Role of zero-point transverse oscillations in the Langevin description of nuclear fission

Speaker

dr Pavel V. Kostryukov

Voronezh State University

OUTLINE

1. Introduction to the Problem

- Successful application of the Langevin approach for describing fission products
- Main problem: determination of effective temperature in low-energy fission

2. Vibrational Modes

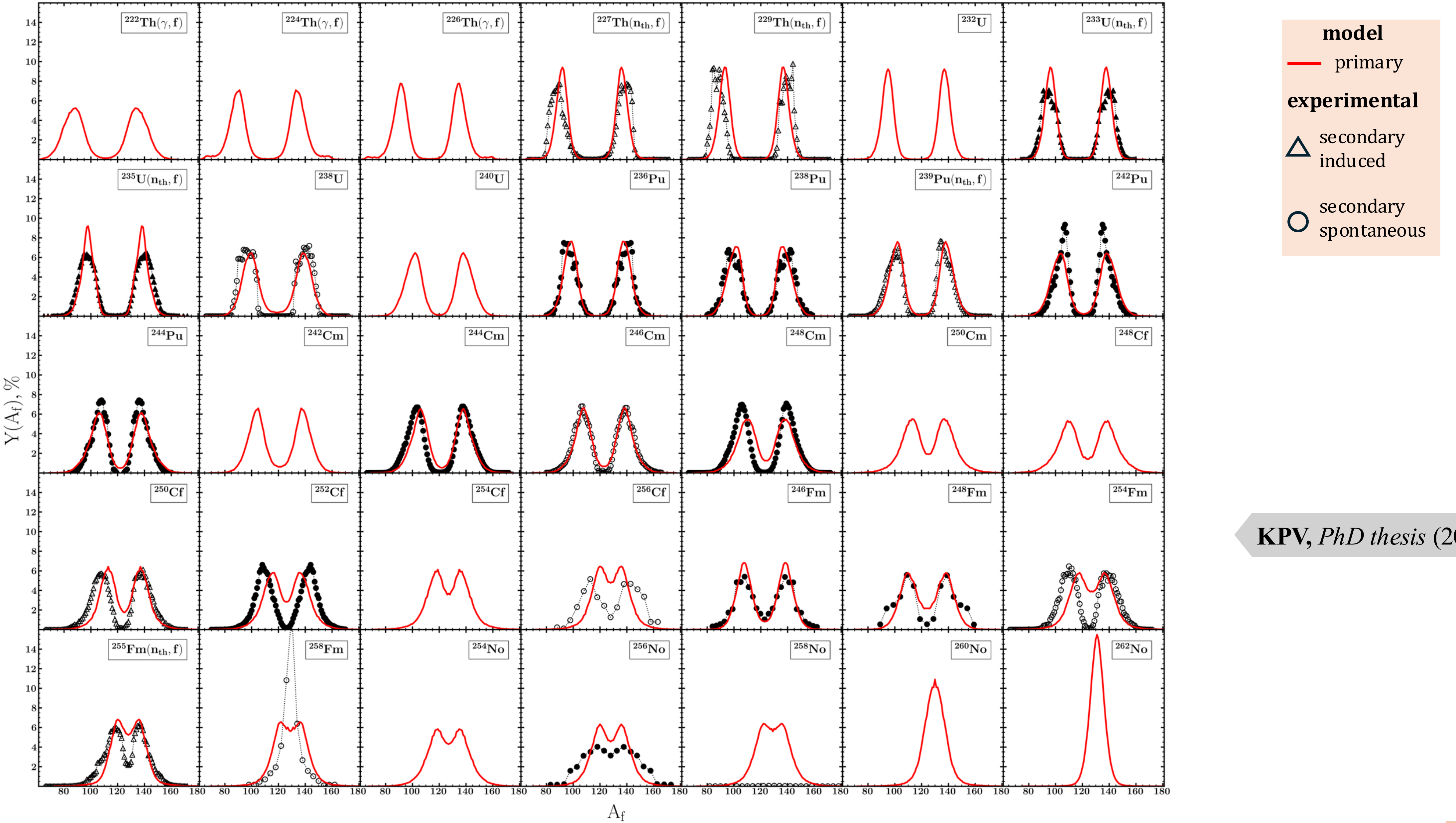
- Analysis of transverse oscillations (bending, wriggling) during fission
- Quantum estimation of zero-point oscillation energies
- Dependence of mode energies on fragment configurations

3. Results

- Comparison with experimental data
- Comparison between dynamic and static $\hbar\omega_0$ approaches
- Introduction of higher-order mode

