Unitarity of amplitude and the halo effect in pp scattering at LHC energies

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LXXV International Conference «NUCLEUS - 2025. Nuclear physics and elementary particle physics. Nuclear physics technologies»

Saint-Petersburg State University, St.Petersburg, July 1-6, 2025

A simple representation of the unitarity condition

The amplitude of pp scatering at high energies

$$a(\mathbf{k}) \equiv 4\pi\, A(s,t) = T(s,t)/J\ .$$

$$J \equiv 4\sqrt{(p_1p_2)^2 - m_1^2m_2^2} = 2\sqrt{s(s-4m^2)} \approx 2s$$

$$t = -\mathbf{k}^2 = -\mathbf{k}_\perp^2$$

$$\sigma^{tot} = \frac{2}{J}\,\mathrm{Im}\,T(s,t=0) = 8\pi\,\mathrm{Im}\,A(s,t=0) = 2\,\mathrm{Im}\,a(\mathbf{k}=0)$$

- the optical theorem

$$\sigma^{el} = \int \frac{d\sigma^{el}}{dt}dt = \int \frac{|T(s,t)|^2}{16\pi s^2}dt = 4\pi \int |A(s,t)|^2 dt = \int |a(\mathbf{k})|^2 \frac{d^2\mathbf{k}}{(2\pi)^2}$$

$$\sigma^{tot} \geq \sigma^{el}$$

$$2\,\mathrm{Im}\,a(\mathbf{k}=0) \geq \int |a(\mathbf{k})|^2 \frac{d^2\mathbf{k}}{(2\pi)^2}$$

Impact parameter representation:

$$a(\mathbf{b}) \equiv \int \frac{d^2\mathbf{k}}{(2\pi)^2}\, a(\mathbf{k})\ \exp(-\mathrm{i}\mathbf{k}\cdot\mathbf{b})$$

$$a(\mathbf{k}) \equiv \int d^2\mathbf{b}\, a(\mathbf{b})\ \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{b})$$

A simple representation of the unitarity condition

$$\sigma^{tot} = \int d^2b\, 2\,\mathrm{Im}\, a(b) \equiv \int d^2b\, \sigma^{tot}(b) \qquad \sigma^{el} = \int d^2b\, |a(b)|^2 \equiv \int d^2b\, \sigma^{el}(b)$$

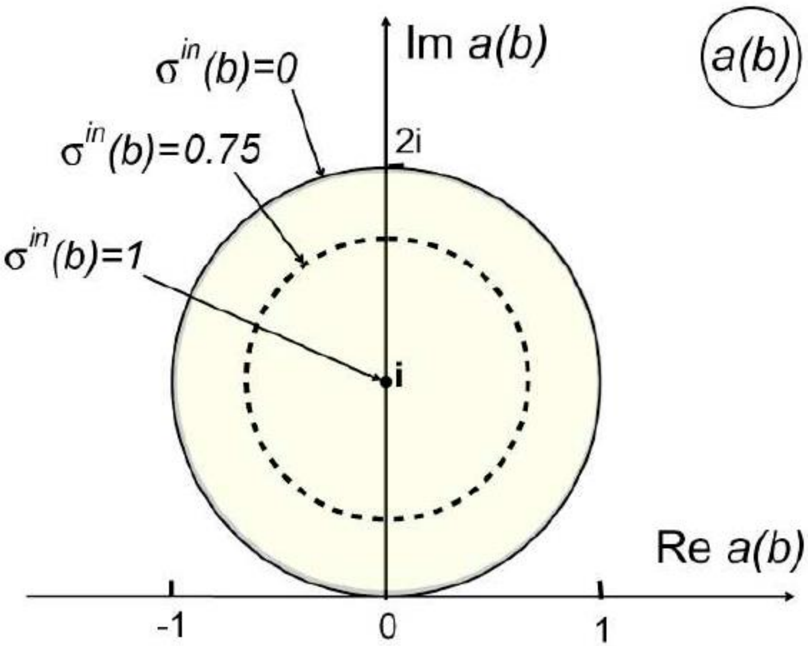
$$\sigma^{in} \equiv \sigma^{tot} - \sigma^{el} = \int d^2b\, [2\,\mathrm{Im}\, a(b) - |a(b)|^2] \geq 0$$

- the consequence of the optical theorem

$$l \simeq p\,b \qquad l_{max} \simeq p\,R \gg 1 \qquad p = \sqrt{s}/2$$

$$\sigma^{in}(b) \equiv \sigma^{tot}(b) - \sigma^{el}(b) = 2\,\mathrm{Im}\, a(b) - |a(b)|^2 \geq 0$$

- the unitarity condition



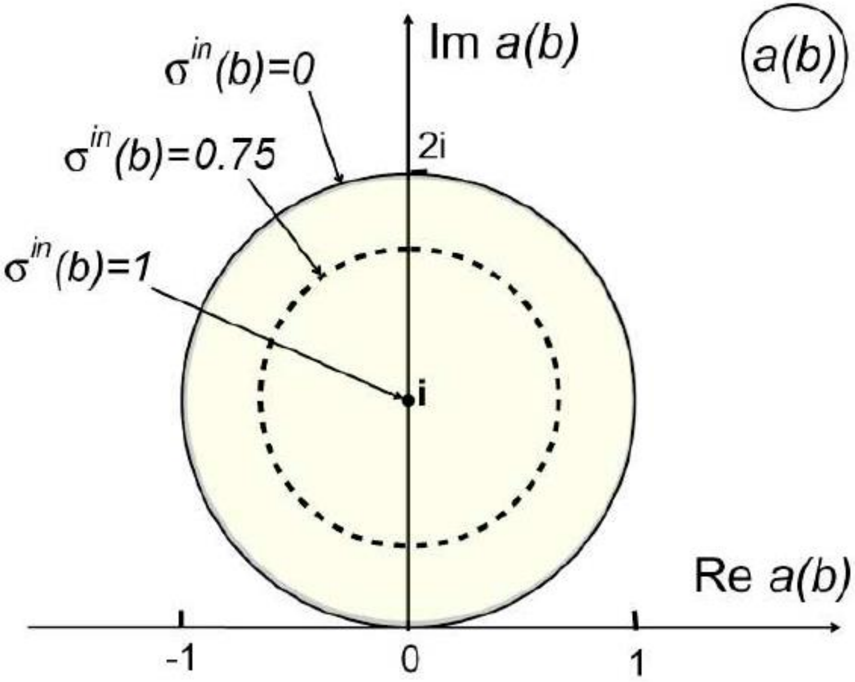
$$\boxed{2\,\mathrm{Im}\, a(b) \geq |a(b)|^2}$$

