rightarrow npd) = A_Y^d(pd^\uparrow \rightarrow pd),$$

$$A_{YY} = (dd_2^\uparrow \rightarrow npd) = A_{YY}(pd^\uparrow \rightarrow pd)$$

$d_1^\uparrow$: Vector Polarized

$$A_Y^d(d_1^\uparrow d \rightarrow npd) = \frac{2}{3} A_Y^p(p^\uparrow d \rightarrow pd)$$

Both d_1 and d_2 deuterons are vector or tensor polarized:

$$C_{Y,Y}(d^\uparrow d^\uparrow \rightarrow npd) = \frac{2}{3} C_{y,y}(p^\uparrow d^\uparrow \rightarrow pd)$$

$$C_{Y,YY}(d^\uparrow d^\uparrow \rightarrow npd) = \frac{1}{3} C_{y,yy}(p^\uparrow d^\uparrow \rightarrow pd)$$

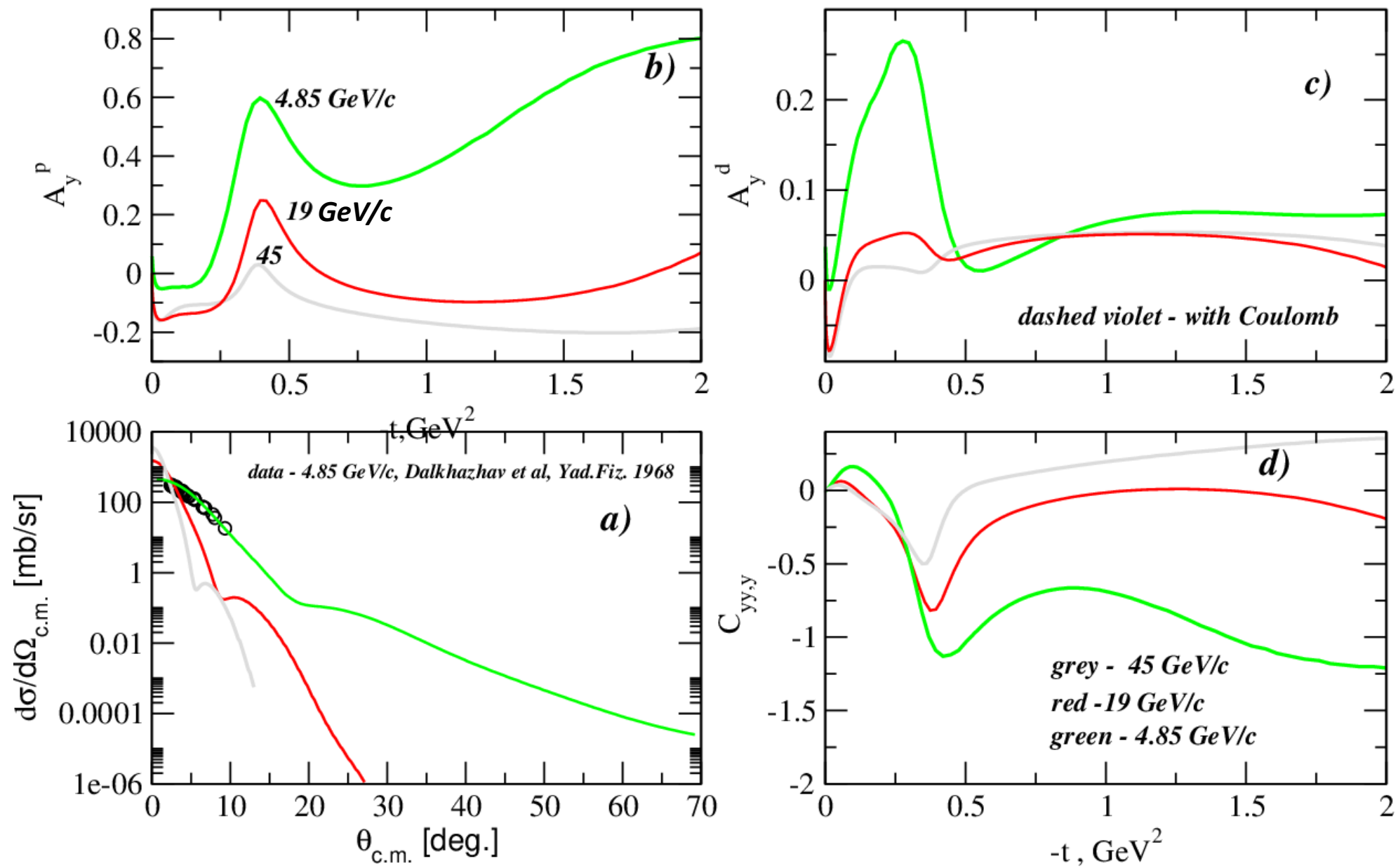
Yu. N. Uzikov, e-Print: [2506.17799](https://arxiv.org/abs/2506.17799) [nucl-th]

Yu.N. Uzikov, A.A. Temerbayev, Phys. Part. Nucl. 55 (2024) 895.