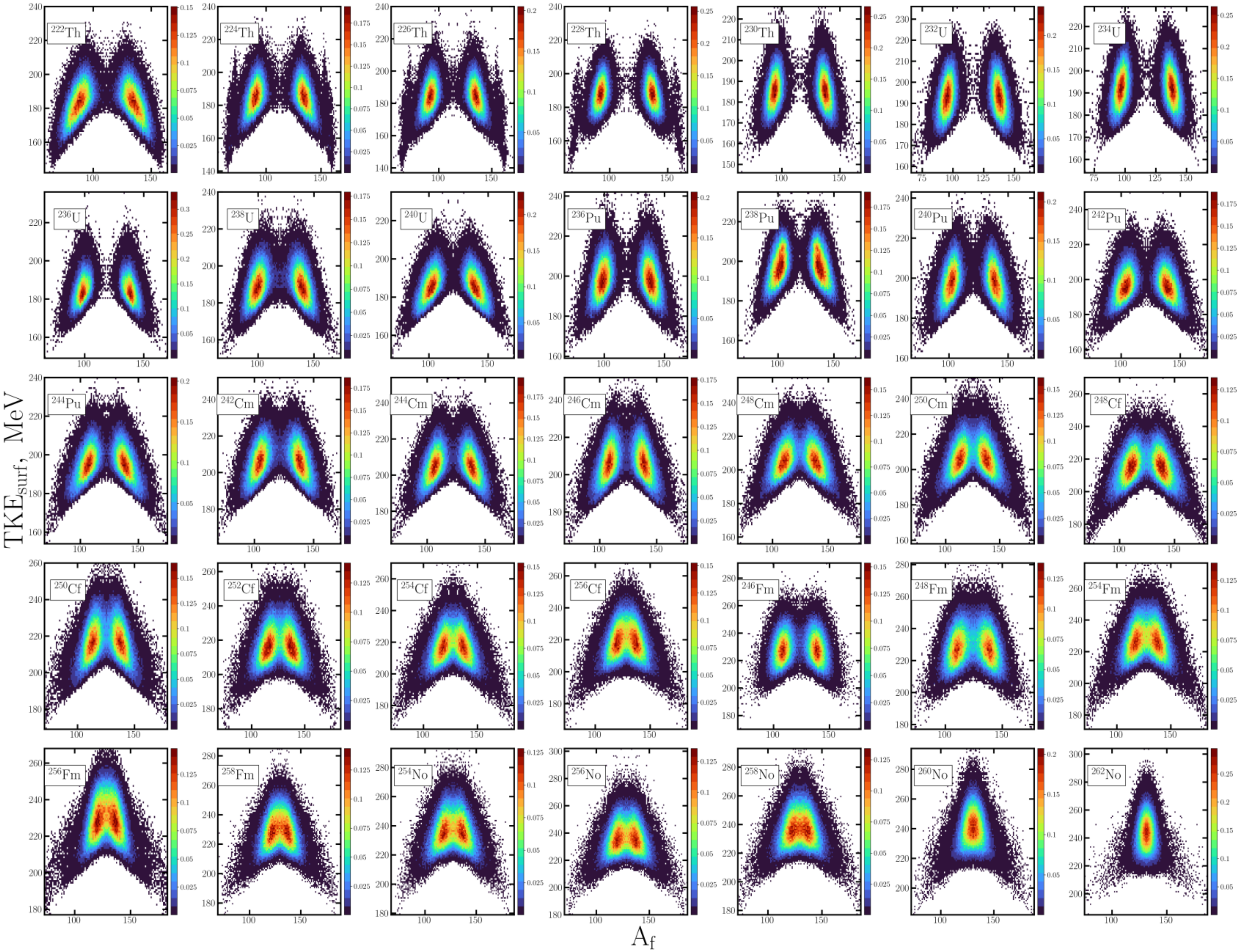
4. Summary

Introduction into the problem



KPV, *PhD thesis* (2023)

Introduction into the problem



KPV, *PhD thesis* (2023)

Introduction into the problem

$$\begin{cases} \dfrac{dq_i}{dt} = \sum_j [\mathcal{M}^{-1}]_{ij} p_j \\ \dfrac{dp_i}{dt} = -\dfrac{1}{2}\sum_{jk} \dfrac{\partial[\mathcal{M}^{-1}]_{jk}}{\partial q_i} p_j p_k - \dfrac{\partial F}{\partial q_i} - \sum_{jk} \gamma_{ij}[\mathcal{M}^{-1}]_{jk} p_k + \sum_j g_{ij}\xi_j \\ E^* = a(\boldsymbol{q})T^2 = E_{total} - \dfrac{1}{2}\sum_{jk}[\mathcal{M}^{-1}]_{ij} p_j p_k - V(\boldsymbol{q}) \end{cases}$$

Y. **Abe**, et al., **J. de Phys.**, **47**, 329 (1986)

q_i is generalized coordinates of deformation space $\boldsymbol{q}$
p_i is conjugated momentum for q_i

$\mathcal{M}_{ij}(\boldsymbol{q})$, $\gamma_{ij}(\boldsymbol{q})$ are **inertia** and **friction tensors**, $F(\boldsymbol{q})$ is **free energy potential** related with mic-mac potential $V(\boldsymbol{q})$ and **temperature of the system** T as

$$F(\boldsymbol{q}) \equiv V(\boldsymbol{q}) - a(\boldsymbol{q})T^2$$

$a(\boldsymbol{q})$ is **level-density parameter**

Introduction into the problem

$$\begin{cases} \dfrac{dq_i}{dt} = \sum_j [\mathcal{M}^{-1}]_{ij} p_j \\ \dfrac{dp_i}{dt} = -\dfrac{1}{2}\sum_{jk} \dfrac{\partial [\mathcal{M}^{-1}]_{jk}}{\partial q_i} p_j p_k - \dfrac{\partial F}{\partial q_i} - \sum_{jk} \gamma_{ij} [\mathcal{M}^{-1}]_{jk} p_k + \sum_j g_{ij}\xi_j \\ E^* = a(\boldsymbol{q})T^2 = E_{total} - \dfrac{1}{2}\sum_{jk} [\mathcal{M}^{-1}]_{ij} p_j p_k - V(\boldsymbol{q}) \end{cases}$$

q_i is generalized coordinates of deformation space $\boldsymbol{q}$

p_i is conjugated momentum for q_i

Y. **Abe**, et al., **J. de Phys.**, **47**, 329 (1986)

$\mathcal{M}_{ij}(\boldsymbol{q})$, $\gamma_{ij}(\boldsymbol{q})$ are **inertia** and **friction tensors**, $F(\boldsymbol{q})$ is **free energy potential** related with mic-mac potential $V(\boldsymbol{q})$ and **temperature of the system** T as