$$\boxed{0 \leq \sigma^{tot}(b) \leq 4\,, \qquad 0 \leq \sigma^{el}(b) \leq 4}$$

$$\boxed{0 \leq \sigma^{in}(b) \leq 1(!)}$$

- allows for probabilistic interpretation

Black disk and two types of gray disk



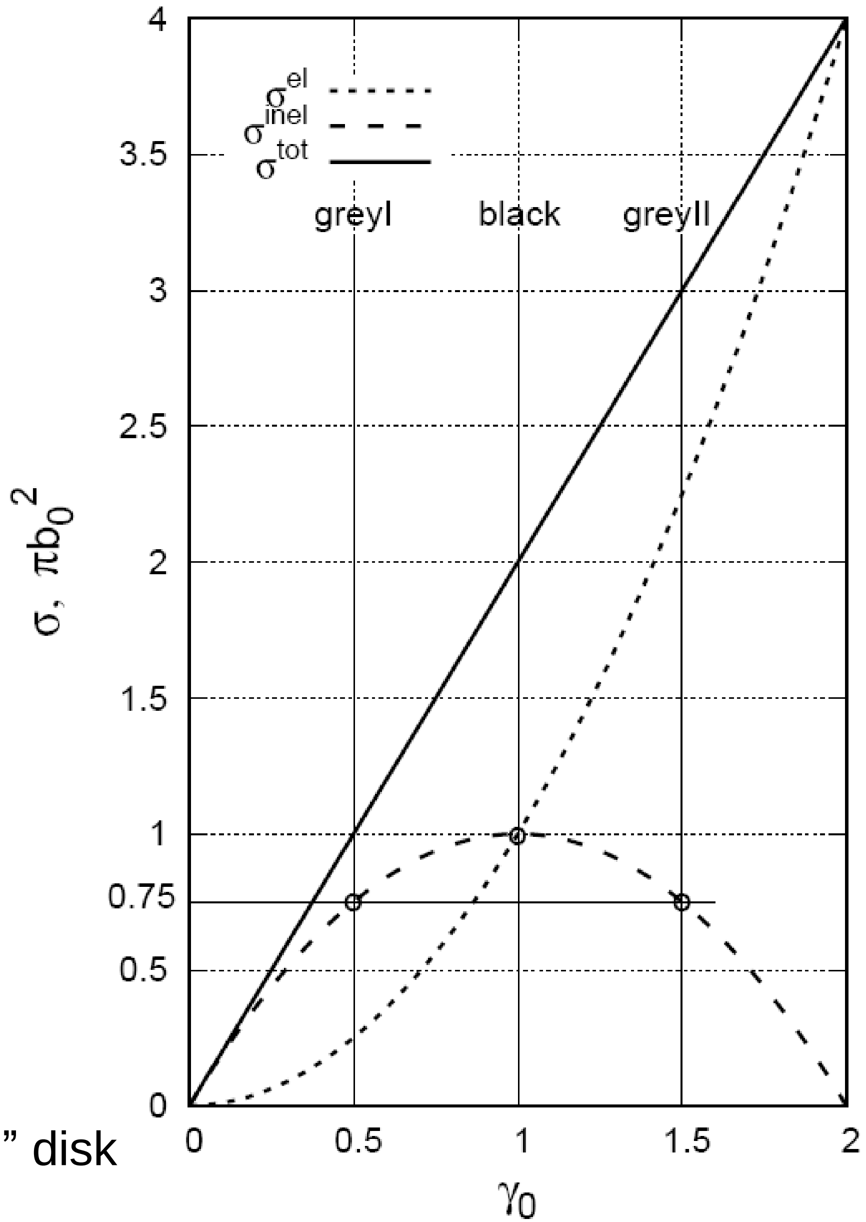
Purely imaginary amplitude:

$$a(b) = \mathrm{i}\,\gamma(b) \qquad 0 \le \gamma(b) \le 2$$

$$\sigma^{tot}(b) = 2\gamma(b)\ , \qquad \sigma^{el}(b) = \gamma^2(b)$$

$$\sigma^{in}(b) = \sigma^{tot}(b) - \sigma^{el}(b) = \gamma(b)[2 - \gamma(b)]$$

$$\gamma(b) = \gamma_0\,\theta(b_0 - |\mathbf{b}|) \qquad 0 \le \gamma_0 \le 2$$



$\gamma_0 = 2$ - “shining” disk

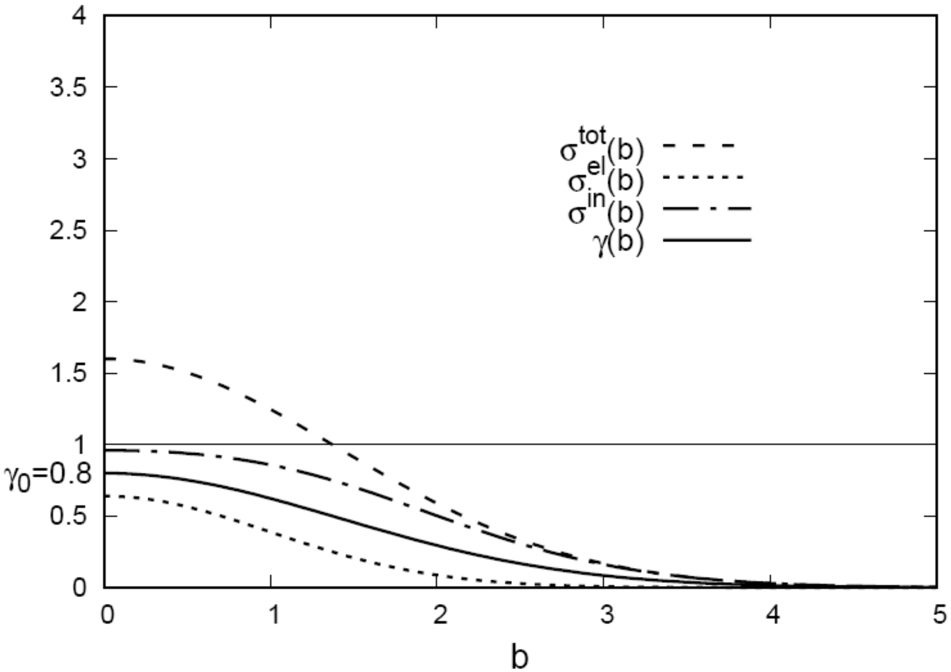
Pure imaginary Gaussian amplitude

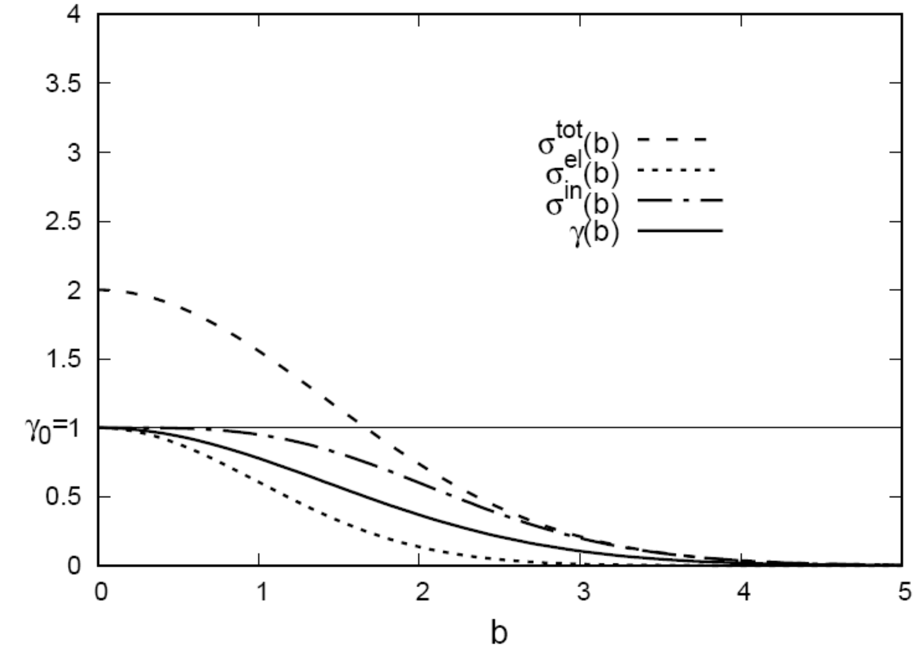
$$a(b) = \mathrm{i}\,\gamma(b)$$

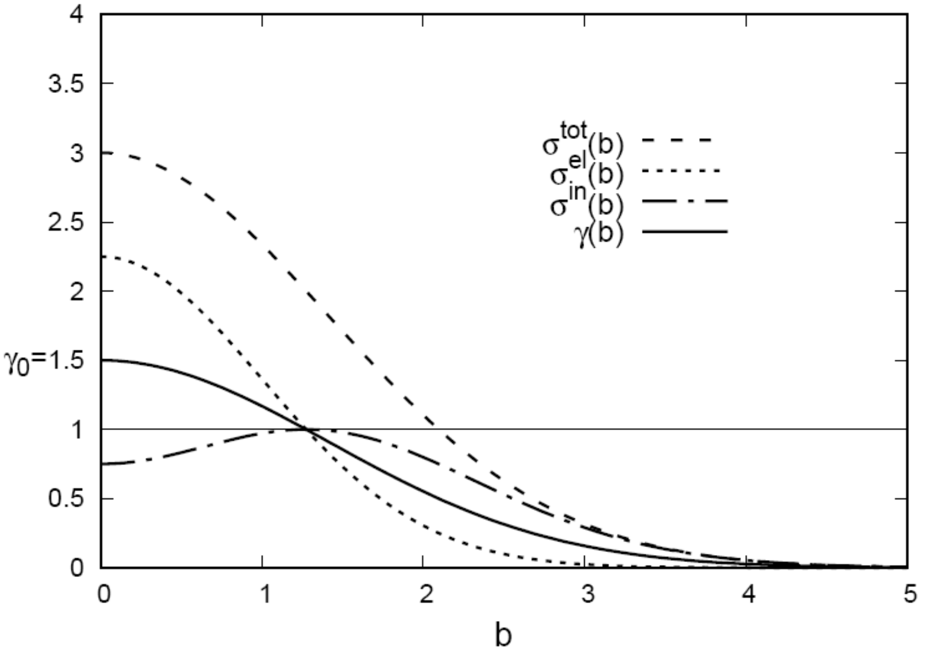
$$\gamma(b) = \gamma_0 \exp(-b^2/b_0^2)$$

Only two parameters

$$0 \le \gamma_0 \le 2$$







Pure imaginary Gaussian amplitude

$$a(b) = \mathrm{i}\,\gamma(b)$$

$$\gamma(b) = \gamma_0 \exp(-b^2/b_0^2)$$

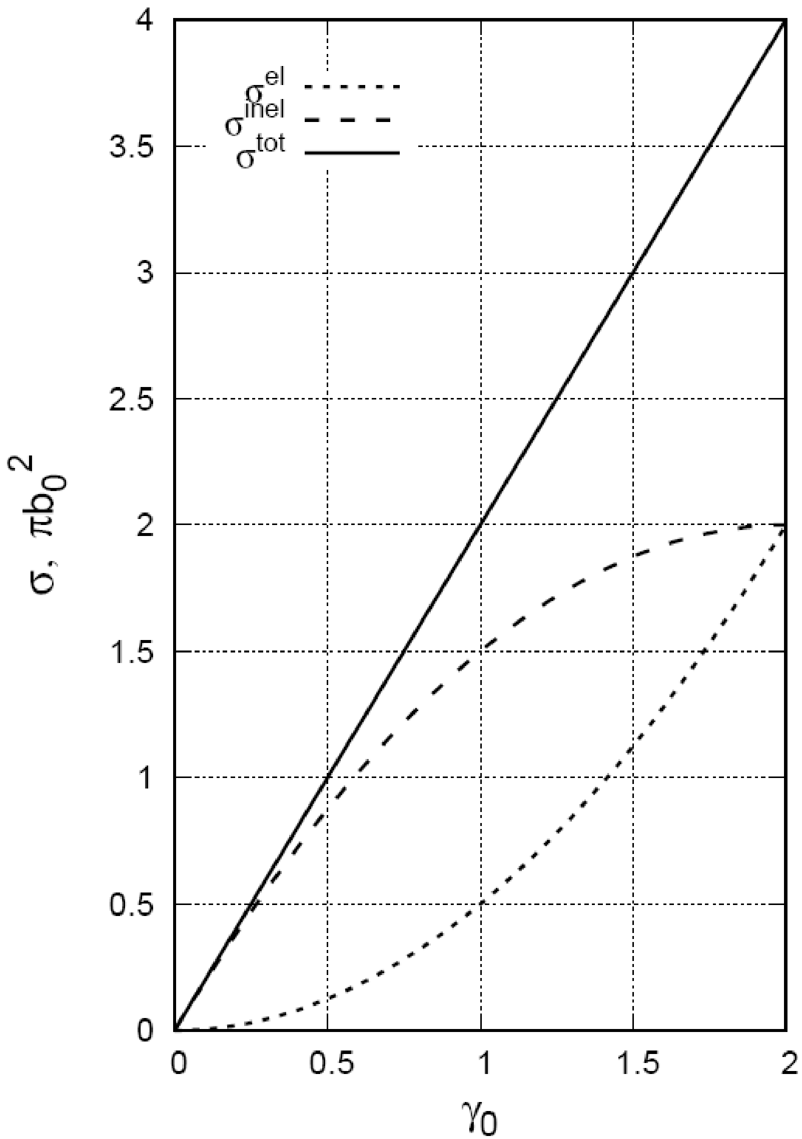
Only two parameters

$$0 \le \gamma_0 \le 2$$

$$\boxed{\gamma_0 = 4\frac{\sigma^{el}}{\sigma^{tot}} = 4\alpha\ ,}$$

$$\alpha \equiv \frac{\sigma^{el}}{\sigma^{tot}}$$

$$b_0^2 = 2B = \frac{\sigma^{tot\,2}}{8\pi\sigma^{el}} = \frac{\sigma^{tot}}{8\pi\alpha}\ .$$



