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*Experimental and Theoretical Investigation of Nuclear Reactions*

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# CROSS SECTIONS OF MUON CATALYZED FUSION REACTIONS

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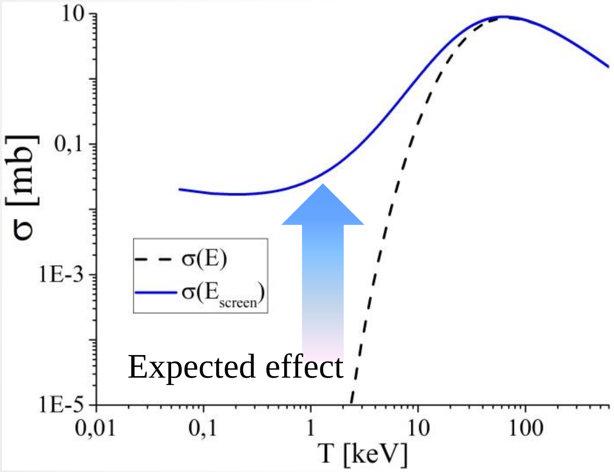
# An acceleration of the nuclear reaction rate?

## Electron Screening

$$\sigma(E_{screen}) \approx \sigma(E) \cdot \exp\left(\frac{\pi \cdot \eta \cdot U_{screen}}{E}\right)$$

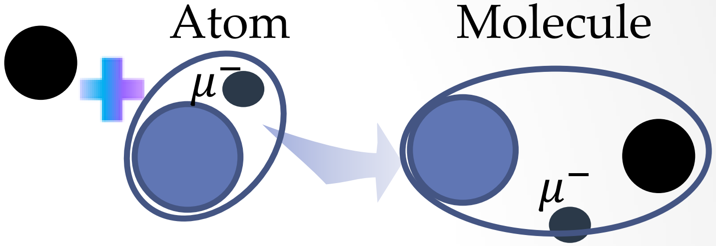
$U_{screen} \in [65, 220]$ eV for $^3He(d,p)^4He$




- ✓ Experimental evidence is available

## Muon catalyzed fusion

[Alvarez, 1957; Jones, 1986]




«Prompt fusion»

$$\langle \sigma v \rangle_{\mu dt} \approx 1.7 \cdot 10^{-11}\ \mathrm{cm^3/s}$$

[Kamimura, PRC107(2023)034607]

$$\langle \sigma v \rangle_{dt}^{max} \approx 10^{-15}\ \mathrm{cm^3/s}$$

- ✓ Low-temperature catalysis has been experimentally confirmed
- ✓ **The expected effect for reactions without the formation of molecules is unknown**

## What is all this for?

- ✓ Thermonuclear driver
- ✓ Boosting the yield of nuclear processes

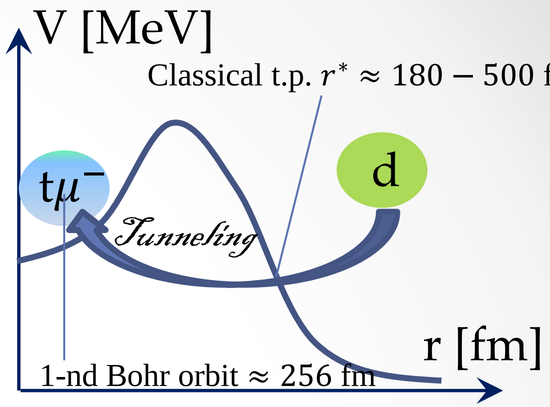
# $\mu^-$ in nuclear reaction zone

Known rate estimates raise questions :

[L.I. Ponomarev, Contemporary Physics (1990) 31:4, 219-245]:

$$\lambda_f \approx \frac{1}{4\pi a_\mu^{\;3}} \exp\left[-\pi(2Mr^*)^{1/2}\right] \quad ?? \rightarrow \quad \lambda_f^{\text{in-flight}} \approx \left(\frac{a_\mu}{a_0}\right)^3 \lambda_f \approx 10^{-7}\lambda_f$$

$a_0, a_\mu$ – Bohr radii, $e^-$ – atom and $\mu^-$ – atom




The number $N$ of catalyzed fusion for $n_d = 5 \cdot 10^{20}$ см$^{-3}$:

$$N \approx \tau_{\mu^-} \cdot \sigma_{dt} \cdot n_d \cdot v_{\mu^- t} = \begin{cases} 0.017 \cdot \sigma_{dt}[\text{barn}], at\ T_d = 1\ \text{keV} \\ 0.054 \cdot \sigma_{dt}[\text{barn}], at\ T_d = 10\ \text{keV} \end{cases}$$

$$d + (t\mu^-) \rightarrow d + \mu^- + t$$
$$E_{th} = 4.45\ \text{keV}$$
$$d + (t\mu^-) \rightarrow n + \mu^- + {}^4\text{He}$$
$$\min(E_{\mu^- {}^4\text{He}}) = 17.58\ \text{MeV}$$

$$d + ({}^3\text{He}\mu^-) \rightarrow d + \mu^- + {}^3\text{He}$$
$$E_{th} = 17.82\ \text{keV}$$
$$d + ({}^3\text{He}\mu^-) \rightarrow p + \mu^- + {}^4\text{He}$$
$$\min(E_{\mu^- {}^4\text{He}}) = 18.34\ \text{MeV}$$

- For **cold** fusion $T \leq 100\ K°$, $N \leq 150$

[Jones, Nature 321 (1986) 127] → $\sigma_{dt} \leq 3000$ [barn]

- In-flight reactions (with muonic atom $t\mu^-$)