$$F(\boldsymbol{q}) \equiv V(\boldsymbol{q}) - a(\boldsymbol{q})T^2$$

$a(\boldsymbol{q})$ is **level-density parameter**

Introduction into the problem

Langevin random force term

$$\sum_j g_{ij}\xi_j$$

Introduction into the problem

Langevin random force term

$$\sum_j g_{ij}\xi_j$$

White noise function with properties

$$\langle \xi_j \rangle = 0, \langle \xi_j^2 \rangle = 2$$

Introduction into the problem

White noise function with properties

$$\langle \xi_j \rangle = 0, \langle \xi_j^2 \rangle = 2$$

Langevin random force term

$$\sum_j g_{ij} \xi_j$$

g_{ij} **is a strength tensor** defined by Einstein relation

$$D_{ij} \stackrel{\text{def}}{=} \gamma_{ij} T = \sum_k g_{ik}\, g_{jk}$$

Introduction into the problem

White noise function with properties

$$\langle \xi_j \rangle = 0, \langle \xi_j^2 \rangle = 2$$

Langevin random force term

$$\sum_j g_{ij} \xi_j$$

g_{ij} **is a strength tensor** defined by Einstein relation

$$D_{ij} \stackrel{\text{def}}{=} \gamma_{ij} T = \sum_k g_{ik} \, g_{jk}$$

In case of low-energy fission $T \leq 1$ MeV in Langevin approach uses effective temperature

$$T^* = E_0 \coth \frac{E_0}{T} \qquad \text{where} \quad E_0 = \frac{\hbar \omega_0}{2}$$

K. **Pomorski** and H. **Hofmann**, J. Physique **42** 381(1981)

Introduction into the problem

Langevin random force term

$$\sum_j g_{ij}\xi_j$$

White noise function with properties
$$\langle \xi_j \rangle = 0, \langle \xi_j^2 \rangle = 2$$

g_{ij} **is a strength tensor** defined by Einstein relation

$$D_{ij} \stackrel{\text{def}}{=} \gamma_{ij} T = \sum_k g_{ik}\, g_{jk}$$

In case of low-energy fission $T \leq 1$ MeV in Langevin approach uses effective temperature

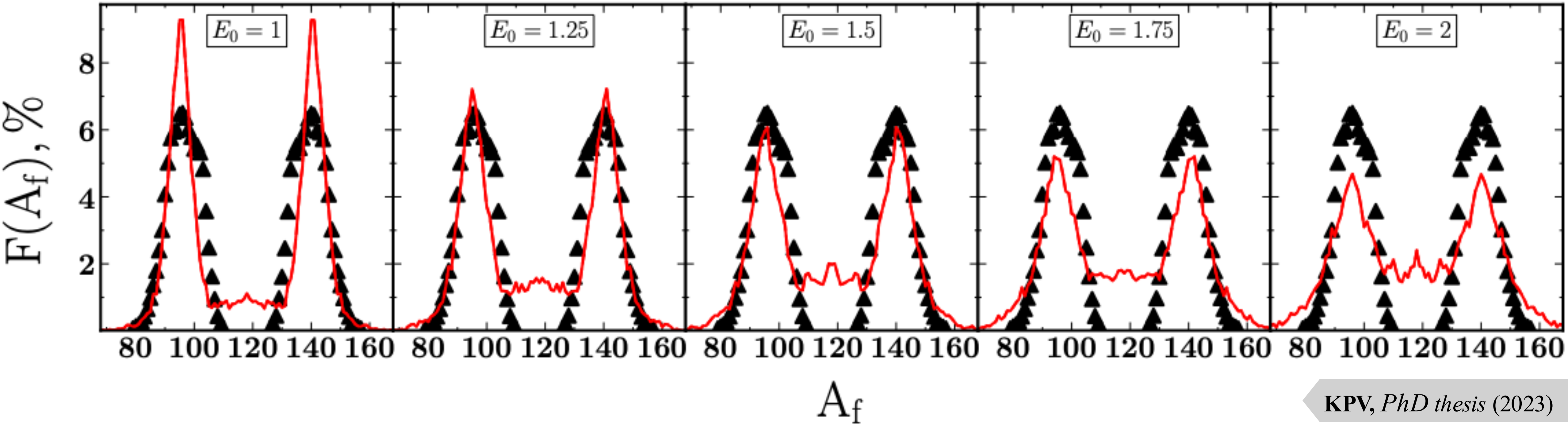
$$T^* = E_0 \coth \frac{E_0}{T}$$ where $$E_0 = \frac{\hbar\omega_0}{2}$$ **but which value?**