Comparison of pure imaginary Gaussian amplitude with experiment

$$a(b) = \mathrm{i}\,\gamma(b)$$

$$\gamma(b) = \gamma_0 \exp(-b^2/b_0^2)$$

$$0 \le \gamma_0 \le 2$$

$$B \equiv -\frac{d}{d\,q^2} \ln |\tilde{a}(q)|^2 \Big|_{q=0}$$

Only two parameters

$$\gamma_0 = 4\frac{\sigma^{el}}{\sigma^{tot}} = 4\alpha\ ,$$

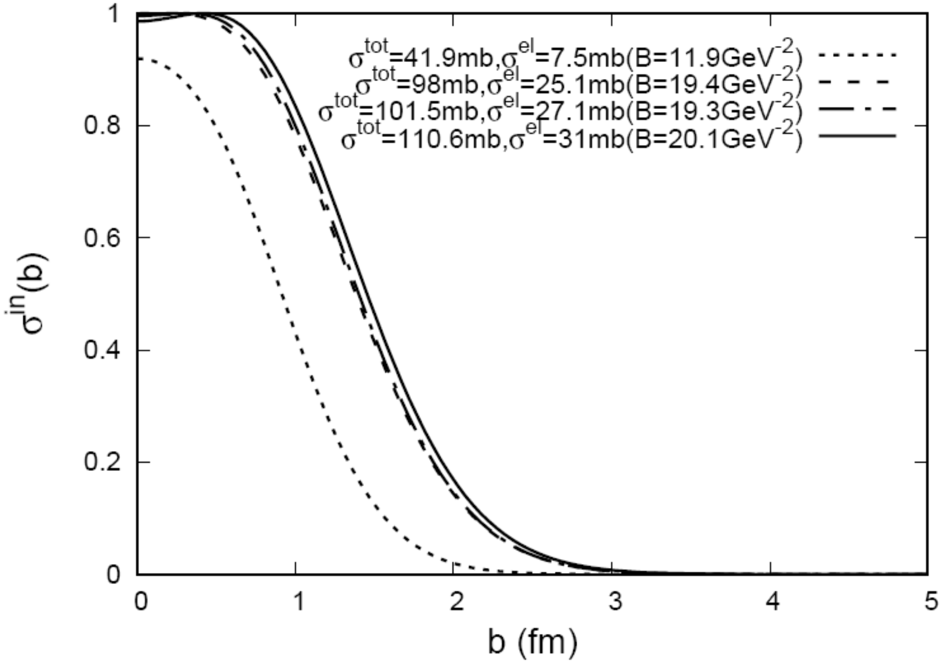
$$\alpha \equiv \frac{\sigma^{el}}{\sigma^{tot}}$$

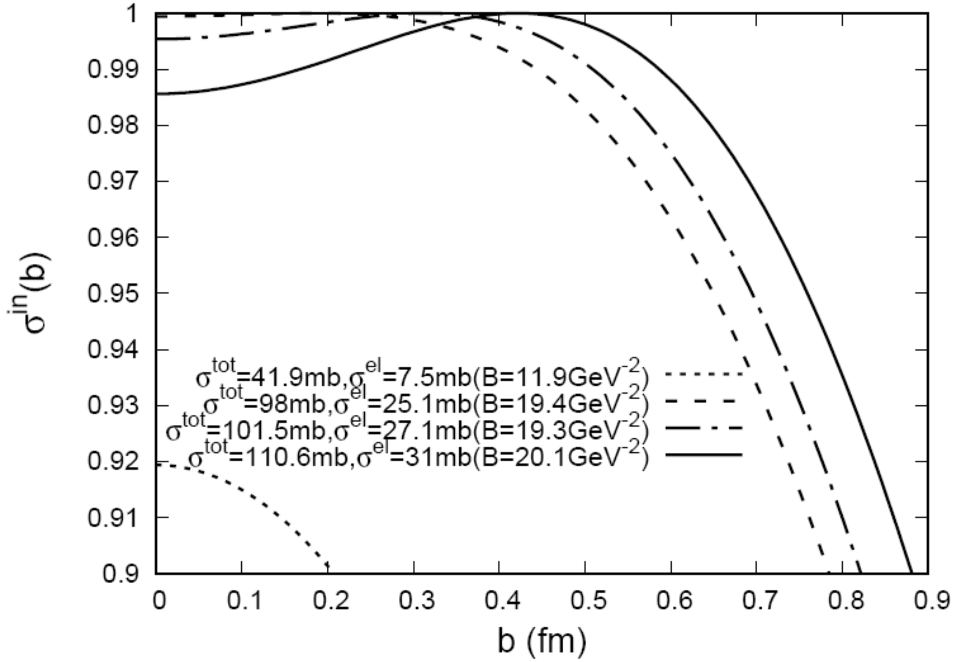
$$b_0^2 = 2B = \frac{\sigma^{tot\,2}}{8\pi\sigma^{el}} = \frac{\sigma^{tot}}{8\pi\alpha}\ .$$

$\sqrt{s}\ (GeV)$	$\sigma^{tot}\ (mb)$	$\sigma^{el}\ (mb)$	σ^{el}/σ^{tot}	$\frac{\mathrm{Re}\,\tilde{a}(q=0)}{\mathrm{Im}\,\tilde{a}(q=0)}$	$B(GeV^{-2})$
45	41.9	7.5	0.179	0.062	11.9
62	43.6	7.7	0.177	0.095	12.5
200	51.5	10	0.194		
540	61.5	13	0.211		15.5
900	68	15	0.221		
1800	77	18	0.234	0.132	16.5
2760	84.7 ± 3.3	21.8 ± 1.4	0.257	0.145	17.1
7000	98.0 ± 2.5	25.1 ± 1.1	0.256	0.145	19.8
8000	$101.7\pm2.9[102.9\pm2.3]$	27.1 ± 1.4	0.266 ± 0.006	$0.14[0.12\pm0.03]$	19.9 ± 0.3
13000	$109.5[110.6\pm3.4]$	$30.7[31.0\pm1.7]$	0.281 ± 0.009	$0.1[0.14]$	20.4

Comparison of pure imaginary Gaussian amplitude with experiment

Dremin, I. M., JETP Letters 99 (2014) 243;100 (2014) 491; Physics-Uspekhi 58 (2015) 61;
Dremin, I. M., Torus or black disk?, Bulletin of the Lebedev Physics Institute 42 (2015) 21.

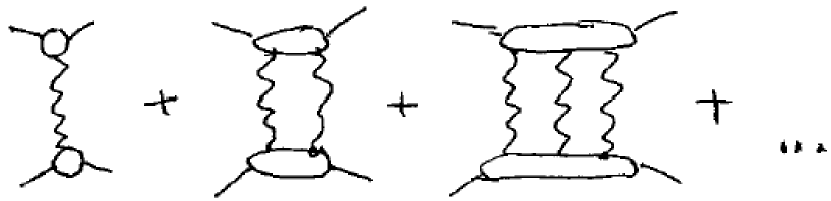




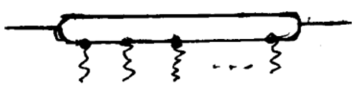
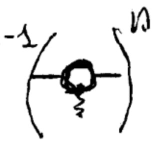
T. Csorgo, for the TOTEM Collaboration,
Recent Results from the CERN LHC Experiment TOTEM, ArXiv: 1903.06992 [hep-ex] (2019).
A.P. Samokhin, Physics Letters B 786 (2018) 100.
Uzhinsky, V., Galoyan, A., Description of the Totem experimental data on elastic pp scattering at 7 TeV in the framework of unified systematic of elastic scattering data, arXiv:1111.4984 [hep-ph].

Gribov-Regge approach

K.Werner, Phys. Rep. 232 (1993) 87.
V.N. Kovalenko, A.M. Puchkov, V.V. Vechernin, D.V. Diatchenko,
Restrictions on pp scattering amplitude imposed by first diffraction minimum data obtained by TOTEM at LHC, IEEE Xplore, IEEE Conference Publications, 7354853; arXiv:1506.04442 [hep-ph], 2015.

