DIRECT EXTRACTION OF VECTOR-MESON- PRODUCTION AMPLITUDES FROM EXPERIMENTAL DATA

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LXXV International Conference “Nucleus-2025”,

Saint Petersburg, 1.07-6.07.2025

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Introduction

- Hard exclusive vector-meson electroproduction provides information both on the reaction mechanism and target structure.
- Production of vector mesons by heavy photons is one of two basic processes for extraction of Generalized Parton Distributions (GPDs) of nucleon and nucleus from data.
- Usual method of data processing is Spin-Density-Matrix-Element (SDME) method. The SDMEs are coefficients in angular distribution of final particles in vector-meson production by heavy photons. SDMEs are considered as free fit parameters.
- Any SDME can be expressed via amplitudes of process $\gamma^*+T\rightarrow V+T'$,

where γ^* denotes virtual photon, T is a target nucleonic system (nucleus or nucleon), V is vector meson, T' denotes final state of nucleonic system.

- Direct extraction from the angular distribution of helicity-amplitude ratios, which are considered as free fit parameters, is called the amplitude method.

Introduction

- Some amplitudes contain linear contribution of GPDs. Not all helicity amplitudes of vector-meson production can be now calculated in theoretical models.
- Any SDME contains contributions of all amplitudes. Some SDMEs contain contributions of many amplitudes to the numerator of the formula for the SDME. In case of disagreement of model prediction for the SDME with data, it is difficult to establish which amplitude is predicted wrong in the model.
- Therefore to obtain GPDs, we are to extract all the amplitudes from data in the amplitude method, single out those amplitudes which can be calculated in the model and fit them using free parameters of GPD.

Coherent vector-meson production on spinless targets

Subprocesses: e⟶e' + γ^*; $\gamma^*(\beta) + S \to V(\lambda) + S$: helicity amplitude $F_{\lambda\beta}$; $V = \rho^{o} \to \pi^+ + \pi^-$.
S is spinless nucleus retained intact; β and λ are helicities in center-of-mass system.

Amplitudes obey symmetry relations due to parity conservation $F_{-\lambda-\beta} = (-1)^{\lambda-\beta} F_{\lambda\beta}$.

Independent amplitudes: F_{11}, F_{10}, F_{1-1}, F_{01}, F_{00}.

Von-Neumann formula for spin-density matrix of vector meson $NR_{\lambda\nu} = \sum_{\alpha\beta} F_{\lambda\alpha} F^*_{\nu\beta}\, \rho_{\alpha\beta}$,
where ρ is spin-density matrix of virtual photon known from QED which depends on beam polarization $\boldsymbol{P_b}$, N is normalization factor, $N = |F^2_{11}| + |F^2_{01}| + |F^2_{-11}| + \varepsilon\{ |F^2_{10}| + |F^2_{00}| + |F^2_{-10}| \}$, .
and ε is flux ratio of longitudinally ($\beta = 0$) to transversely polarized ($\beta = \pm 1$) virtual photons.

Angular distribution of decay pions ($\rho^0 \to \pi^+ + \pi^-$)

$W(\Phi,\theta,\varphi) = \Sigma_{\lambda\nu} Y_{1\lambda}(\theta,\varphi)\, Y^*_{1\nu}(\theta,\varphi)\, R_{\lambda\nu}(\Phi,\varepsilon)$,

where $Y_{1\lambda}(\theta,\varphi)$ are spherical harmonics depending on polar θ and azimuthal φ angles of π^+ three-momentum in vector-meson-rest frame. Angular distribution $W(\Phi,\theta,\varphi)$ depends on Schilling-Wolf matrix elements $r^n_{\lambda\nu}$ being combinations of spin-density-matrix elements $R_{\lambda\nu}$ of vector meson.

Coherent vector-meson production on spinless targets

Formulas for SDMEs $r^n_{\lambda\nu}$ for unpolarized targets in terms of helicity amplitudes were obtained for nucleons by K. Schilling and G. Wolf in 1973 (Nucl. Phys. , B61, 381) and are also valid for spinless targets.
They look like $r^n_{\lambda\nu} = f^n_{\lambda\nu}\,(\,F_{\lambda\beta}\,)$, where $f^n_{\lambda\nu}$ is a function of amplitudes and are basic equations for finding helicity amplitude ratios. Exact solutions of equations are found by author in paper SIM Phys. of Part. and Nucl. Letters 21 (2024) 34.