K. **Pomorski** and H. **Hofmann**, J. Physique **42** 381(1981)

Introduction into the problem

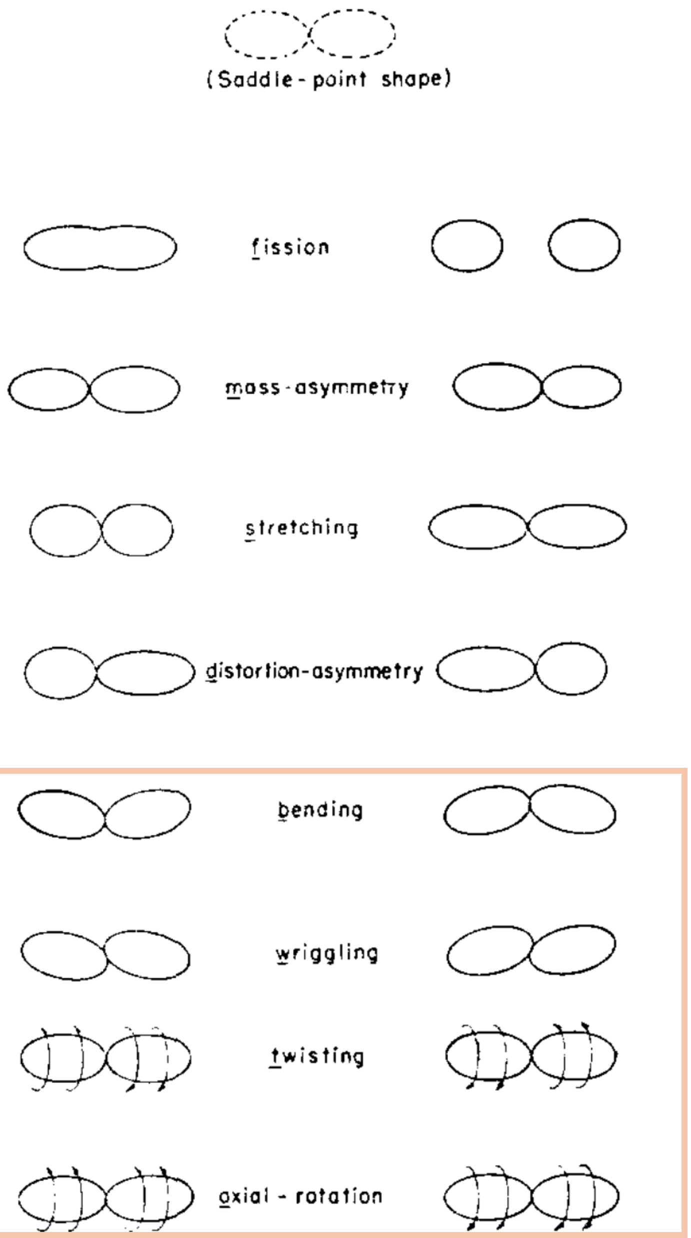
Publication	M.D.Usang et al, PRC 96, 064617 (2017)	L.-L. Liu et al, PRC 99, 044614 (2019)	K. Pomorski et al, PRC 107, 054616 (2023)
E_0, MeV	1.2	1	1.5

For the reaction $^{235}\mathrm{U(n, f)}$



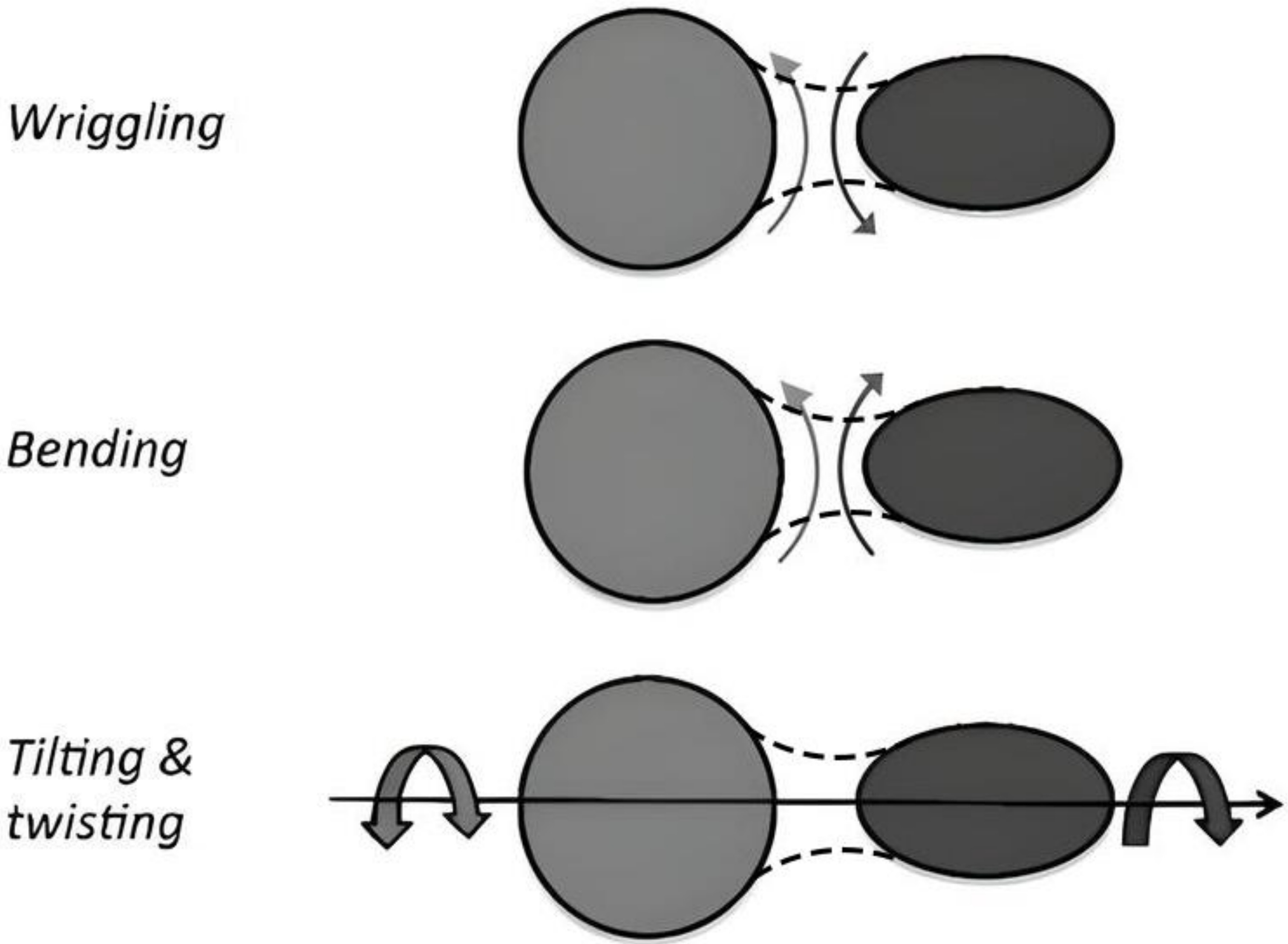
KPV, *PhD thesis* (2023)