$A(s,t) =$ 

$$A(s,t) = \sum_{n=1}^{\infty} A_n(s,t)$$

 $= C^{n-1}$ 

+ quasi-eikonal approximation (!)

$$N_n(k_1, \ldots, k_n) = C_n \prod_{i=1}^{\infty} N(k_i^2) \qquad C_n = C^{n-1}$$

$$A_n(s,t) = \frac{\mathrm{i}^{n-1} \pi^{1-n} C_n}{n!} \int \prod_{i=1}^{n} \mathrm{d}^2 k_i \, \delta^{(2)}\left(k - \sum k_i\right) \prod_{j=1}^{n} N(k_j^2) D(s, k_j^2)$$

$$D(s,t) = \eta(\alpha(t)) \left(\frac{s}{s_0}\right)^{\alpha(t)-1} \qquad \eta(\alpha(t)) = -\frac{\exp(-\mathrm{i}\frac{\pi\alpha(t)}{2})}{\sin(\frac{\pi\alpha(t)}{2})} ;$$

$$\alpha(t) = 1 + \Delta + \alpha' t = 1 + \Delta - \alpha' k^2, \qquad D(s,t) = \eta(\alpha(t)) \ \exp(\Delta y) \exp(-\alpha' y k^2)$$

Gribov-Regge approach – impact parameter space

$$A(s,t) = \frac{\mathrm{i}}{4\pi}\int \mathrm{d}^2 b \, \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{b})\gamma(s,b)$$

$$\gamma(s,b) = C^{-1}\{1 - \exp[-C\omega(s,b)]\}$$

$$\omega(s,b) := \frac{1}{\mathrm{i}\pi}\int \mathrm{d}^2 k \, N(k^2)D(s,k^2)\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{b}),$$

$$\omega(s,b) = \frac{N_0}{r^2}\, e^{y\Delta}\, e^{-\frac{b^2}{4r^2}}$$

$$N(k^2) = N_0 \exp(-R^2 k^2),$$

$$r^2 \equiv R^2 + \alpha' y\,, \qquad y = \ln(s/s_0)$$

$$z = \frac{2N_0 C}{r^2}\, e^{y\Delta} = \frac{2N_0 C}{r^2}\left(\frac{s}{s_0}\right)^{\Delta}$$

$$A_n(s,t) = \frac{\mathrm{i}N_0\exp(\Delta y)}{n!}\left(\frac{-N_0 C\exp(\Delta y)}{\pi}\right)^{n-1}$$

$$\times \int \prod_{i=1}^{n} \mathrm{d}^2 k_i \, \delta(k - \sum k_i) \prod_{i=1}^{n} \exp\{-[R^2 + \alpha'(y + \tfrac{1}{2}\mathrm{i}\pi]k_i^2\},$$

$$A_n(s,t) = \frac{\mathrm{i}N_0\exp(\Delta y)}{n\, n!}\left(\frac{-N_0 C\exp(\Delta y)}{R^2 + \alpha' y}\right)^{n-1} \exp[-(R^2 + \alpha' y)k^2/n]$$

$$A(s,t) = \mathrm{i}\, N_0\, e^{y\Delta}\sum_{n=1}^{\infty}\frac{(-z/2)^{n-1}}{n\cdot n!}\exp(-\mathrm{k}^2\, r^2/n)\ .$$

Gribov-Regge approach – the *a(b)* amplitude

$$a(\mathbf{b}) = \frac{1}{\pi}\int d^2\mathbf{k}\, A(s,t)\, \exp(-\mathrm{i}\mathbf{k}\cdot\mathbf{b}) \qquad a(b) = -\frac{\mathrm{i}}{C}\sum_{n=1}^{\infty}\frac{(-z/2)^n}{n!}e^{-\frac{nb^2}{4r^2}}\ ,$$

$$a(b) = \frac{\mathrm{i}}{C}\left[1-\exp\left(-\frac{z}{2}e^{-\frac{b^2}{4r^2}}\right)\right] = \frac{\mathrm{i}}{C}\left[1-e^{-X}\right] \qquad X \equiv \frac{z}{2}\,e^{-\frac{b^2}{4r^2}}\ .$$

$$a(b) = \mathrm{i}\gamma(s,b) \qquad X = C\,\omega(s,b)$$

$$\sigma^{tot}(b) = \frac{2}{C}\left[1-e^{-X}\right] \qquad \sigma^{el}(b) = \frac{1}{C^2}\left[1-e^{-X}\right]^2 = \frac{1}{C^2}\left[2(1-e^{-X})-(1-e^{-2X})\right]$$

$$\sigma^{tot} = \int d^2b\,\sigma^{tot}(b) = \frac{2\pi}{C}\int_0^{\infty} db^2\,\left[1-\exp\left(-\frac{z}{2}e^{-\frac{b^2}{4r^2}}\right)\right]$$

$$\sigma^{tot} = \frac{2}{C}(4\pi r^2)\int_0^{z/2}\frac{dx}{x}\,(1-e^{-x}) = \frac{2}{C}(4\pi r^2)\Phi_1(z/2)$$

$$\sigma^{el} = \frac{1}{C^2}(4\pi r^2)[2\,\Phi_1(z/2)-\Phi_1(z)]$$

Gribov-Regge approach – cross sections

$$\frac{\sigma^{el}}{\sigma^{tot}} = \frac{1}{C}\left[1 - \frac{\Phi_1(z)}{2\Phi_1(z/2)}\right]$$

$$B = 2r^2 \frac{\Phi_2(z/2)}{\Phi_1(z/2)}$$

$$\Phi_m(z) \equiv \int_0^z \frac{dx}{x^m}\left(1 - e^{-x}\right) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^n}{n!\, n^m} .$$

$$\Phi_1(z) = \ln z + \gamma_E + E_1(z)$$

$$\gamma_E = 0,5772$$

$$E_1(z) \equiv \int_z^\infty e^{-x} \frac{dx}{x} < \frac{1}{z} e^{-z}$$

$$f(z) = \frac{1}{z}\Phi_1(z) = \frac{1}{z}\int_0^z \frac{dx}{x}\left(1 - e^{-x}\right) = \sum_{n=1}^{\infty} \frac{(-z)^{n-1}}{n!\, n}$$

Gribov-Regge approach – possibility of halo?

G.H. Arakelyan, A. Capella, A.B. Kaidalov, Yu.M. Shabelski, *Eur. Phys. J. C* 26 (2002) 81.
Capella, A; Ferreiro, E.G. Eur. Phys. J. C. 72 (2012) 1936.

V.N. Kovalenko, A.M. Puchkov, V.V. Vechernin, D.V. Diatchenko,
Restrictions on pp scattering amplitude imposed by first diffraction minimum data obtained by TOTEM at LHC, IEEE Xplore, IEEE Conference Publications, 7354853; arXiv:1506.04442 [hep-ph], 2015.

Vechernin V.V., Belokurova S.N. Theor. Math. Phys. 216 (2023) 1299.

V.A. Abramovsky, N.V. Abramovskaya and N.V. Evstigneeva,
Multipomeron theory in the Gribov approach,
International Journal of Modern Physics A 31 (2016) 16450132.