$$\frac{F_{00}}{F_{11}} = \frac{1}{\epsilon\sqrt{8}} \frac{\left(r^{04}_{00} + r^{1}_{00}\right)\left(\mathrm{Im}\{\mathrm{r}^2_{1-1}\} - \mathrm{r}^1_{11} - \mathrm{r}^1_{1-1} + \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^3_{1-1}\}\right)}{\left(r^1_{1-1} - \mathrm{Im}\{\mathrm{r}^2_{1-1}\}\right)\left(\mathrm{Im}\{\mathrm{r}^6_{10}\} - \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^7_{10}\}\right)}$$

$$\frac{F_{1-1}}{F_{11}} = \frac{r^1_{11} - i\cdot\mathrm{Im}\{\mathrm{r}^3_{1-1}\}}{r^1_{1-1} - \mathrm{Im}\{\mathrm{r}^2_{1-1}\}}.$$

$$\frac{F_{10}}{F_{11}} = \frac{\mathrm{Im}\{\mathrm{r}^6_{1-1}\} - \mathrm{r}^5_{1-1} - \mathrm{i}\cdot\left(\mathrm{r}^8_{11} - \mathrm{Im}\{\mathrm{r}^7_{1-1}\}\right)}{\sqrt{2}\left(r^1_{1-1} - \mathrm{Im}\{\mathrm{r}^2_{1-1}\}\right)}.$$

$$\frac{F_{01}}{F_{11}} = \frac{2\left(\mathrm{Im}\{\mathrm{r}^2_{10}\} - \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^3_{10}\}\right)}{r^1_{1-1} - \mathrm{Im}\{\mathrm{r}^2_{1-1}\} + \mathrm{r}^1_{11} + \mathrm{i}\cdot\mathrm{Im}\{\mathrm{r}^3_{1-1}\}}.$$

In order to obtain moduli and phase differences between helicity amplitudes, data on differential cross section $\frac{d\sigma}{\mathrm{dt}}$ is to be added to the angular distribution.

Vector-meson production on nucleons: Incoherent production on unpolarized targets

Helicity amplitudes: $\text{Natural} - \text{Parity} - \text{Exchange (NPE) amplitudes } \mathrm{T}_{\lambda \pm \frac{1}{2} \beta \frac{1}{2}}$ correspond to exchanges with pomeron, rho, omega, f_2 $-$ reggeons etc. ($\frac{1}{2}$ and - $\frac{1}{2}$ correspond to nucleon helicity).

Unnatural-Parity-Exchange (UPE) amplitudes $U_{\lambda \pm \frac{1}{2} \beta \frac{1}{2}}$ correspond to pion and a_1 exchanges etc.

- There is no interference of contributions of NPE and UPE amplitudes to Schilling-Wolf spin-density-matrix elements for unpolarized nucleus targets. This means that if ratio $U_{\lambda \pm \frac{1}{2} \beta \pm \frac{1}{2}} / T_{\lambda \pm \frac{1}{2} \beta \pm \frac{1}{2}} \sim' \Upsilon_U$ is small parameter, then contribution to SDMEs is very small $\sim {\Upsilon_U}^2$.
- There is not interference of contributions to Schilling-Wolf SDMEs of nucleon-spin-flip and non-spin-flip helicity amplitudes. If ratio $T_{\lambda \frac{1}{2} \beta - \frac{1}{2}} / T_{\lambda \frac{1}{2} \beta \frac{1}{2}} \sim' \Upsilon_{SF}$ is small parameter, then fractional contribution of spin-flip amplitudes to SDMEs is proportional to square of small parameter $\sim {\Upsilon_{SF}}^2$.
- Main amplitudes : F_{11}, F_{10}, F_{1-1}, F_{01}, $F_{00} \rightarrow T_{1\frac{1}{2}1\frac{1}{2}}$, $T_{1\frac{1}{2}0\frac{1}{2}}$, $T_{1\frac{1}{2}-1\frac{1}{2}}$, $T_{0\frac{1}{2}1\frac{1}{2}}$, $T_{0\frac{1}{2}0\frac{1}{2}}$. $U_{\lambda \pm \frac{1}{2} \beta \frac{1}{2}} \equiv 0$.
  Dominant helicity-amplitude ratio is $T_{1\frac{1}{2}1\frac{1}{2}} / T_{0\frac{1}{2}0\frac{1}{2}}$.

  The largest amplitude ratio violating SCHC (S-Channel Helicity Violation) is $T_{0\frac{1}{2}1\frac{1}{2}} / T_{0\frac{1}{2}0\frac{1}{2}}$.

Non-negativity of angular distribution of final particles for spinless targets

- In SDME method, SDMEs should obey some inequalities to garantee that angular distribution W(Φ,θ,φ) is not negative.

This problem is discussed in details in paper by M. Gavrilova, O. Teryaev, Phys. Rev. D99 (2019) 076013.

- In amplitude method for nonzero amplitudes, W(Φ,θ,φ) ≥ 0 (SIM, Phys. Atom. Nucl. 87 (2024) 505).