Numerical results for *pd*- elastic

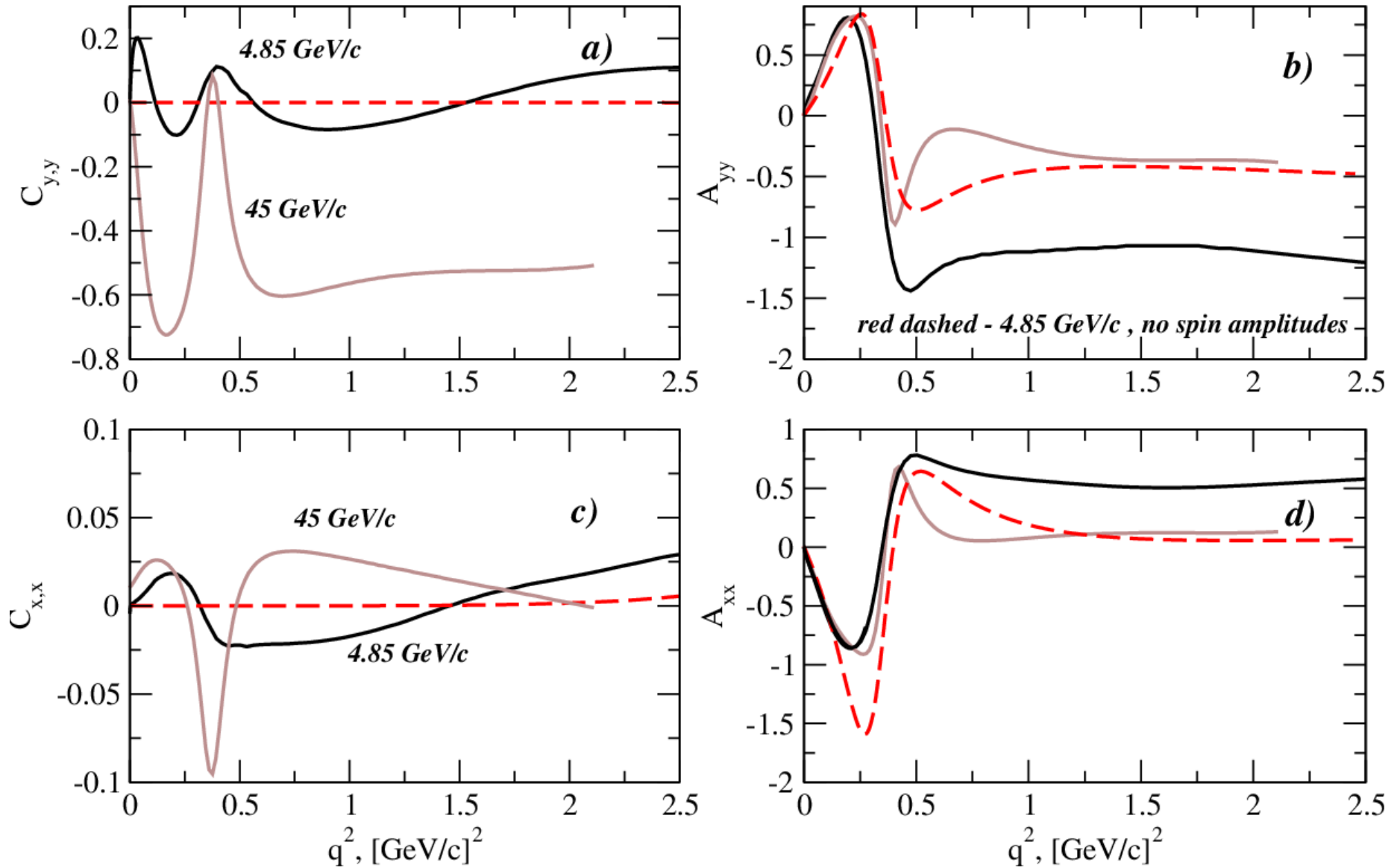
full black , full green $p=4.8$ GeV/c new05.data; grey,,violet - $p=45$ GeV/c new45.data