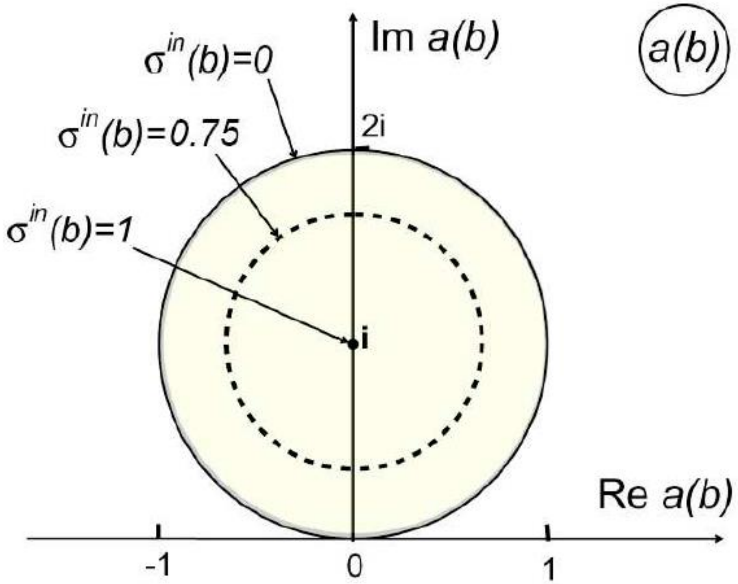
$$C = 0.8 \ ?$$

$$C = 1 + \frac{\sigma^{dif}}{\sigma^{el}}$$ - follows from the AGK rules for non-enhanced Regge diagrams

$$a(0) = \frac{\mathrm{i}}{C}\left[1 - \exp\left(-\frac{z}{2}\right)\right]$$

$$z = \frac{2N_0 C}{r^2}\, e^{y\Delta} = \frac{2N_0 C}{r^2}\left(\frac{s}{s_0}\right)^{\Delta}$$

$$r^2 \equiv R^2 + \alpha' y \ , \qquad y = \ln(s/s_0)$$



Modified quasi-eikonal approximation for Regge approach

$$N_n(k_1, k_2, ..., k_n) = C_n((k_1 + ... + k_n)^2) \prod_{i=1}^{n} N(k_i^2)$$

$$C_n \to C_n((k_1 + ... + k_n)^2) = C_n(t) \qquad t = -k^2 = -(k_1 + ... + k_n)^2$$

$$C_n(t) = C^{n-1}(t) = [C(t)]^{n-1}$$

$$A_n(s,t) = \frac{1}{4\pi i C(t)} \int d^2b \exp(i\mathbf{k}\cdot\mathbf{b}) \frac{[-C(t)\omega(s,b)]^n}{n!} \qquad t = -\mathbf{k}^2$$

- not a Fourier transform

$$A(s,t) = \frac{i}{4\pi} \int d^2b \exp(i\mathbf{k}\cdot\mathbf{b}) \gamma_t(s,b)$$

$$\gamma_t(s,b) \equiv \frac{1}{C(t)} \{1 - \exp[-C(t)\omega(s,b)]\}$$

$$A(s,t) = \frac{i}{2} \int_0^{\infty} J_0(kb)\, \gamma_t(s,b)\, b\, db$$

$\omega(s,b)$ - remains unchanged

$$J_0(z) = 2\pi \int_0^{2\pi} d\phi \exp(iz\cos\phi)$$

$$\sigma^{el} = \int_0^{-\infty} \frac{d\sigma^{el}}{dt} dt$$

$$\frac{d\sigma^{el}}{dt} = \frac{|T(s,t)|^2}{16\pi s^2} = 4\pi |A(s,t)|^2 = \pi \left| \int_0^{\infty} J_0(\sqrt{-t}\, b)\, \gamma_t(s,b)\, b\, db \right|^2$$

σ^{tot} - old formulae with *C => C(0)*

$$B = \frac{d}{dt} \ln \frac{d\sigma^{el}}{dt} \bigg|_{t=0}$$

Summary

- Simple **Gaussian** approximation **can reproduce** the **halo** effect in pp collisions at ultra-high energies.

- Gribov-Regge approach within the **standard quasi-eikonal** approximation **cannot reproduce the halo** effect in pp collisions at ultra-high energies.

- A **modified quasi-eikonal approximation** for the Gribov-Regge approach is **proposed**.

$$\sigma^{tot} \sim \frac{1}{C(0)} , \qquad \sigma^{el} \sim \frac{1}{C^2(t)} , \qquad \frac{\sigma^{el}}{\sigma^{tot}} \sim \frac{C(0)}{C^2(t)}$$

The author is grateful to Saint-Petersburg State University for a research project 103821868.

Backup slides

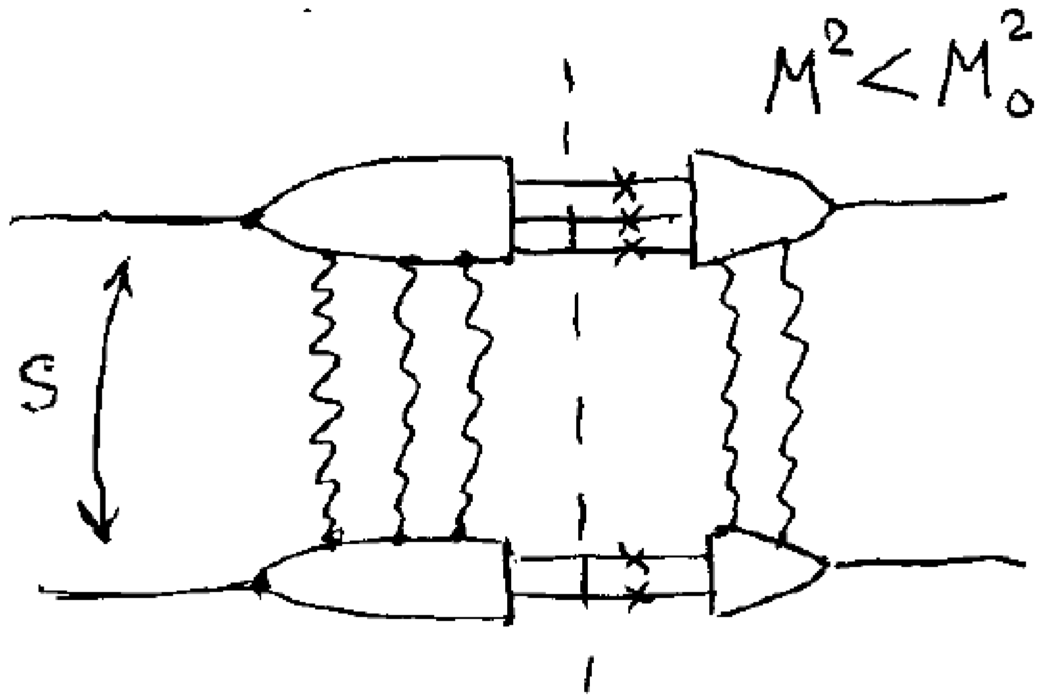


Рис. 2: Вклад промежуточных состояний с инвариантной массой M меньшей какого-то фиксированного значения, не зависящего от начальной энергии s, $M^2 \leq M_0^2$, дающий вклад в σ^{dif}. Возникает при разрезании амплитуды pp рассеяния между померонами.