Oscillations



J. R. **Nix** and W. J. **Swiatecki**, Nucl Phys **71**, 1 (1965)

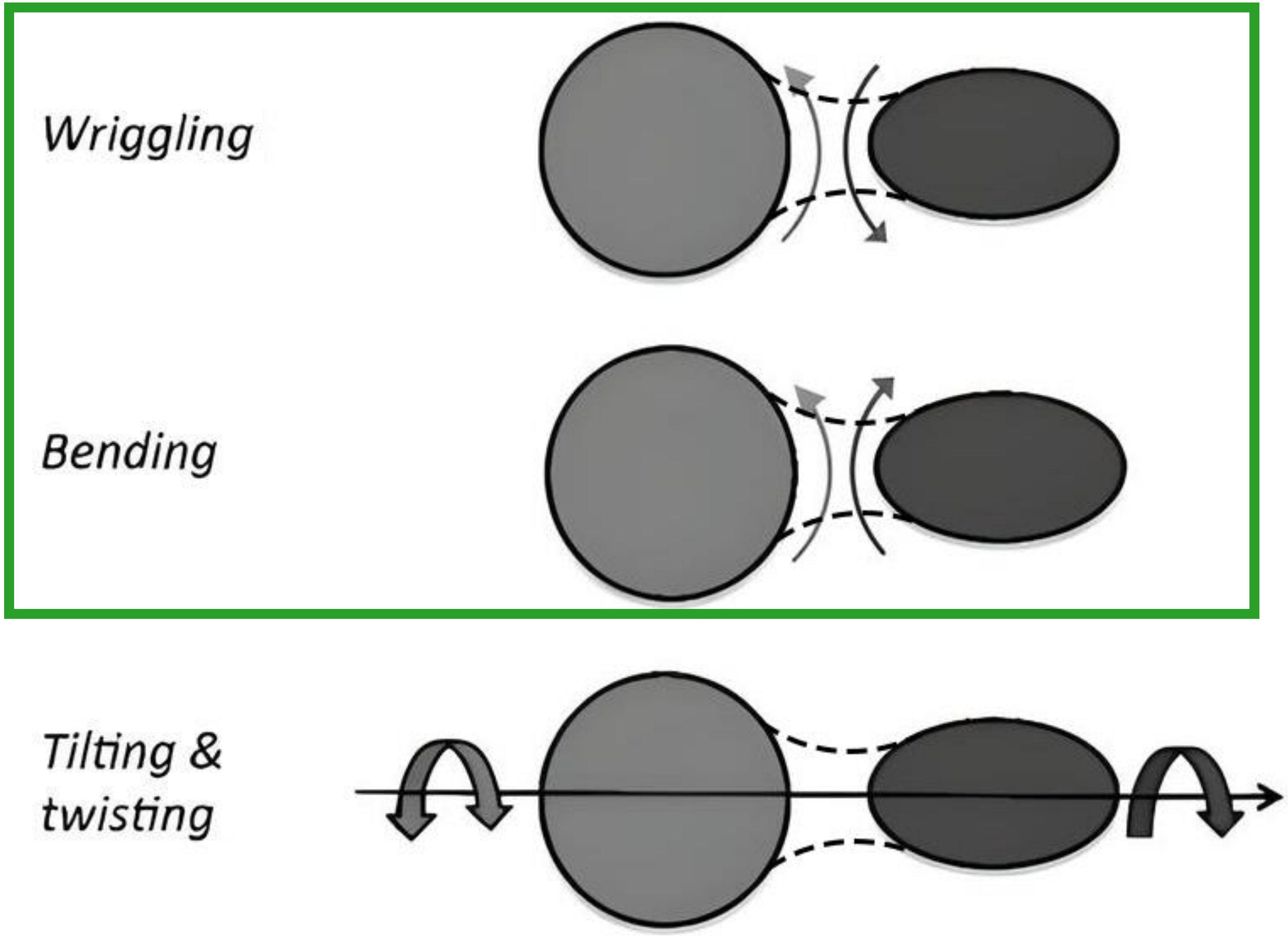
Oscillations



Relaxation time τ, zs
$\sim 0.1 \div 1$
$\sim 0.1 \div 1$
$\sim 0.1 \div 10$

J. R. **Nix** and W. J. **Swiatecki**, Nucl Phys **71**, 1 (1965)

Oscillations



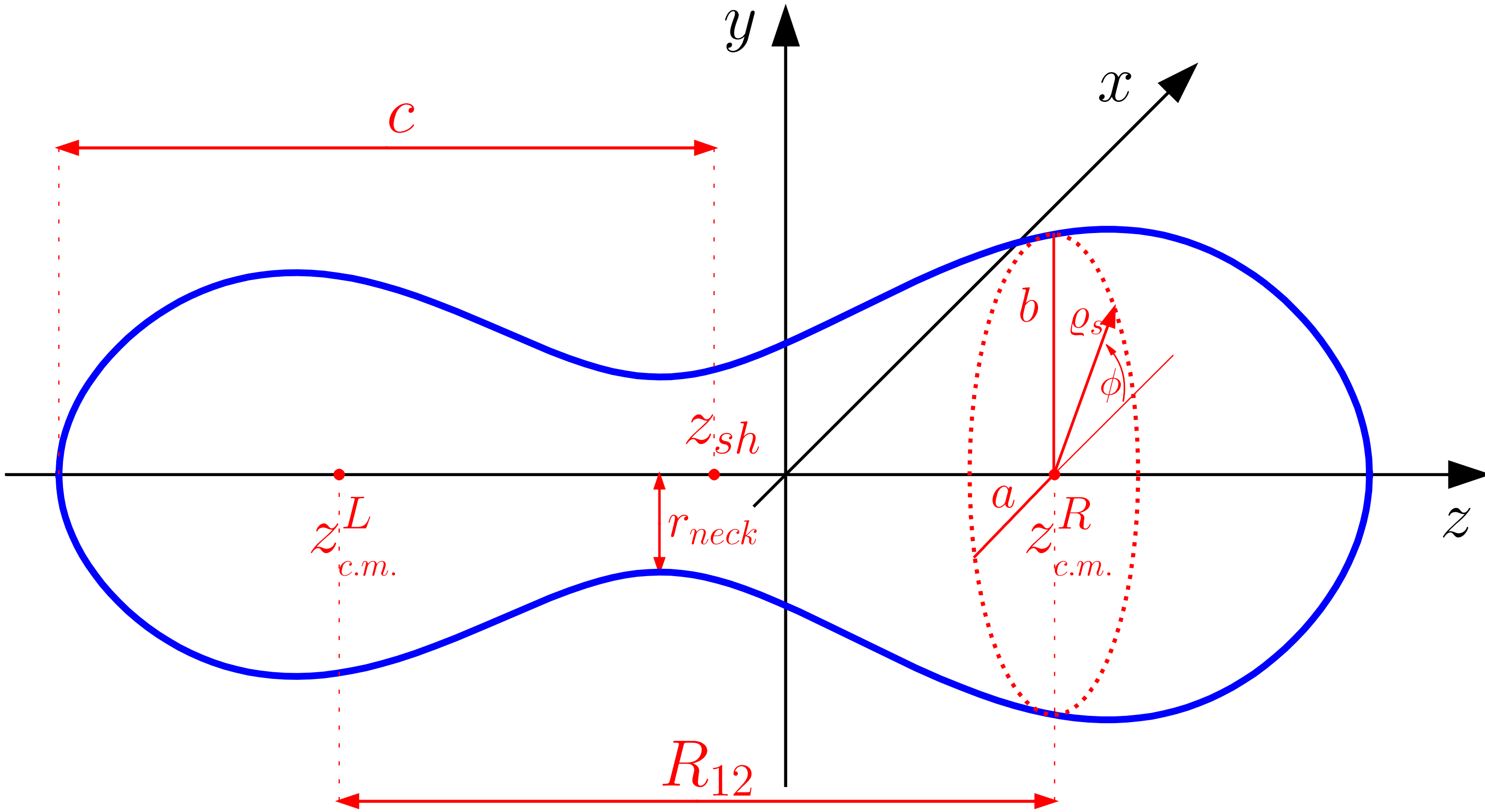
Relaxation time τ, zs
$\sim 0.1 \div 1$
$\sim 0.1 \div 1$
$\sim 0.1 \div 10$

J. R. **Nix** and W. J. **Swiatecki**, Nucl Phys **71**, 1 (1965)

Fourier shape parametrization

$$\rho_s^2(u) = \sum_n \left[a_{2n} \cos\left(\frac{2n-1}{2}\pi u\right) + a_{2n+1} \sin(n\pi u) \right], \qquad u = \frac{z - z_{sh}}{z_0}$$

C. Schmitt et al.**, PRC 95, 034612 (2017)**



Fourier shape parametrization

$$\rho_s^2(u) = \sum_n \left[a_{2n} \cos\left(\frac{2n-1}{2} \pi u \right) + a_{2n+1} \sin(n\pi u) \right], \qquad u = \frac{z - z_{sh}}{z_0}$$