gray - SS+DS 45 GeV/c, new45.data, madis.f



pd- elastic with A. Sibirtsev (2010) amplitudes

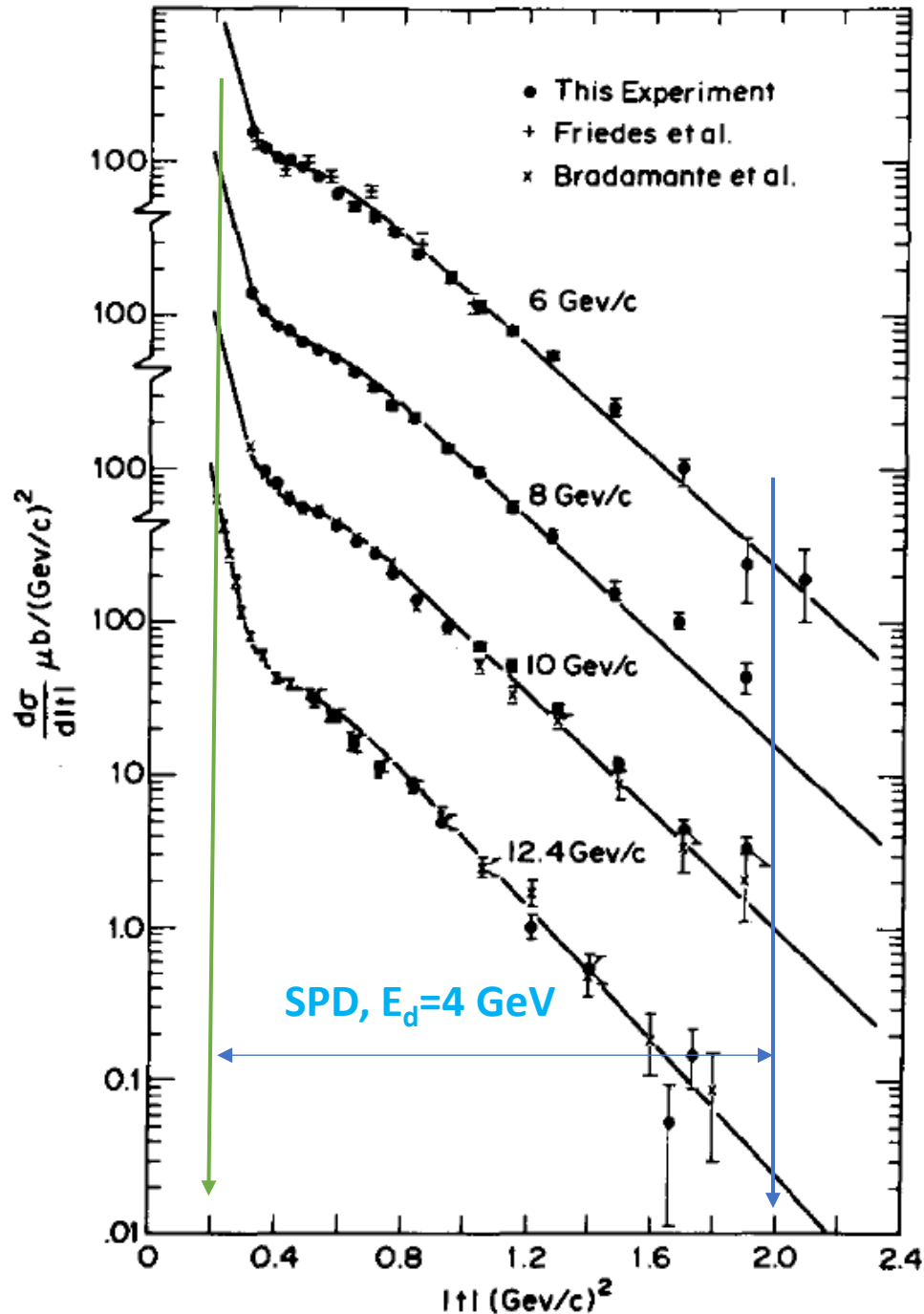
dashed red - without spin dependent pN amplitudes 4.85 GeV/c