Eigenvalues of spin-density matrix of virtual photon: $\eta_1 > \eta_2 > \eta_3 = 0$, where

$\eta_1 = (1+\varepsilon+\sqrt{g})/2$, $\eta_2 = (1+\varepsilon-\sqrt{g})/2$, $\eta_3 = 0$, with $g = 4\varepsilon^2 + P_b^2\,(1-\varepsilon)(1+3\varepsilon) > 0$ for $0<\varepsilon<1$.

It is easy to show that $\eta_1 > \eta_2 > 0$, $\eta_3 = 0$ if $0<\varepsilon<1$, $P_b^2 < 1$.

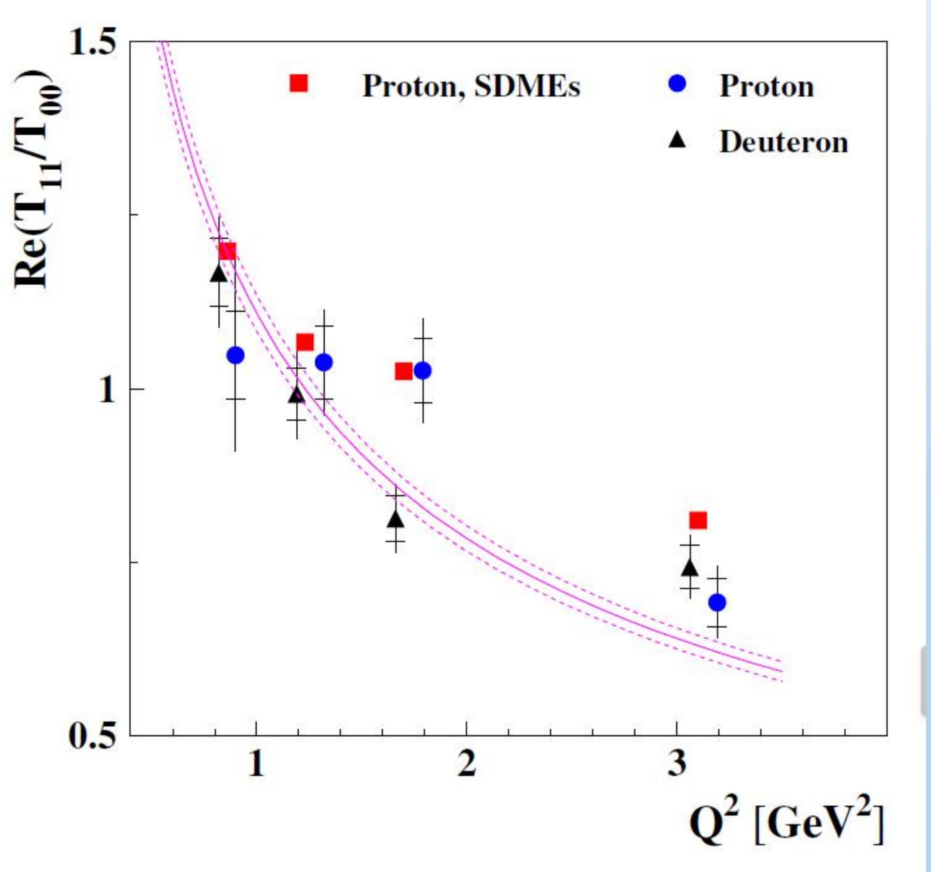
3. If X_n^{α} are three eigenvectors for n=1, 2, 3 of virtual-photon spin-density matrix, then