**If $\sigma_{dt}$ = max($\sigma_{dt}$) ≡ 5 [barn]**, then

$N \leq 0.085$ at $T_d = 1$ keV and $N \leq 0.27$ at $T_d = 10$ keV

**If $N \approx 100$**, then $\sigma_{dt} \approx 6000$ [barn] at $T_d = 1$ keV

and $\sigma_{dt} \approx 2000$ [barn] at $T_d = 10$ keV

✓ Non-overlapping temperature ranges for cold fusion and in-flight fusion reactions

✓ Expected catalysis of in-flight fusion reactions resides in the realm of muonic atom ionization

# Computation of reaction cross sections

Let's start with reactions: $d^3\mathrm{H} \rightarrow n^4\mathrm{He}$, $d^3\mathrm{He} \rightarrow p^4\mathrm{He}$

- **Res** (One resonance model)

$$\frac{d\sigma}{d\Omega} = \frac{1}{4p_i^2} \cdot \frac{2J+1}{3(2I+1)} \cdot \frac{\Gamma_i \Gamma_f \cdot 10 \cdot (\hbar c)^2}{(E-E_R)^2 + \frac{1}{4}\left(\Gamma_i + \Gamma_f\right)^2}$$

$$\Gamma_i = 2\gamma^2 \sqrt{\xi E_f} \cdot \exp(-\pi\eta)\,, \xi = \frac{2\mu R^2}{(\hbar c)^2}, \Gamma_f = 2\gamma^2 \frac{\sqrt{\left(\xi E_f\right)^5}}{9 + 3\xi E_f + \left(\xi E_f\right)^2} \cdot \exp(-\pi\eta)$$

- **Pot** (Simple potential model)

$$\lambda_l^J \Xi_l(p_f)\Xi_l(p_i) \equiv v_{22},$$

$$[{}^1S_{1/2}, {}^2P_{1/2}, {}^2P_{3/2}, {}^2D_{3/2}]$$

$$\frac{V_R}{\mu_R^2 + p_f^2 + p_i^2 - 2p_f p_i y_{p_f p_i}} \equiv \begin{cases} v_{11} - elastic \\ v_{21} - thermonuclear \\ v_{12} - rearrangement \end{cases}$$

$$\Xi_l(p) = \frac{c_1 p^l}{(p^2 + b_1^2)^{l+1}} + \frac{c_2 p^{l+2}}{(p^2 + b_2^2)^{l+2}}$$

$$\begin{cases} t_{11} = v_{11} + v_{11}G_0 t_{11} + v_{12}G_0 t_{21} \\ t_{21} = v_{21} + v_{21}G_0 t_{11} + v_{22}G_0 t_{21} \end{cases}$$

- **A-Cont** (Analytical continued model)

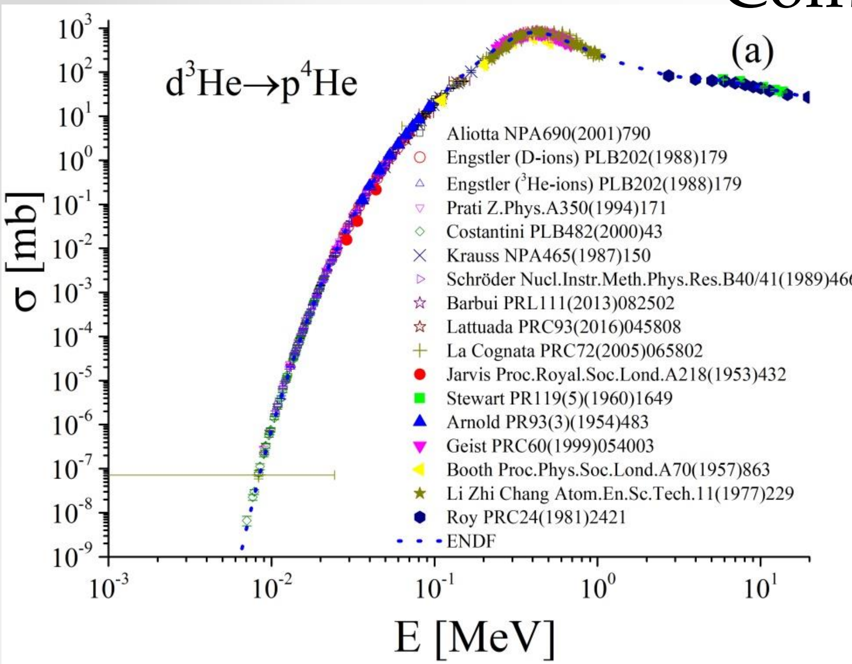
$$\tilde{v}_{\beta\alpha}(\vec{p}_f, \vec{p}_i) = v_{\beta\alpha}(\vec{p}_f, \vec{p}_i) + \frac{i\mu\sqrt{2\mu z}}{\pi} \sum_{\gamma} v_{\beta\gamma}\left(\vec{p}_f, \sqrt{2\mu z} \cdot \hat{\vec{p}}\right) \times$$