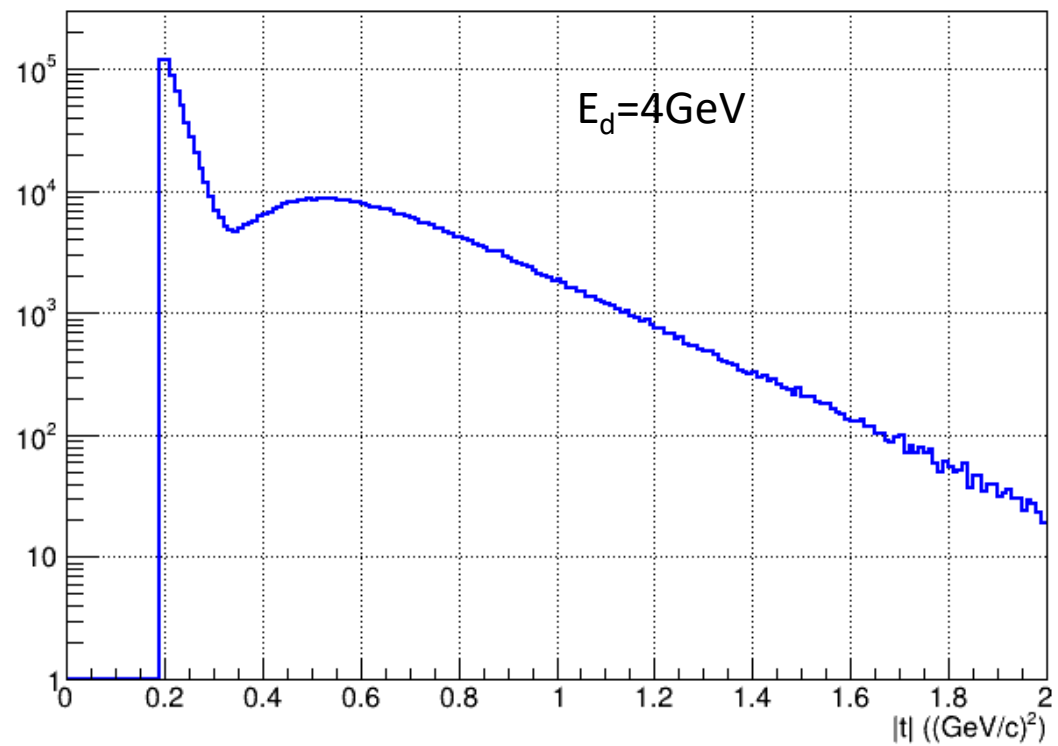
P. Chanowski et al. PLB 61 (1976)
nd-elastic scattering

MC simulation for SPD ($E_d=4$ GeV):

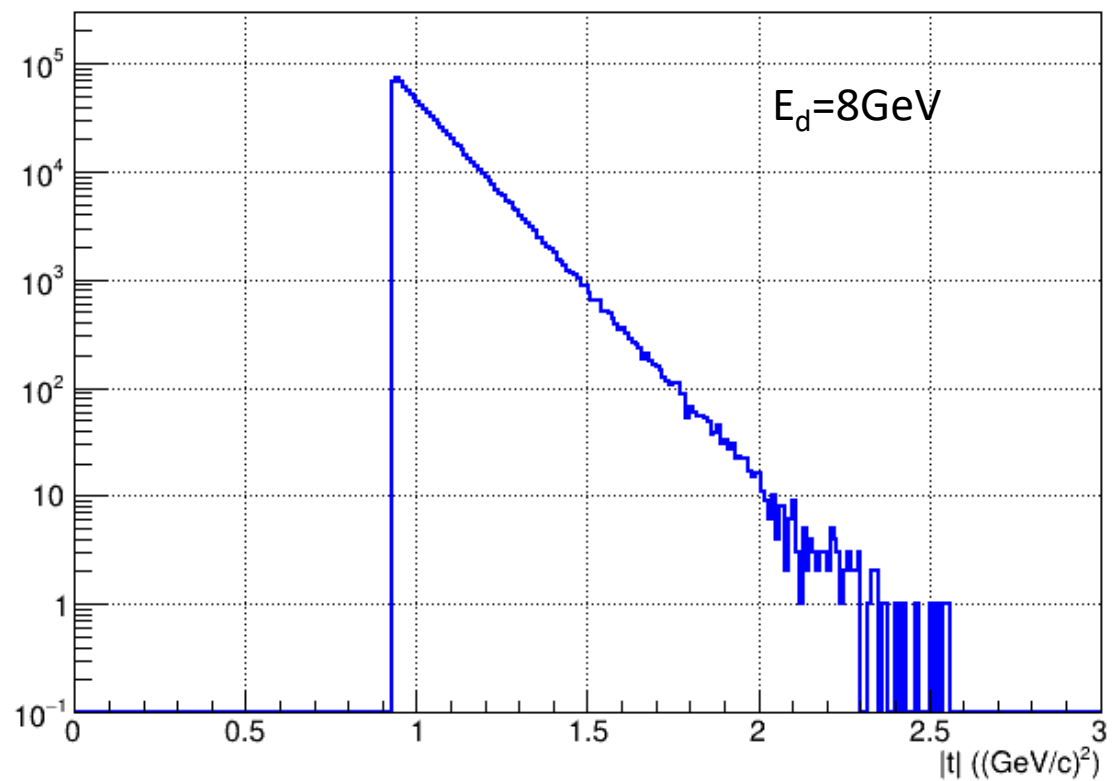
- Kinematic fit with invariant mass constraint (FUMILI fit with 'heavy terms' (p,d tracks only))
- Can not distinguish between signal and background - not useful on its own
- Kinematic fit with Lagrange multipliers with 3-momenta constraints (BES-III technique adapted by I. Denisenko)
- Takes into account neutron directions from ZDC
- SPD cannot reliably detect tracks at $\theta < 7^\circ$, limit on $|t|$
 - At $E_d=8$ GeV $\rightarrow |t|=0.9-2.3$ GeV²
 - At $E_d=6$ GeV $\rightarrow |t|=0.5-2.1$ GeV²