C. Schmitt et al., **PRC 95, 034612 (2017)**

$$\eta = \frac{b-a}{a+b}$$

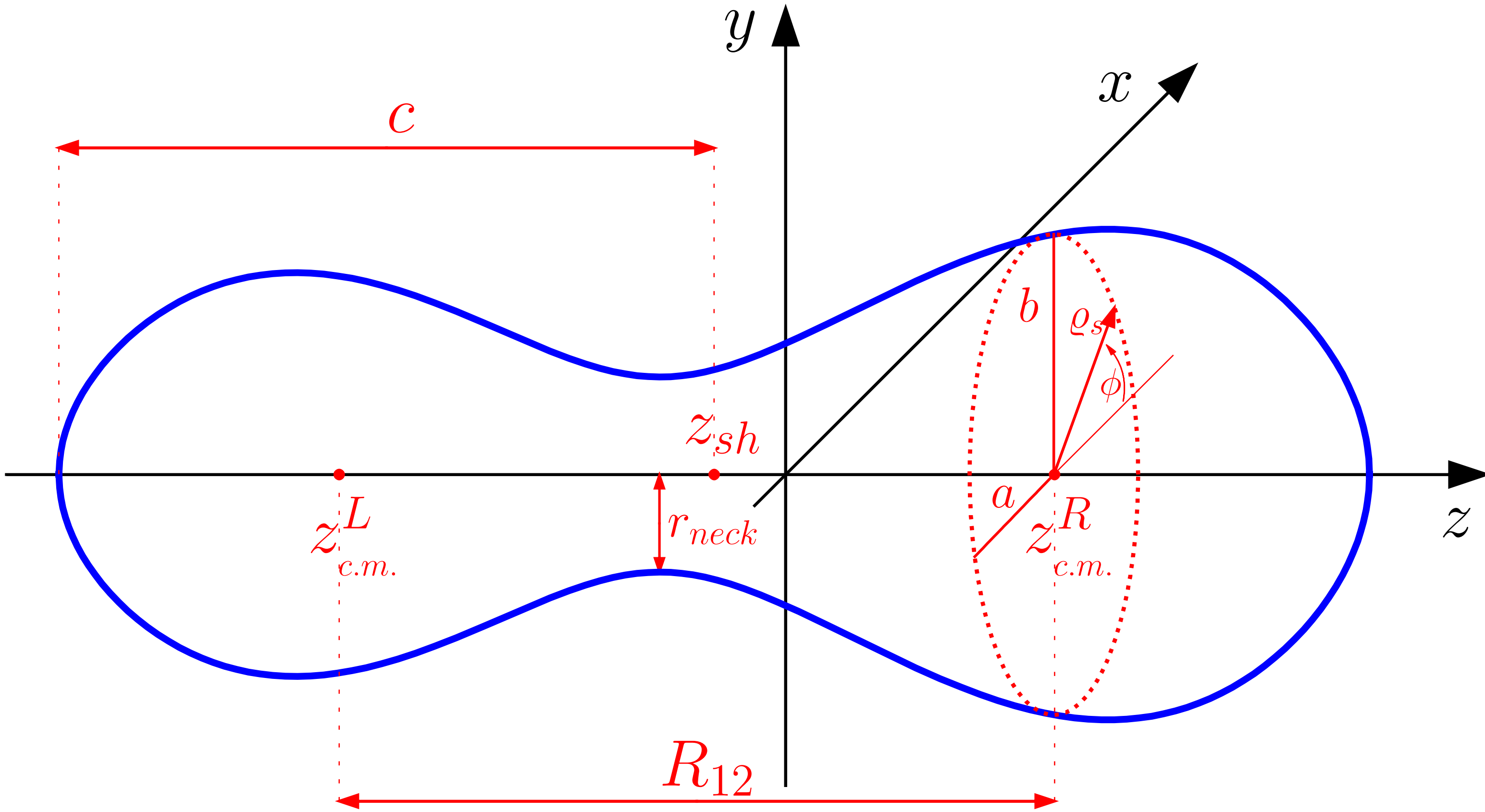
$$q_2 = a_2^0/a_2 - a_2/a_2^0$$

$$q_3 = a_3$$

$$q_4 = a_4 + \sqrt{(q_2/9)^2 + (a_4^0)^2}$$

$$q_5 = a_5 - \frac{a_3}{10}(q_2 - 2)$$

$$q_6 = a_6 - \sqrt{(q_6/100)^2 + (a_6^0)^2}$$



Fourier shape parametrization

$$\rho_s^2(u) = \sum_n \left[a_{2n} \cos\left(\frac{2n-1}{2}\pi u\right) + a_{2n+1} \sin(n\pi u) \right], \qquad u = \frac{z - z_{sh}}{z_0}$$

C. Schmitt et al., PRC 95, 034612 (2017)

$\eta = \frac{b-a}{a+b}$ *non-axiality*

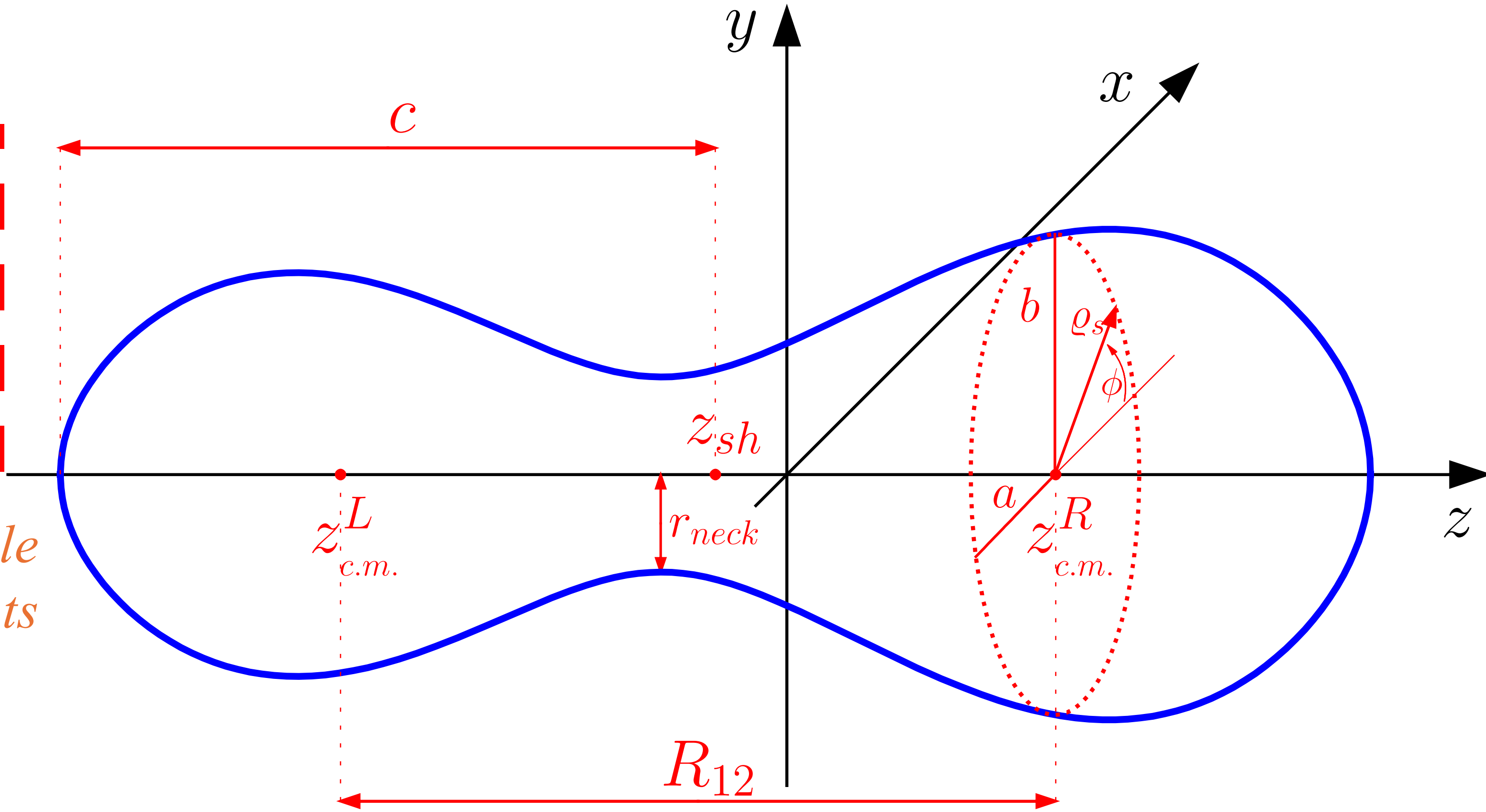
$q_2 = a_2^0/a_2 - a_2/a_2^0$ *elongation*