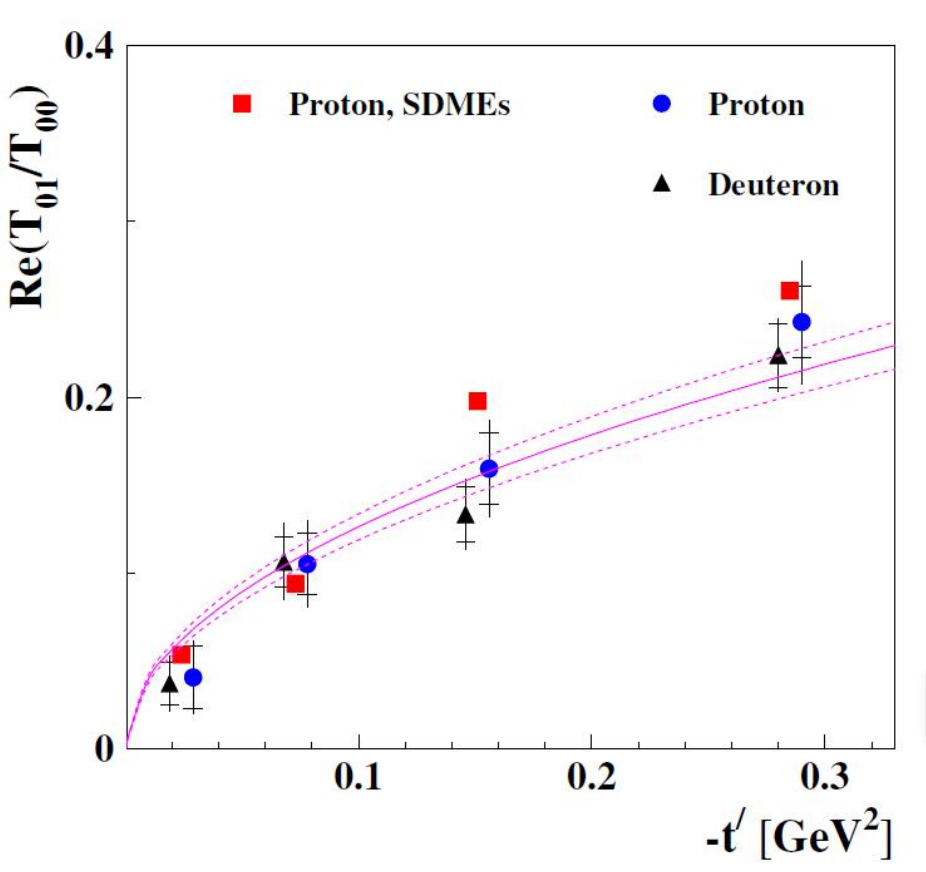
$\rho_{\alpha\beta} = \sum_{n=1}^{3} \eta_n X_n^{\alpha} (X_n^{\beta})^*$,

$NW(\Phi,\theta,\varphi) = \sum_{n=1}^{3} \eta_n \,|\, G_n \,|^2 \geq 0$, where $G_n = \sum_{\lambda,\alpha} Y_{1\lambda}(\theta,\varphi)\, F_{\lambda\alpha}\, X_n^{\alpha}(\Phi)$.

Since N>0 and NW is non-negative, the angular distribution W is non-negative in amplitude method for any non-zero helicity amplitudes of vector-meson production on spinless nuclei. Non-negativity of the angular distributions of final particles can be proved for production on nucleons in analogous way.

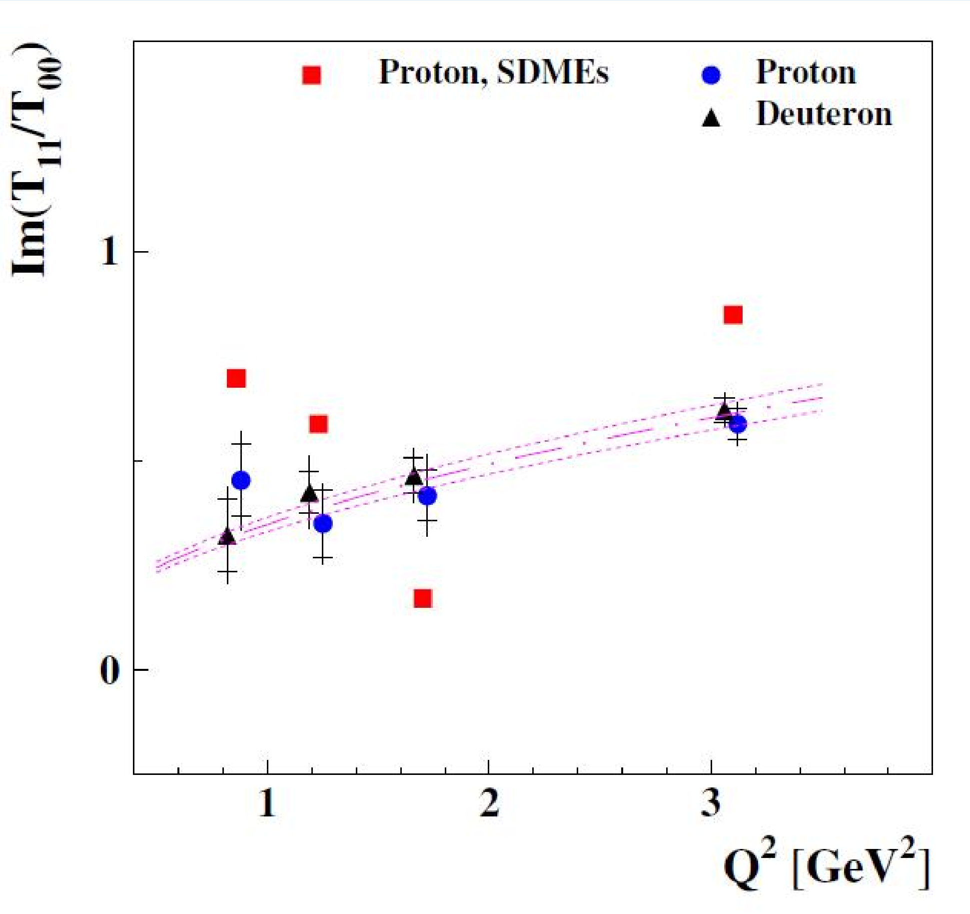
Numerical Calculation of Helicity Amplitude Ratios in Terms of SDMEs