mom transfer to d



mom transfer to d



SOME ESTIMATES at $E_d=4$ GeV

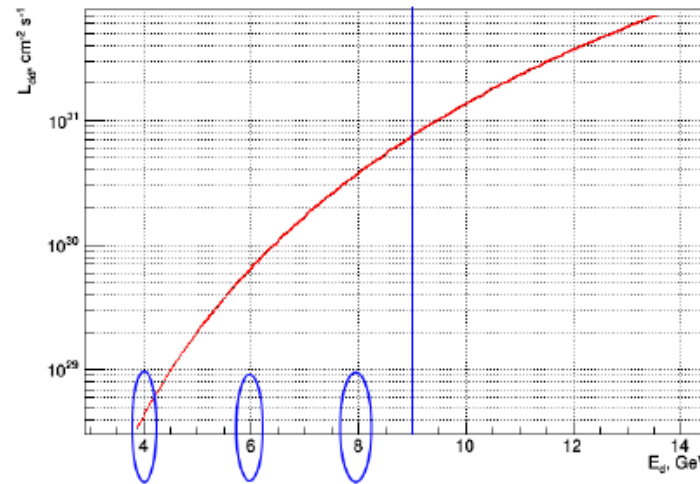
- Pythia8 : $\sigma^{tot} \sim 38$ mb (not very clear, if this is p+p cross-section, nuclear cross-section $\sigma_{dd}^{tot} \sim 160$ mb)
- (Custom) generated signal event : $\sigma_{dd}^{sig} \sim 39.5 \mu\text{b}$
- Assuming $\mathcal{L}_{dd} \sim 4.5 \times 10^{28} \text{ cm}^{-2} \text{ s}^{-1}$ for $\sqrt{S_{dd}} = 8$ GeV : PER MONTH ($\frac{10^7}{12}$ s)
 - ① 10 billion all d+d events
 - ② 1.5 million signal events produced
 - ③ 74k signal events detected (5% detection efficiency)
- Asymmetries for various spin combinations (vector and tensor) can be measured for the first time in polarized d+d collisions

SUMMARY

- NN-NN spin amplitudes are important for hadron physics and for physics BSM
- Can be studied at the first stage of SPD.
- Diffraction-> large cross section. One should not lose this chance.
- **Spin observables $A_y, A_{yy}, C_{y,y}, C_{y,yy}$ of the reaction $dd \rightarrow npd$ are directly related to those for $pd \rightarrow pd$ in the IA :**
 $A_y^p, A_y^d, C_{y,y}, C_{y,yy}$ - highly sensitive to pN spin amplitudes,
 A_{yy}, A_{xx} - robust observables, suitable for tensor polarimetry.
- The study can be extended to the dd- elastic scattering at SPD.
What has to be done:
 - Estimation for FSI effects.
 - Calculations of $A_y, A_{yy}, C_{y,y}, C_{y,yy}$ for pd - pd with different pN models.
 - Theory of dd-elastic scattering with full spin-dependence.