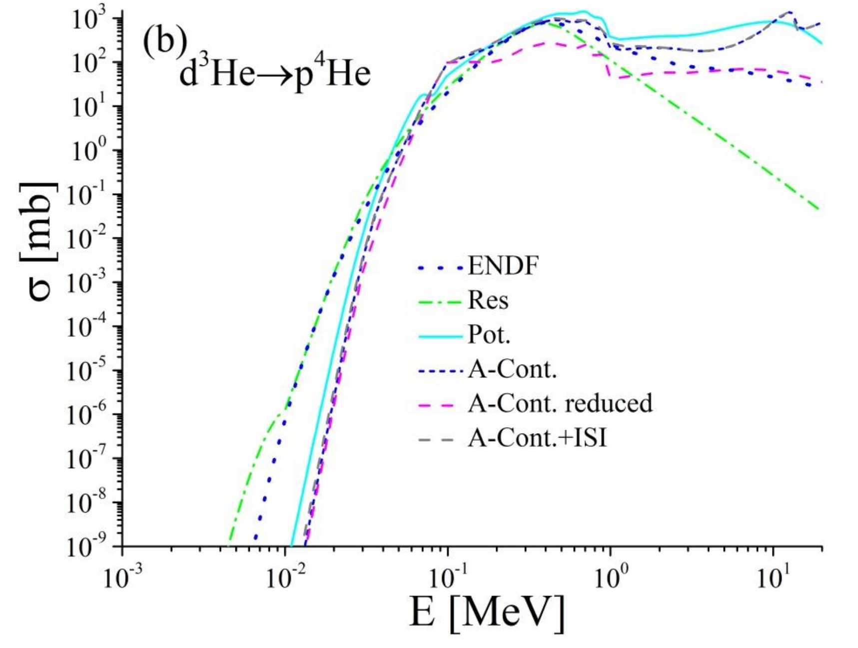
$$\sum_{\delta} \left[\delta_{\delta\gamma} - \frac{i\mu}{\pi}\sqrt{2\mu z} \cdot v_{\delta\gamma}\left(\sqrt{2\mu z} \cdot \hat{\vec{p}}, \sqrt{2\mu z} \cdot \hat{\vec{p}}\right)\right]_{\gamma\delta}^{-1} \times v_{\delta\alpha}\left(\sqrt{2\mu z} \cdot \hat{\vec{p}}, \vec{p}_i\right)$$

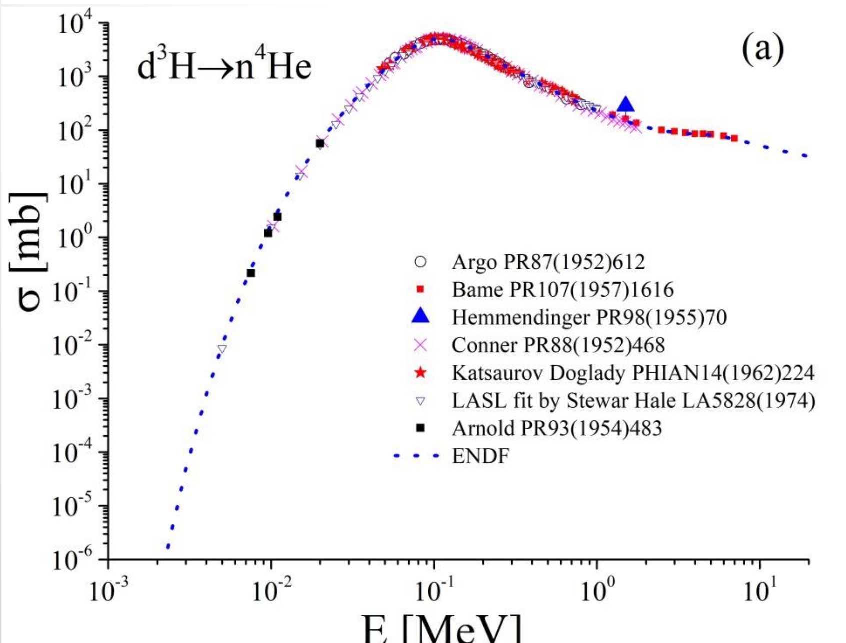
Lippman Schwinger equation is numerically solved with potentials $\tilde{v}_{11}$, $\tilde{v}_{12}$, $\tilde{v}_{21}$

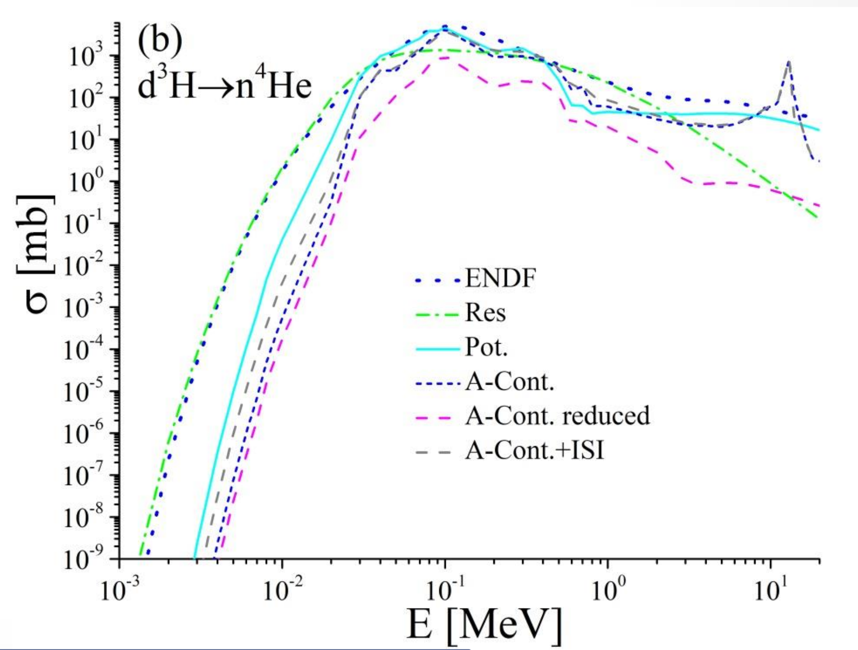
[Yu.V. Orlov, JETP letters 33(1981)380]