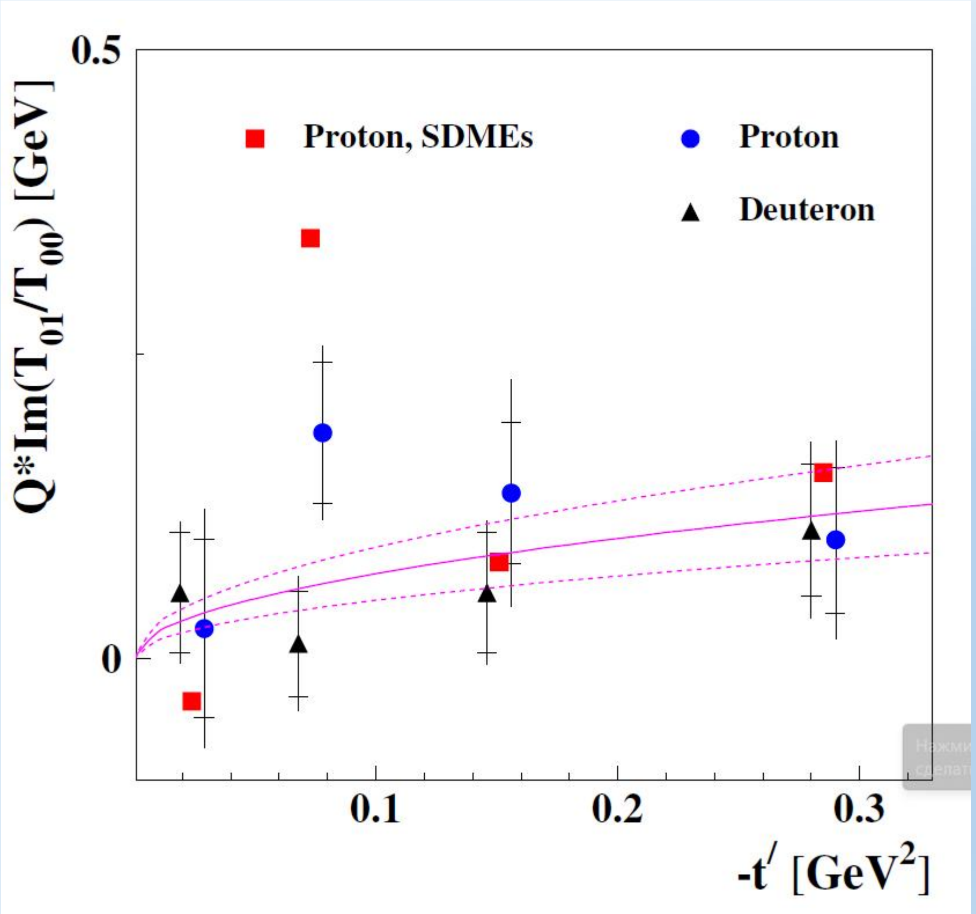




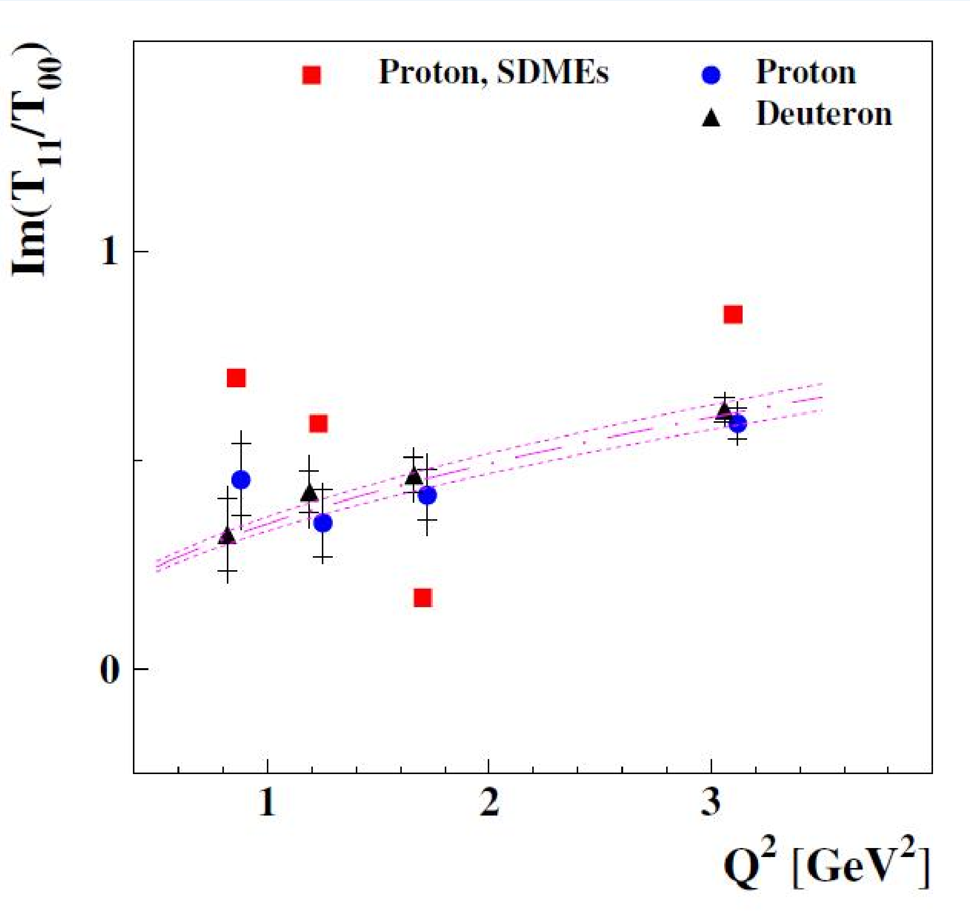
Circles and triangles are results of amplitude analysis of HERMES data in EPJ C71, 1609 (2011).