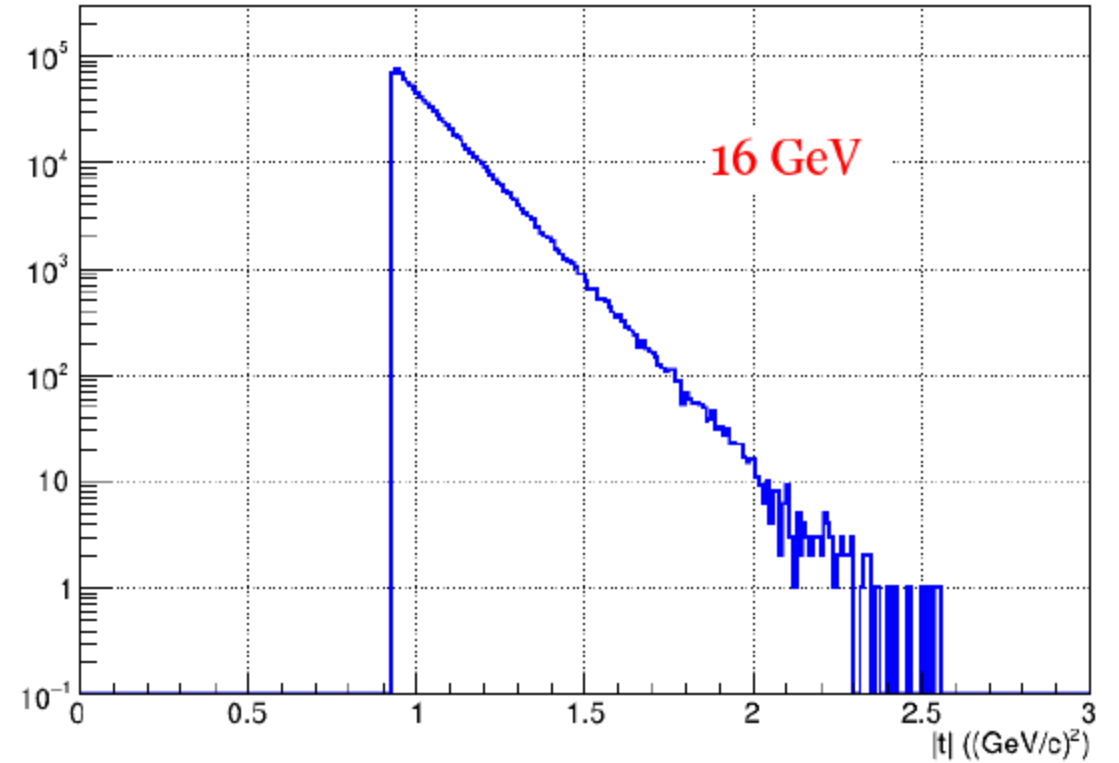
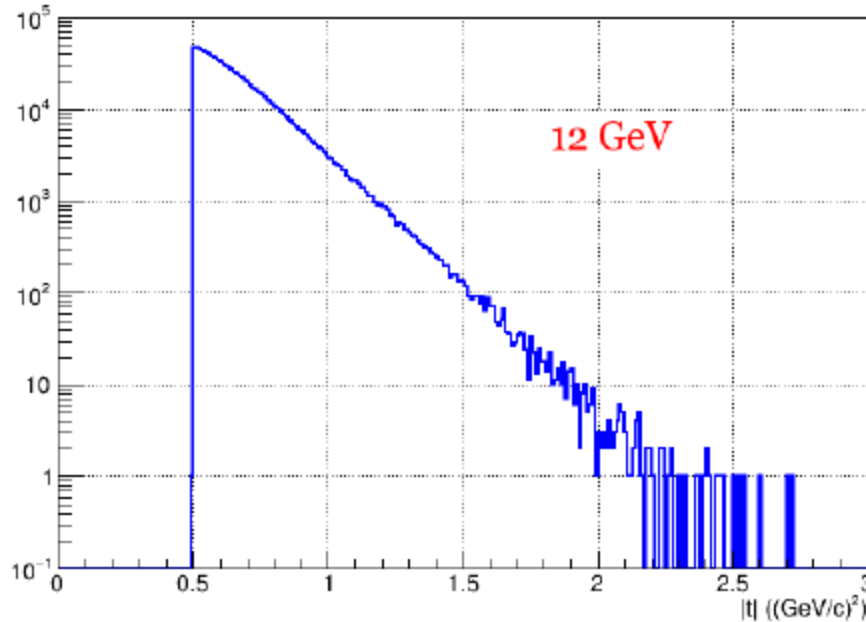
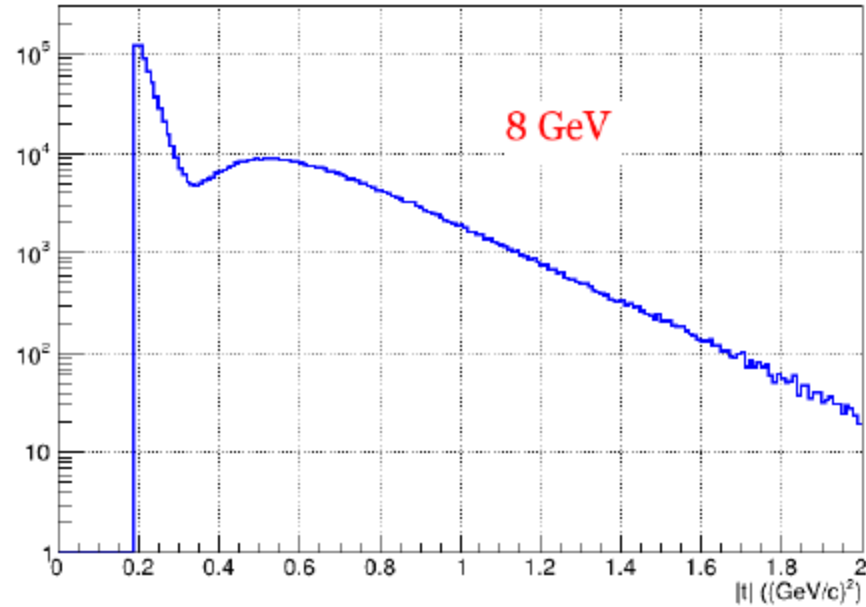
THANK YOU FOR
ATTENTION!