# Computation of reaction cross sections



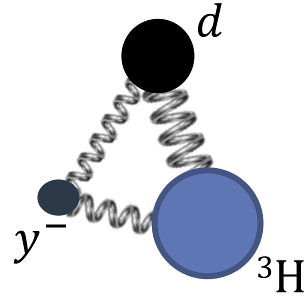



# Precise three-body dynamics

For processes like as $d + (y^{-\,3}\mathrm{H}) \rightarrow n + y^{-} + {}^{4}\mathrm{He}$, where $y^{-} \in [e^{-}, \mu^{-}]$ there are partitions and channels representation:

$$\begin{matrix}\alpha \\ \beta\end{matrix}\begin{pmatrix} 1 & 2 & 3 \\ d & y^{-} & {}^{3}\mathrm{H} \\ n & y^{-} & {}^{4}\mathrm{He}\end{pmatrix}$$

$$T_i \rightarrow T_i^{\alpha\beta}$$




All accessible channels are reflected in the following representations:

$$t_1 \rightarrow t_1^{\alpha\beta} = \begin{pmatrix} t_1^{y^{-\,3}\mathrm{H}\rightarrow y^{-\,3}\mathrm{H}} & 0 \\ 0 & t_1^{y^{-\,4}\mathrm{He}\rightarrow y^{-\,4}\mathrm{He}}\end{pmatrix}$$

$$t_2 \rightarrow t_2^{\alpha\beta} = \begin{pmatrix} t_2^{d^3\mathrm{H}\rightarrow d^3\mathrm{H}} & t_2^{d^3\mathrm{H}\rightarrow n^4\mathrm{He}} \\ t_2^{n^4\mathrm{He}\rightarrow d^3\mathrm{H}} & t_2^{n^4\mathrm{He}\rightarrow n^4\mathrm{He}}\end{pmatrix}$$

$$t_3 \rightarrow t_3^{\alpha\beta} = \begin{pmatrix} t_3^{y^{-}d\rightarrow y^{-}d} & 0 \\ 0 & t_3^{y^{-}n\rightarrow y^{-}n}\end{pmatrix}$$

**System of Faddeev equations with thermonuclear fusion**

$$\begin{pmatrix} T_1^{11} \\ T_1^{21} \\ T_2^{11} \\ T_2^{21} \\ T_3^{11} \\ T_3^{21} \end{pmatrix} = \begin{pmatrix} t_1^{11}(\phi_2+\phi_3) \\ t_1^{21}(\phi_2+\phi_3) \\ t_2^{11}(\phi_3+\phi_1) \\ t_2^{21}(\phi_3+\phi_1) \\ t_3^{11}(\phi_1+\phi_2) \\ t_3^{21}(\phi_1+\phi_2) \end{pmatrix} + \begin{pmatrix} 0 & 0 & t_1^{11}R_0 & 0 & t_1^{11}R_0 & 0 \\ 0 & 0 & 0 & t_1^{22}R_0 & 0 & t_1^{22}R_0 \\ t_2^{11}R_0 & t_2^{12}R_0 & 0 & 0 & t_2^{11}R_0 & t_2^{12}R_0 \\ t_2^{21}R_0 & t_2^{22}R_0 & 0 & 0 & t_2^{21}R_0 & t_2^{22}R_0 \\ t_3^{11}R_0 & 0 & t_3^{11}R_0 & 0 & 0 & 0 \\ 0 & t_3^{22}R_0 & 0 & t_3^{22}R_0 & 0 & 0 \end{pmatrix} \begin{pmatrix} T_1^{11} \\ T_1^{21} \\ T_2^{11} \\ T_2^{21} \\ T_3^{11} \\ T_3^{21} \end{pmatrix}$$

Note:

analogues reasoning regarding rearrangement operators $U_{ij}^{\alpha\beta}$ where $T_i^{\beta\alpha} = \sum_{\gamma} t_i^{\beta\gamma} R_0 U_i^{\gamma\alpha}$ and $U_i^{\beta\alpha} = \sum_k U_{ki}^{\beta\alpha}$ leads to an equation with the same, but transposed, integral kernel and a different inhomogeneous term.

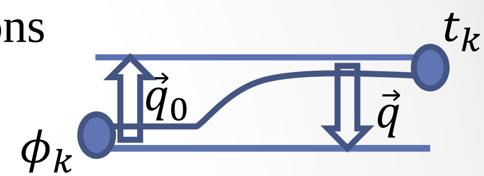
The determinant of the original and transposed matrix is the same - the binding energy of the three-body system does not depend of the method of solution (rearrangement $U_{ij}^{\alpha\beta}$ operators or breakup $T_i$ matrices)

# Precise three-body dynamics

- To achieve the goal (cross sections in the three-body problem)

it is sufficient to find only $T_1^{11}, T_1^{21}$.

- Faddeev equation in the continuum has bound states corresponding to two-body subsystems, **therefore solving the equation by algebraic methods is difficult.**
- The solution is sought in the form of iterations $(C_n)_i^{\alpha\beta}$ where n=0,1,2,…, grouped into Padé approximants $T_i^{\alpha\beta} \approx \sum_n (C_n)_i^{\alpha\beta}$ .
- In present case $t_1^{21}, t_1^{12}, t_3^{12}, t_3^{21}, t_3^{22}$ —are absent under the conditions
- Jacobi momenta are defined in terms of the overall center-of-mass systems momenta