Squares obtained with formulas from PEPAN Lett. 21, 34 (2024) and SDMEs are from EPJ C62, 659 (2009).

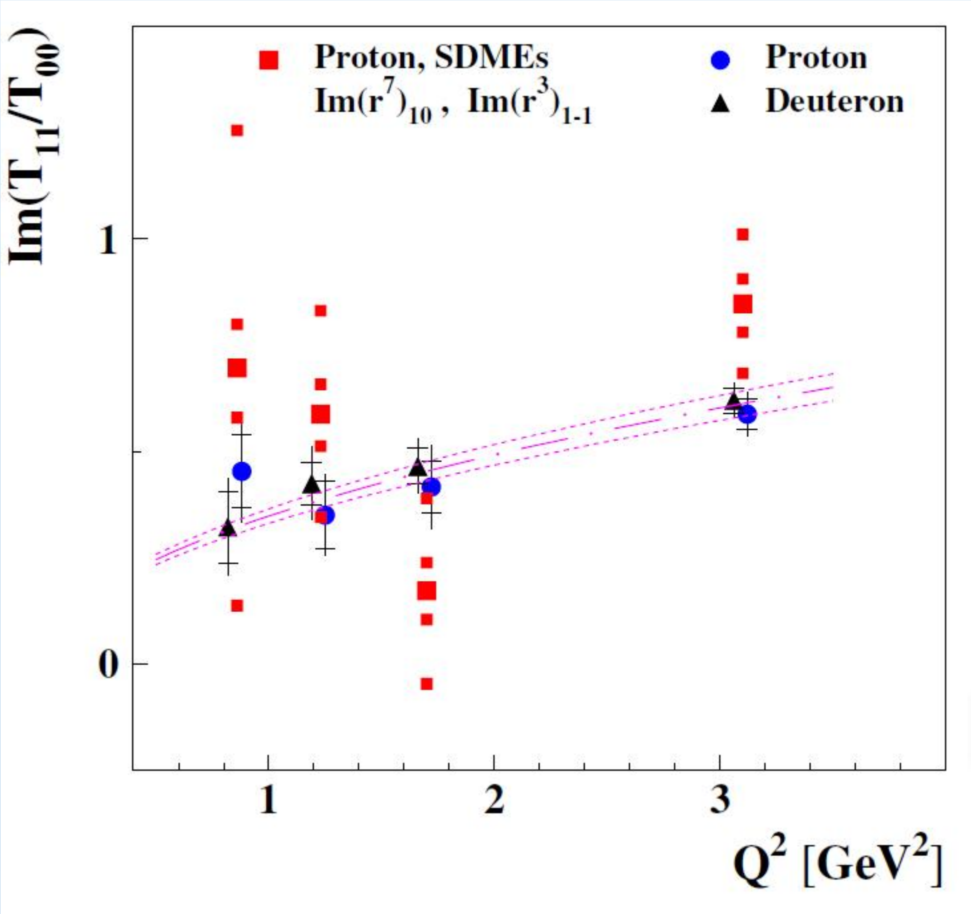
Numerical Calculations of Helicity Amplitude Ratios in Terms SDMEs