Estimate of counts for Different Energies



Event count for 1 month of ‘data recording’ (2σ mass window, one deuteron break up) :

- 8 GeV d+d : $\sigma_{dd}^{sig} \sim 45.6 \mu\text{b}$, $\mathcal{L}_{dd} \sim 4.5 \times 10^{28} \text{ cm}^{-2} \text{ s}^{-1}$, $\epsilon_{det} \sim 5\%$, $\epsilon_{gen} \sim 0.2$
- Estimated detected count ($2\sigma m_n$ window) : 17100
- 12 GeV d+d : $\sigma_{dd}^{sig} \sim 4.5 \mu\text{b}$, $\mathcal{L}_{dd} \sim 6.5 \times 10^{29} \text{ cm}^{-2} \text{ s}^{-1}$, $\epsilon_{det} \sim 10\%$, $\epsilon_{gen} \sim 0.3$
- Estimated detected count ($2\sigma m_n$ window) : 72 900
- 16 GeV d+d : $\sigma_{dd}^{sig} \sim 0.94 \mu\text{b}$, $\mathcal{L}_{dd} \sim 3.8 \times 10^{30} \text{ cm}^{-2} \text{ s}^{-1}$, $\epsilon_{det} \sim 10\%$, $\epsilon_{gen} \sim 0.4$
- Estimated detected count ($2\sigma m_n$ window) : 120 320



- Detected momentum transfer $|t|$
- Starts from 0.2, 0.5, 0.9 (GeV/c)² for 8, 12, 16 GeV collisions respectively

The differential cross section for collision $\vec{1} + \vec{1}$

$$I = I_0 \left[1 + \frac{3}{2} P_y A_y + \frac{3}{2} P_y^T A_y^T + \frac{1}{2} P_y P_{yy}^T C_{y,yy} + \frac{1}{2} P_{yy} P_y^T C_{yy,y} + \frac{9}{4} P_y P_y^T C_{y,y} + \frac{1}{3} P_{yy} A_{yy} + \frac{1}{3} P_{yy}^T A_{yy}^T + \frac{1}{9} P_{yy} P_{yy}^T C_{yy,yy} \right]. \quad (3)$$

Similarly for collision $\vec{1} + \frac{\vec{1}}{2}$

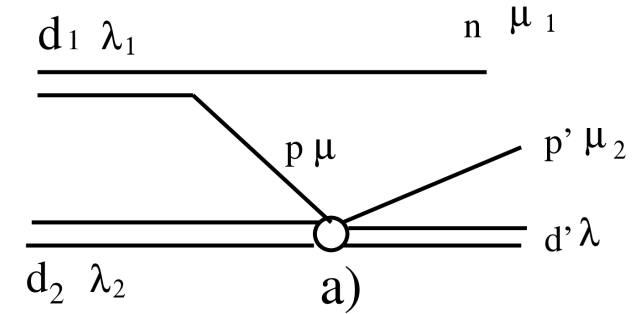
$$I = I_0 \left[1 + \frac{3}{2} P_y A_y + P_y^T A_y^T + \frac{1}{3} P_{yy} A_{yy} + \frac{3}{2} P_y P_y^T C_{y,y} + \frac{1}{3} P_{yy} P_y^T C_{yy,y} \right]. \quad (4)$$