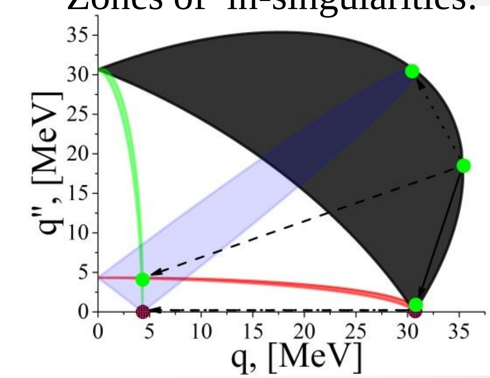

- Scattering energy

$$E = T\frac{m_{23}}{m_{23}+m_1} - |E_{b,23}|$$

- The resolvent energy is shifted

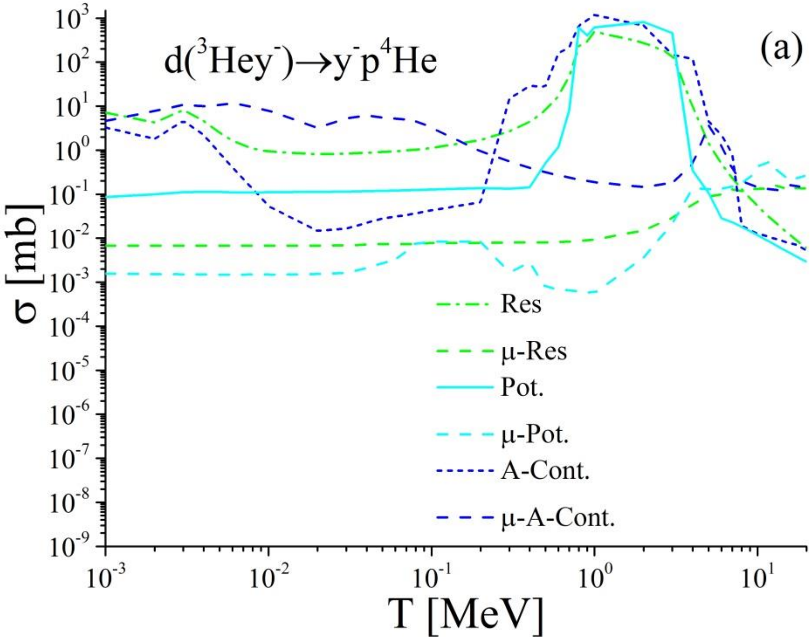
$$z_{i(jk)} = E - \frac{q_{i(jk)}^2}{2M_{i(jk)}}$$

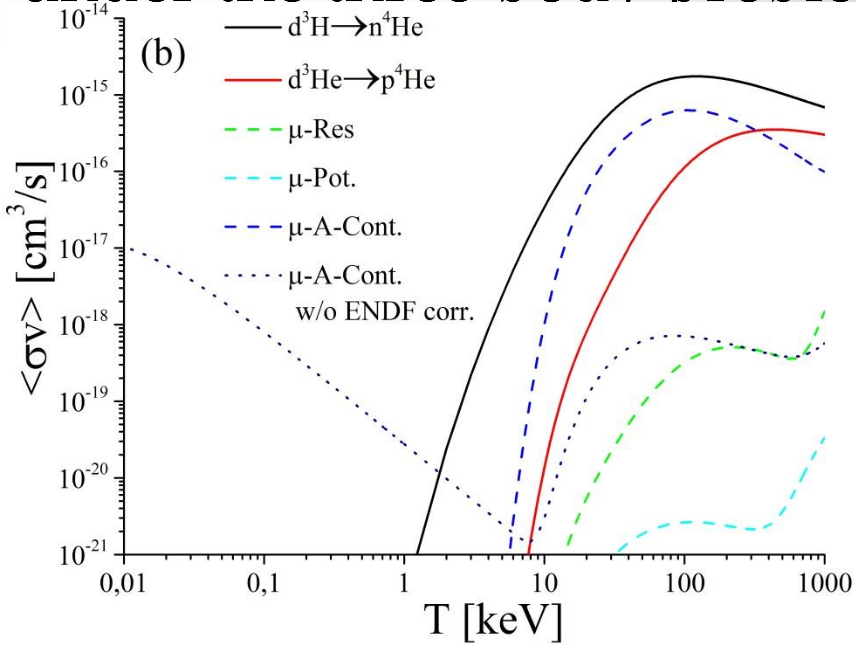
Zones of ln-singularities:

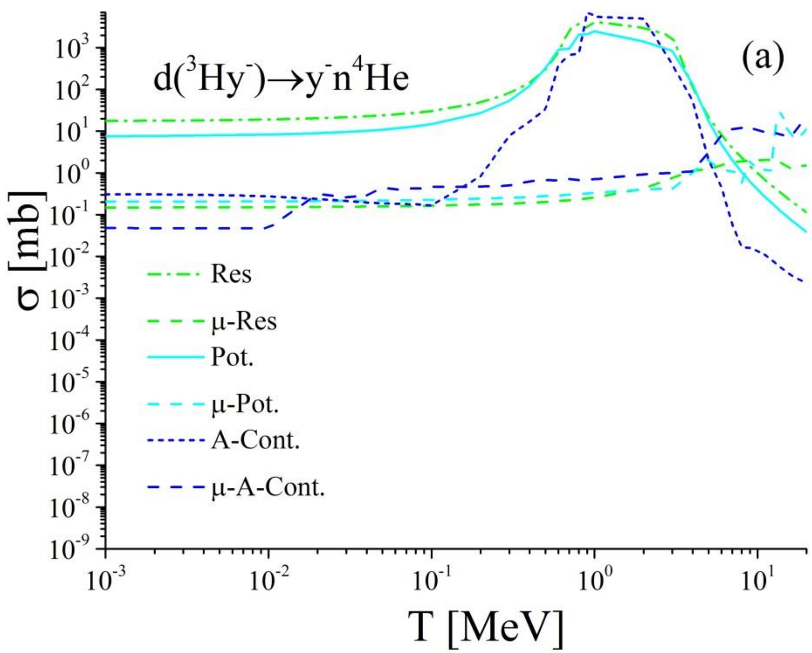


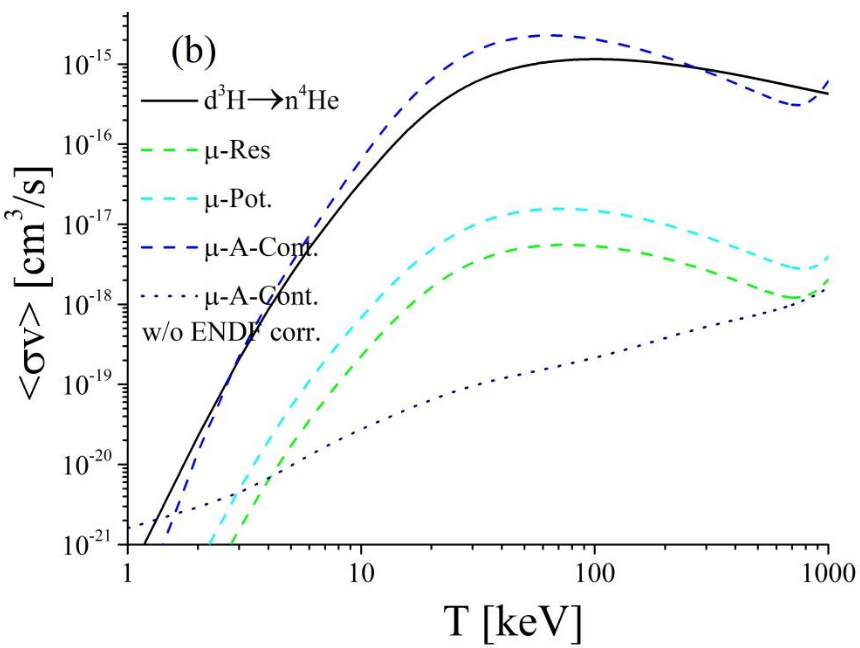

- $U_0(\vec{p},\vec{q};\vec{q}_0) = \langle \vec{p}\vec{q}|T_1|\vec{q}_0\Psi_{1(23)}\rangle + \langle \vec{\bar{p}}\vec{\bar{q}}|T_2|\vec{q}_0\Psi_{2(31)}\rangle + \langle \vec{\bar{\bar{p}}}\vec{\bar{\bar{q}}}|T_2|\vec{q}_0\Psi_{2(31)}\rangle$,
  где $\vec{\bar{p}}(\vec{p},\vec{q}), \vec{\bar{q}}(\vec{p},\vec{q}), \vec{\bar{\bar{p}}}(\vec{p},\vec{q}), \vec{\bar{\bar{q}}}(\vec{p},\vec{q})$ [see details: Few-body systems (2025) 66:24]
- $\frac{d\sigma}{d\omega_{23}d\Omega_{23}d\Omega_1} = \frac{E_1(W-E_1)p_1' p_{23} E_1' E_2' E_3'}{W^2 p_{1(23)}(2\pi)^5} \frac{\sum |U_0(\vec{p},\vec{q};\vec{q}_0)|^2}{(2j_1+1)(2j_2+1)}$