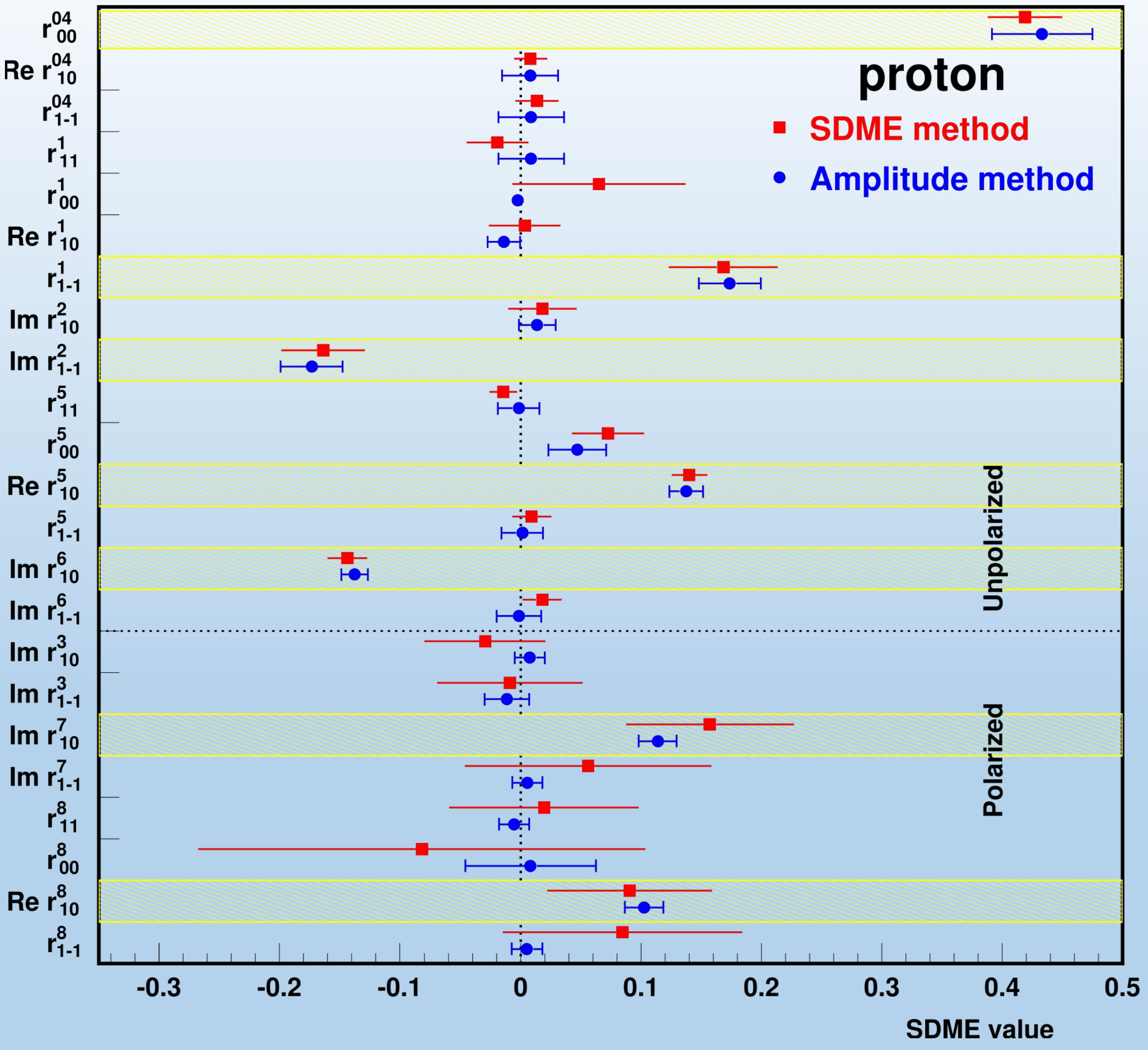




Numerical Calculation of Helicity Amplitude Ratios in Terms of SDMEs

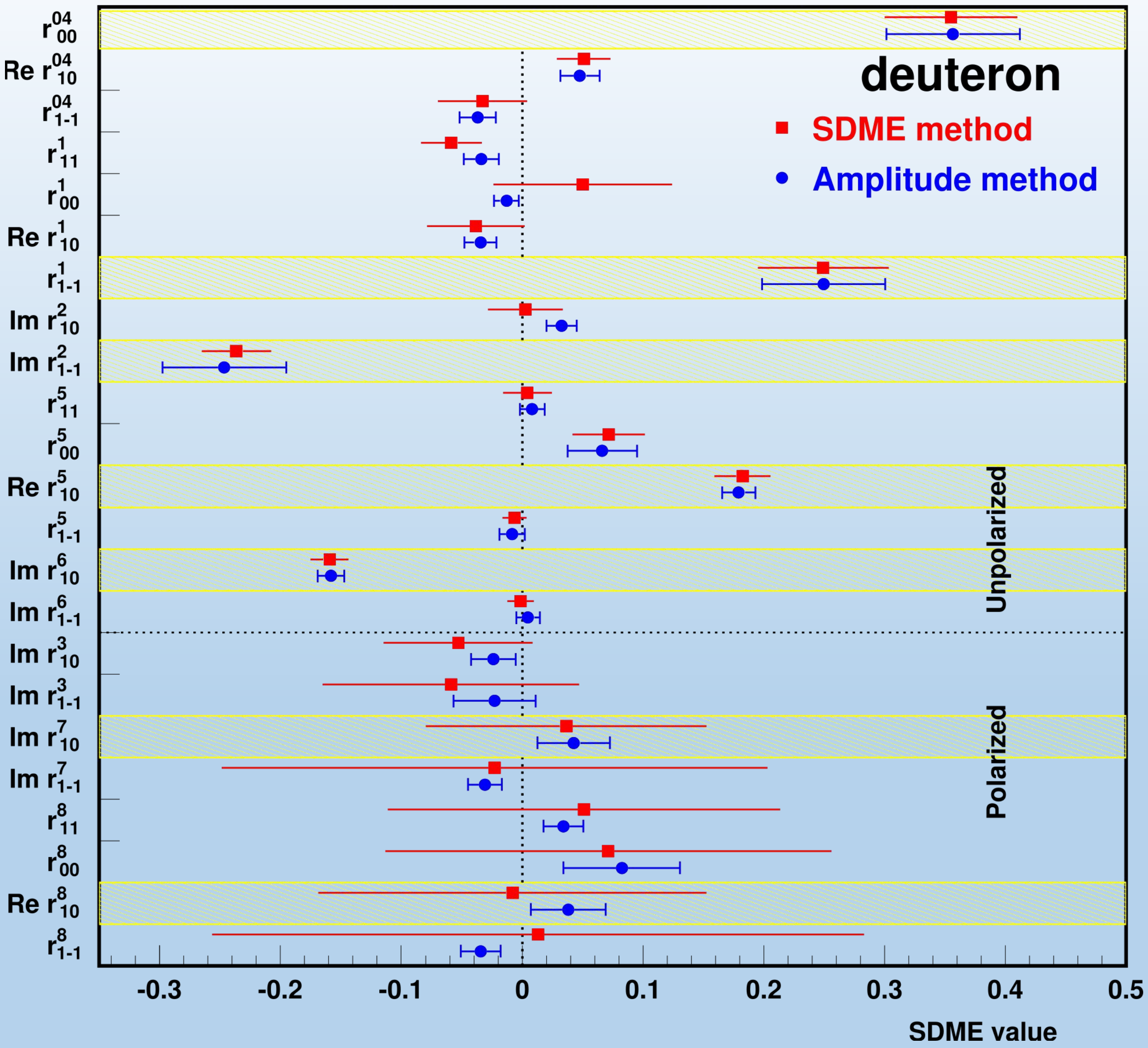






Q^2=3.0 GeV^2 $\nu_T{}^2$ =0.019 GeV^2

HERMES, EPJ C71 (2011) 1609.



Q^2=1.2 GeV^2 $v_T{}^2$ =0.145 GeV^2

HERMES EPJ C71 (2011) 1609.

Conclusions

- It is shown that amplitude method is the best which permits to extract generalized parton distributions from data on vector-meson electroproduction.
- For any set of non-zero helicity amplitudes, angular distribution of final particles is non-negative.
- There are exact formulas for amplitude ratios in terms of SDMEs for spinless targets.
- The formulas become approximate for meson electroproduction on nucleons. They have reasonable accuracy when unnatural-parity-exchange and nucleon-spin-flip amplitudes are much smaller than natural-parity-exchange amplitudes without nucleon-spin flip.
- Accuracy of imaginary parts of amplitude ratios predicted approximate formulas under discussion is worse than real part accuracy. This is due to worse statistical accuracy of polarized SDMEs compared to unpolarized ones. Amplitude method provides imaginary parts of amplitude ratios with reasonable accuracy since they contibute to all SDMEs1414.
- Statistical accuracy of polarized SDMEs should be increased to extract reliably generalized parton distributions.