Vector analyzing power $A_y^{d_2}$:

$$A_y^{d_2}(d_1 \vec{d}_2 = npd) = \frac{d\sigma_{\lambda_2=+1} - d\sigma_{\lambda_2=-1}}{d\sigma_{\lambda_2=+1} + d\sigma_{\lambda_2=0} + d\sigma_{\lambda_2=-1}} = A_y^d(pd \vec{d} \rightarrow pd)$$

Vector analyzing power $A_y^{d_1}$:

$$A_y^{d_1}(\vec{d}_1 d_2 \rightarrow npd) = \frac{d\sigma_{\lambda_1=+1} - d\sigma_{\lambda_1=-1}}{d\sigma_{\lambda_1=+1} + d\sigma_{\lambda_1=0} + d\sigma_{\lambda_1=-1}} = \frac{2}{3} A_y^p(\vec{p}d \rightarrow pd)$$



Tensor analyzing power

$$A_y^d(d_1 \vec{d}_2 \rightarrow npd) = \frac{d\sigma_{\lambda_2=+1} + d\sigma_{\lambda_2=-1} - 2d\sigma_{\lambda_2=0}}{d\sigma_{\lambda_2=+1} + d\sigma_{\lambda_2=0} + d\sigma_{\lambda_2=-1}} = A_{yy}^d(pd \vec{d} \rightarrow pd)$$

$C_{y,y}$ needs four options for dd-collision: (i) $P_y = P_y^T = \frac{2}{3}$ (ii) $P_y = \frac{2}{3}$, $P_y^T = -\frac{2}{3}$ and the same for $P_y = -\frac{2}{3}$. One can find the cross section $I_{\uparrow\uparrow}$ for the option (i), a $I_{\uparrow\downarrow}$ for (ii) from Eq. (4). Similarly for $C_{yy,y}$ we use four options with $P_y = 0$, $P_{yy} = \pm 1$ for d_1 and $P_y = \pm \frac{2}{3}$, $P_{yy} = 0$ for d_2 . One gets

$$C_{y,y} = \frac{(I_{\uparrow\uparrow} - I_{\uparrow\downarrow}) + (I_{\downarrow\downarrow} - I_{\downarrow\uparrow})}{(I_{\uparrow\uparrow} + I_{\uparrow\downarrow}) + (I_{\downarrow\downarrow} + I_{\downarrow\uparrow})}, \quad (5)$$

$$C_{yy,y} = \frac{(I_{+\uparrow} - I_{+\downarrow}) + (I_{-\downarrow} - I_{-\uparrow})}{(I_{+\uparrow} + I_{+\downarrow}) + (I_{-\downarrow} + I_{-\uparrow})}. \quad (6)$$

Yu.N. Uzikov, A.A. Temerbayev, Phys. Part. Nucl. 55 (2024) 895;
e-Print: [2311.12605 \[nucl-th\]](#); [dd-> npnp and hard pN-elastic scattering](#)

AT HIGHER ENERGIES $\sqrt{s_{pN}} = 3-10 \text{ GeV}$

A.Sibirtsev et al., Eur.Phys. J. A 45 (2010) 357

$$\phi_{ai}(s, t) = \pi \beta_{ai}(t) \frac{\xi_i(s, t)}{\Gamma(\alpha(t))}; i = \rho, \omega, a_2, f_2, P; a = 1-5;$$

$$\xi_i(t, s) = \frac{1 + S_i \exp[-i\pi\alpha(t)]}{\sin[\pi\alpha_i(t)]} \left[\frac{s}{s_0} \right]^{\alpha_i(t)},$$

$$\alpha_i(t) = \alpha_i^0 + \alpha_i' t,$$

$$\beta_{1i}(t) = c_{1i} \exp(b_{1i}t),$$

$$\beta_{2i}(t) = c_{2i} \exp(b_{2i}t) \frac{-t}{4m_N^2},$$

$$\beta_{3i}(t) = c_{3i} \exp(b_{3i}t),$$

$$\beta_{4i}(t) = c_{4i} \exp(b_{4i}t) \frac{-t}{4m_N^2},$$

$$\beta_{5i}(t) = c_{5i} \exp(b_{5i}t) \left[\frac{-t}{4m_N^2} \right]^{1/2}.$$

The Regge formalism for pp-helicity amplitudes
at proton beams momenta $p_L = 3-50 \text{ GeV/c}$
includes single-Pomeron exchange and
trajectories ρ, ω, f_2, a_2
pp- data on $d\sigma / dt$, A_N , A_{NN}