# Cross sections and rates under the three-body problem



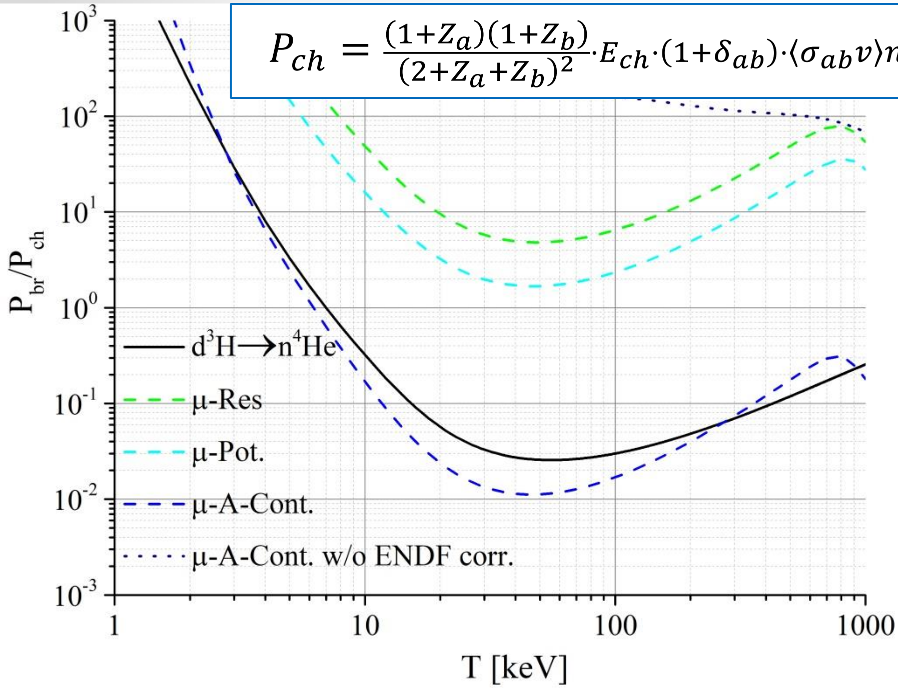



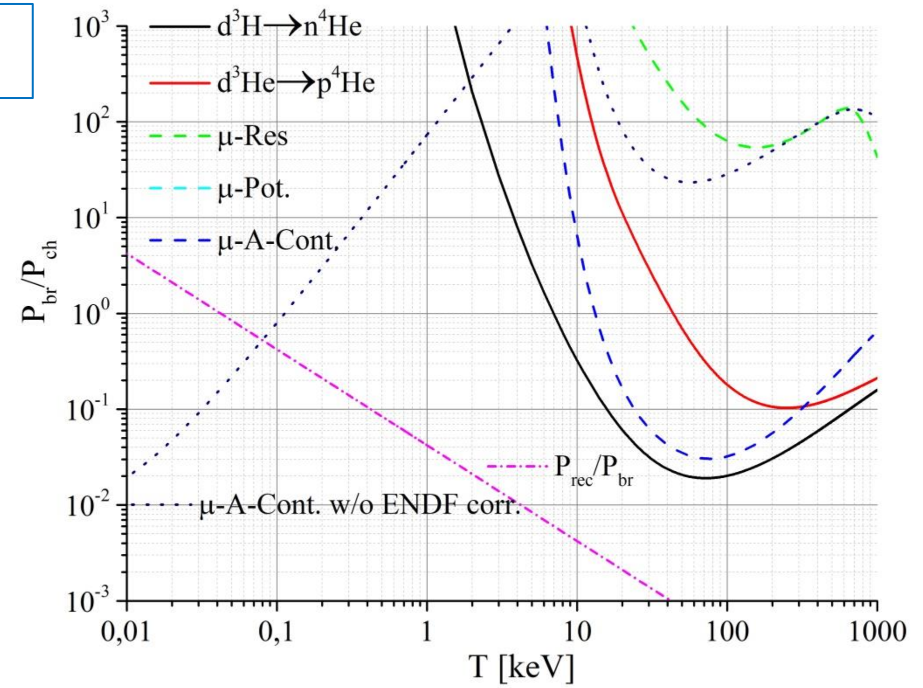



# Reaction power vs bremsstrahlung losses

$$P_{ch} = \frac{(1+Z_a)(1+Z_b)}{(2+Z_a+Z_b)^2} \cdot E_{ch} \cdot (1+\delta_{ab}) \cdot \langle \sigma_{ab} v \rangle n^2$$




**Fusion catalyzed in in-flight reactions is manifests itself**

| [keV] | $d^3\mathrm{H} \to n^4\mathrm{He}$ | $d^3\mathrm{He} \to p^4\mathrm{He}$ | $d(^3\mathrm{H}\mu^-) \to n^4\mathrm{He}\mu^-$ | $d(^3\mathrm{He}\mu^-) \to p^4\mathrm{He}\mu^-$ |
|---|---|---|---|---|
| $T_{in}$ | 6.93 | **43.55** | 6.23 | **13.16/13.15** |
| $T_{opt}$ | 55.56 | 249.8 | 46.15 | 75.23/75.38 |

- Cross section enhancement, a catalytic effect are only predicted by A-Cont. model
- For calculating cross section, it is sufficient to limit by $C_0^{\alpha\beta}$ and $C_1^{\alpha\beta}$ iterations
- Functions $\phi(\vec{p})$ – are eigenfunctions $\underset{z<0}{\Longleftrightarrow} |\vec{p}| = \sqrt{2\mu|E_b|}, E_b$ –atomic binding energy for $z \geq 0$ $\phi(\vec{p})$ –scattering states.

# Thank You!

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RELATED PUBLICATIONS of E.M.:


- Three-dimensional integral Faddeev equations without a certain symmetry // Few-body systems (2025) 66:24, **DOI**:10.1007/s00601-025-01991-z
- Binding energies of $^3H$ and $^3He$ nuclei in three-body Faddeev equations with direct integration //Physics of Atomic Nuclei (2024) 87, №6, pp.682-696, **DOI:** 10.1134/S1063778824700698
- Virtual and resonance states in the three-body $D-\mu^- - T, D-\mu^- - D$ systems // Int. J. Mod. Phys. E (2024) 33, №11, p.2441001, **DOI:** 10.1142/S0218301324410015

# Appendix: Eigenfunctions

- Wave functions of the $y^- d, y^- t, y^{-\ 3,4}$He systems were determined from analytical solutions of the corresponding Coulomb problems

$$\Psi_{nlm}(r,\theta,\phi) = \sqrt{\frac{(n-l-1)!}{2m(n+l)!}} \cdot \left(\frac{2}{na_0}\right)^{\frac{3}{2}} \cdot e^{-\frac{r}{na_0}} \cdot \left(\frac{2r}{na_0}\right)^{l} \cdot L_{n-l-1}^{2l+1}\left(\frac{2r}{na_0}\right) \cdot \mathrm{Y_{lm}}(\theta,\phi)$$

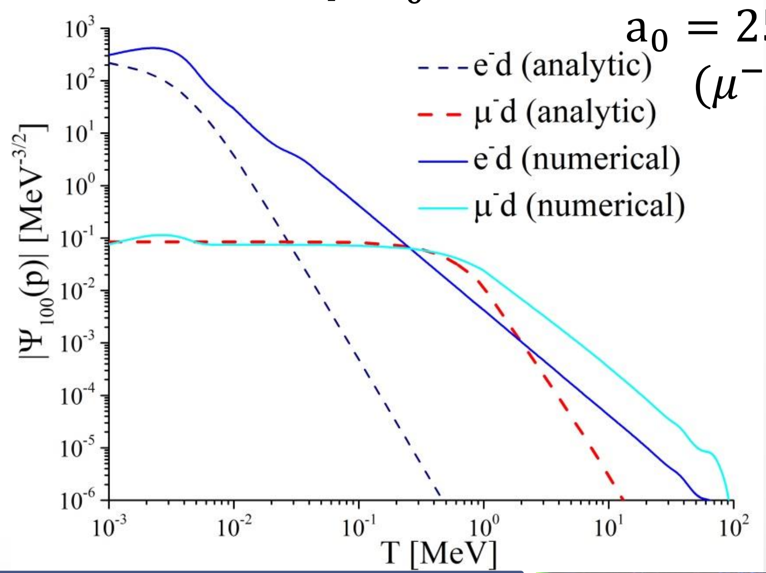
$$\Psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{2}}\left(\frac{2}{a_0}\right)^{\frac{3}{2}} \cdot e^{-\frac{r}{a_0}} \cdot \frac{1}{\sqrt{4\pi}}$$