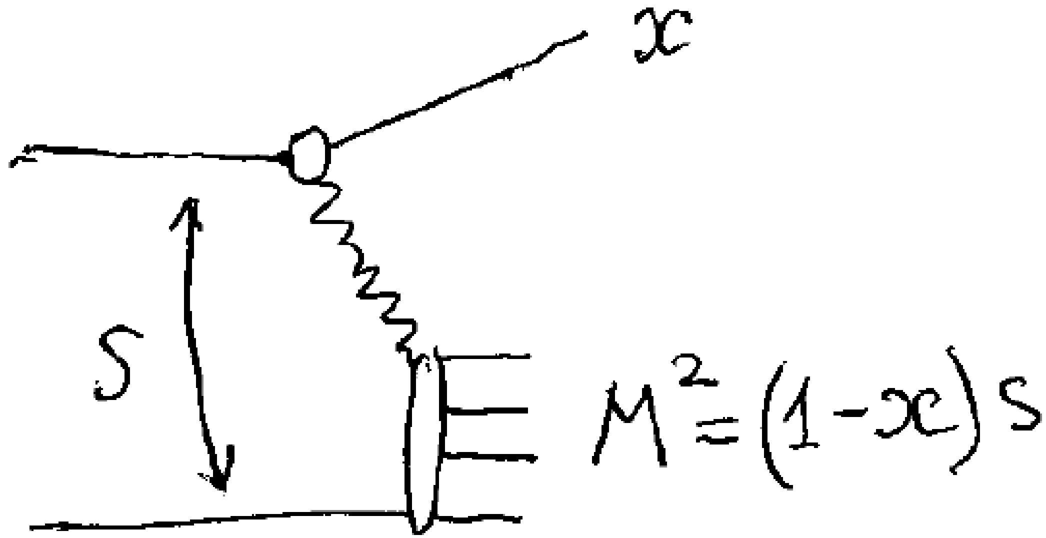


Рис. 3: Пример дифракционного процесса с большой массой $M^2 = (1-x)s$, где $x \to 1$ - фейнмановская переменная рассеившегося начального протона. В рамках реджевского подхода такие процессы могут быть описаны путем добавления вершины 3-померонного взаимодействия.

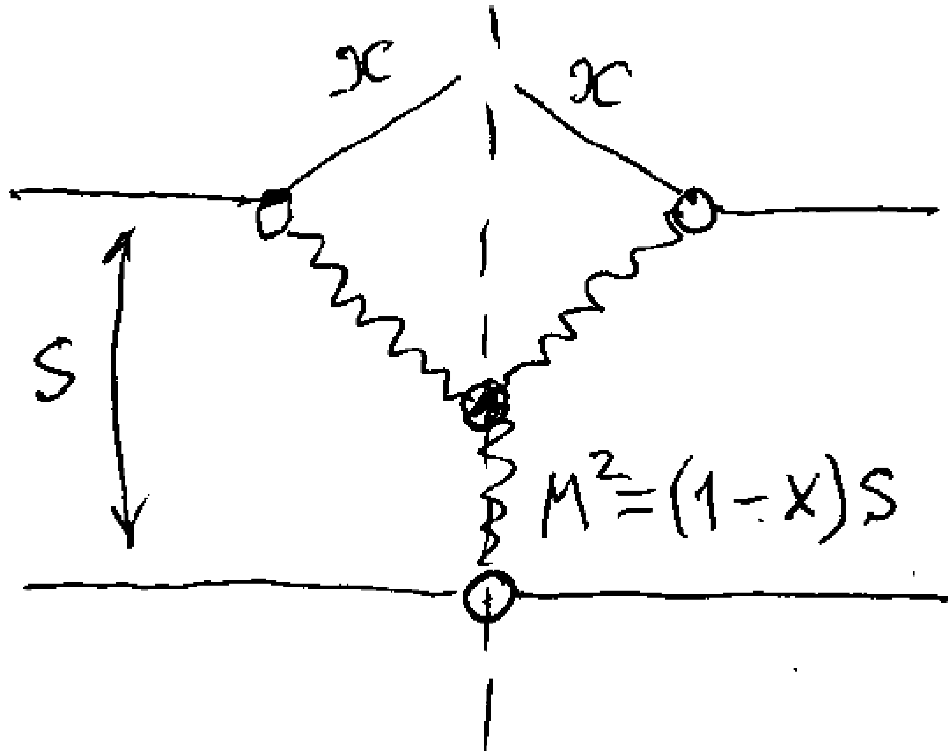


Рис. 4: Реджевская диаграмма с вершиной 3-померонного взаимодействия, описывающая дифракционный процесс с большой массой $M^2 = (1-x)s$, при рассеивании начального протона со значением фейнмановской переменной близкой к 1, $x \to 1$.

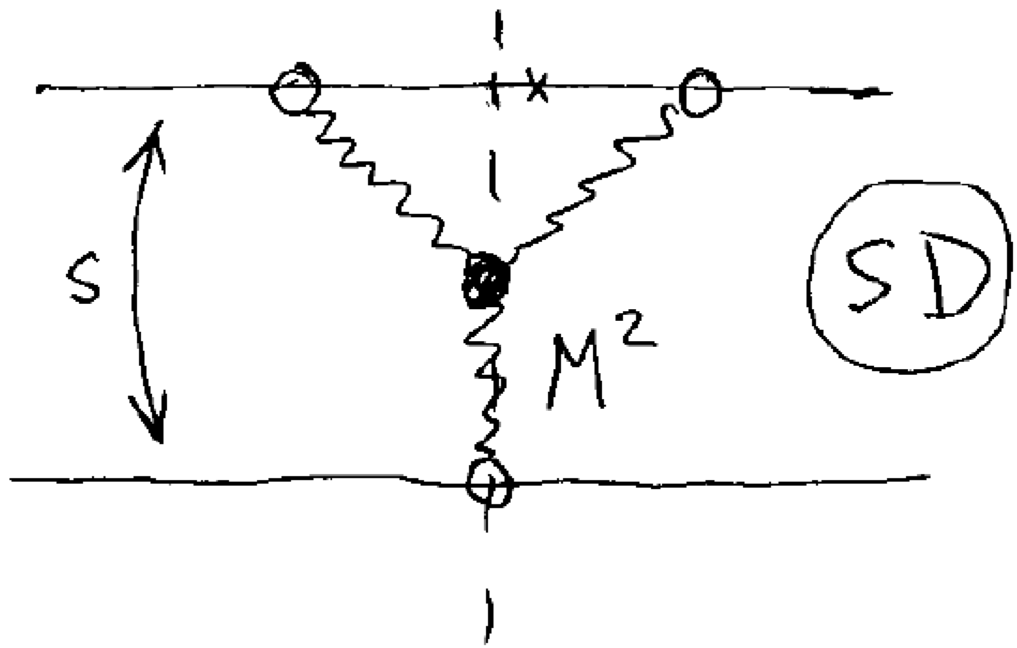


Рис. 5: Усиленая реджеонная диаграмма, описывающая вклад дифракционного процесса с большой массой $m_N^2 \ll M^2 \ll s$ в полное сечение рассеяния.

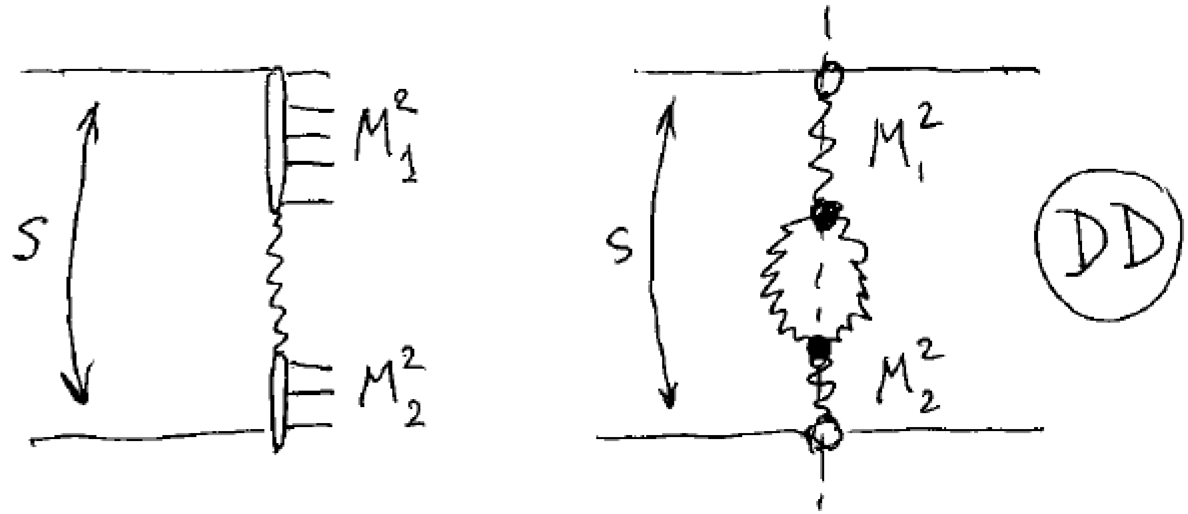


Рис. 6: Вклад в полное сечение и от дважды-дифракционных (Double Difraction) процессов с большими массами M_1^2 и M_2^2 пропорциональными s, и удовлетворяющими условию $m_N^2 \ll M_1^2 \sim M_2^2 \ll s$.

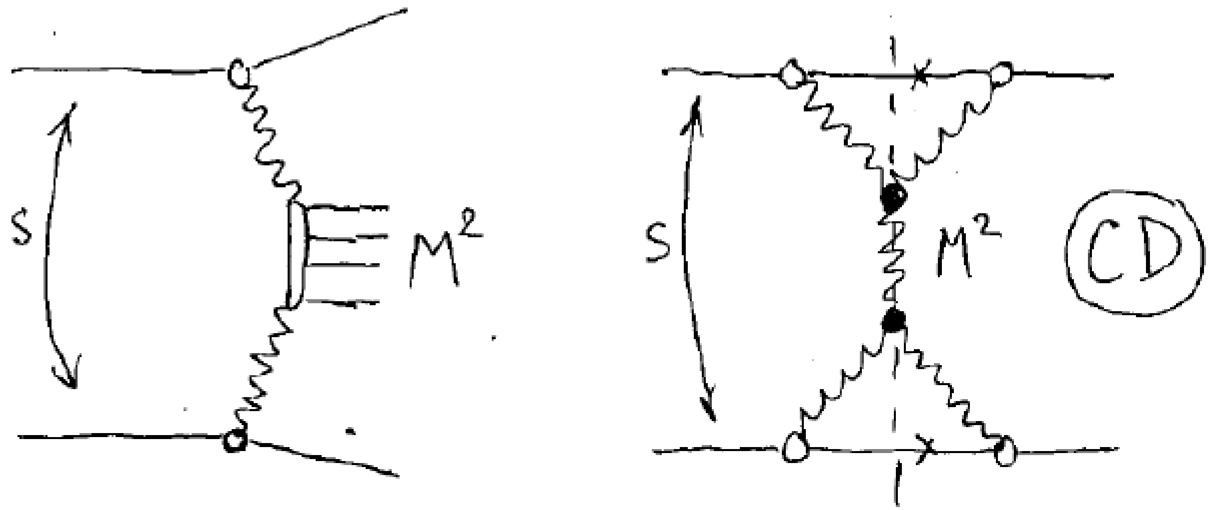


Рис. 7: Вклад в полное сечение и от процессов центральной дифракции (Central Difraction) с большой массой M^2 пропорциональной s, и удовлетворяющей условию $m_N^2 \ll M^2 \ll s$.

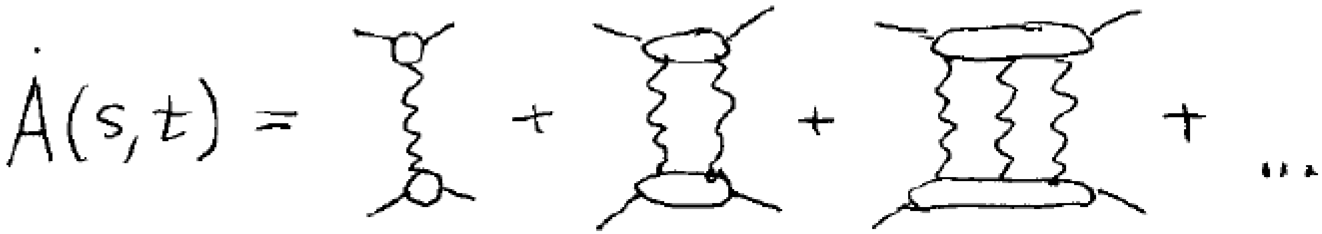


Рис. 8: Диаграммы, дающие вклад в амплитуду упругого pp-рассеяния $A(s,t)$ в стандартном реджевском подходе.

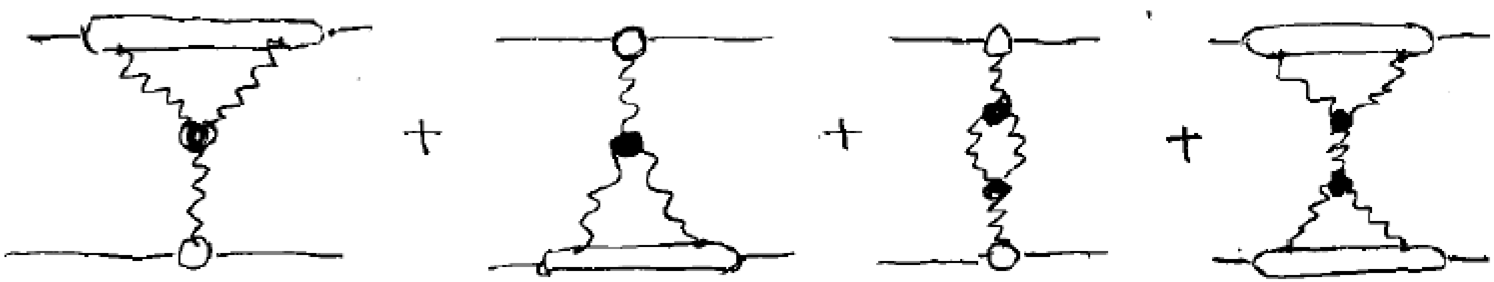


Рис. 9: Вклад в амплитуду упругого pp-рассеяния $A(s,t)$ усиленных диаграмм, содержащих 3-х померонные вершины. Их рассечение позволяет описать вклад дифракционных процессов с большими массами в полное сечение, что приводит к необходимости пересмотра правил АГК.