$$\Psi_{100}(p) = \frac{a_0^{\frac{3}{2}}}{\pi^{\frac{5}{2}}(p^2 a_0^2 + 1)^2}$$

$\mathrm{a_0} = 52917.74$ fm $(\mathrm{e^-d, e^-t})$

$\mathrm{a_0} = 255.927$ fm $(\mu^-\mathrm{d}, \mu^-\mathrm{t})$

| system | Binding energy (n=1/n=2) | Cut off *C* (n=1/n=2) |
|---|---|---|
| $e^- d$ | -13.6 eV/-3.4 eV | -0.265/1.28 |
| $\mu^- d$ | -2.66keV/-0.66keV | -0.885/0.78 |
| $e^- t$ | -13.6 eV/-3.4 eV | -0.265/1.28 |
| $\mu^- t$ | -2.71keV/-0.68keV | -0.885/0.78 |
| $e^{-3}$He | -54.41eV/-13.6eV | -0.621/1.11 |
| $\mu^{-3}$He | -10.84keV/-2.71keV | -0.879/0.755 |
| $e^{-4}$He | -54.41eV/-13.6eV | -0.621/1.11 |
| $\mu^{-4}$He | -10.94keV/-2.73keV | -0.879/0.755 |



# Appendix: Coulomb terms

$$V_R(r) = \frac{Z_1 Z_2 e^2}{r} \exp\left(-\frac{r}{R}\right), \; V^{\phi}(r) = V_C(r) - V_R(r)$$

$$R \equiv R(k,l) = \frac{1}{2k} \exp\left(\frac{C(k,l)}{\eta(k)}\right)$$

$$V_R(p_f, p_i; z) \equiv \frac{2k\eta}{p_f^2 - 2p_f p_i \cos\left(\theta_{p_f p_i}\right) + p_i^2 + \left(\frac{\hbar c}{R}\right)^2}$$

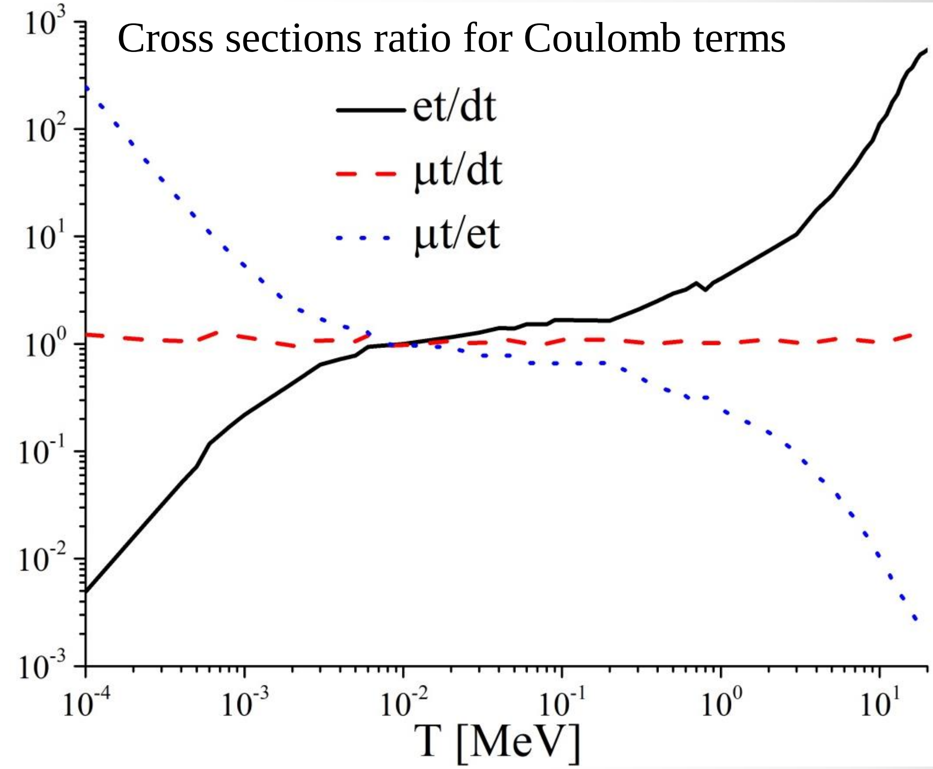
$$\boxed{T^C = T^{\phi} + t^{R\phi}}$$ - Coulomb matrix

$$t^{R\phi} = V^R + V^R G^{\phi} t^{R\phi}$$

$$T^{\phi} = V^{\phi} + V^{\phi} G_0 T^{\phi}$$

$$T^{\phi}(\vec{p},\vec{p}) = T^{\phi}(\vec{p},\vec{q}'') = T^{\phi}(\vec{q}'',\vec{p}) = 0$$

- it is provided by the selection of $R(k,l)$




**Approximations**: $G^{\phi} \to G_0$, *exclusion* of $T^C$ from the short-range Green's function $G_0$
**Coulomb terms are included in**: final p$^4$He interaction with $C = 10^{-4}$, initial state interactions (ISI) in (dt, d$^3$He) with $C = 10^{-4}$, as well as in the lepton-nuclear scattering matrices with a parameter $C = -10^{-2}$.
Initial state interaction:

$$\int d^3q''\, T(\vec{p}_f, \vec{q}'') G_0(q'') T^C(\vec{q}'', \vec{p}_i).$$