$q_3 = a_3$ *right-left asymmetry*

$q_4 = a_4 + \sqrt{(q_2/9)^2 + (a_4^0)^2}$ *neck thickness*

$q_5 = a_5 - \frac{a_3}{10}(q_2 - 2)$ *multipole moments*

$q_6 = a_6 - \sqrt{(q_6/100)^2 + (a_6^0)^2}$



Fourier shape parametrization

$$\rho_s^2(u) = \sum_n \left[a_{2n} \cos\left(\frac{2n-1}{2}\pi u\right) + a_{2n+1} \sin(n\pi u) \right], \qquad u = \frac{z - z_{sh}}{z_0}$$

C. Schmitt et al., PRC 95, 034612 (2017)

WHERE ARE ANGLES BETWEEN PREFRAGMENTS???

$\eta = \frac{b-a}{a+b}$ *non-axiality*

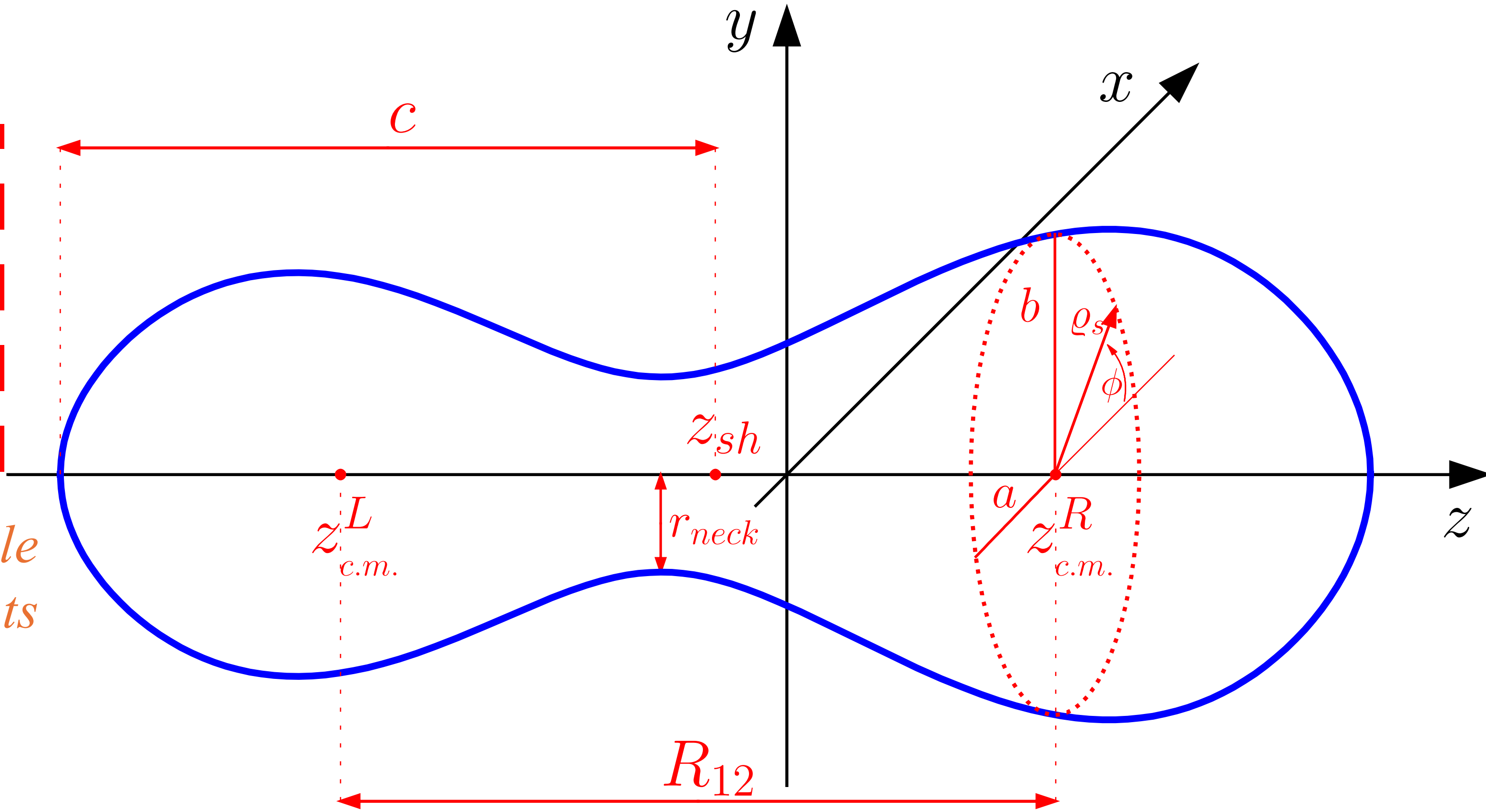
$q_2 = a_2^0/a_2 - a_2/a_2^0$ *elongation*

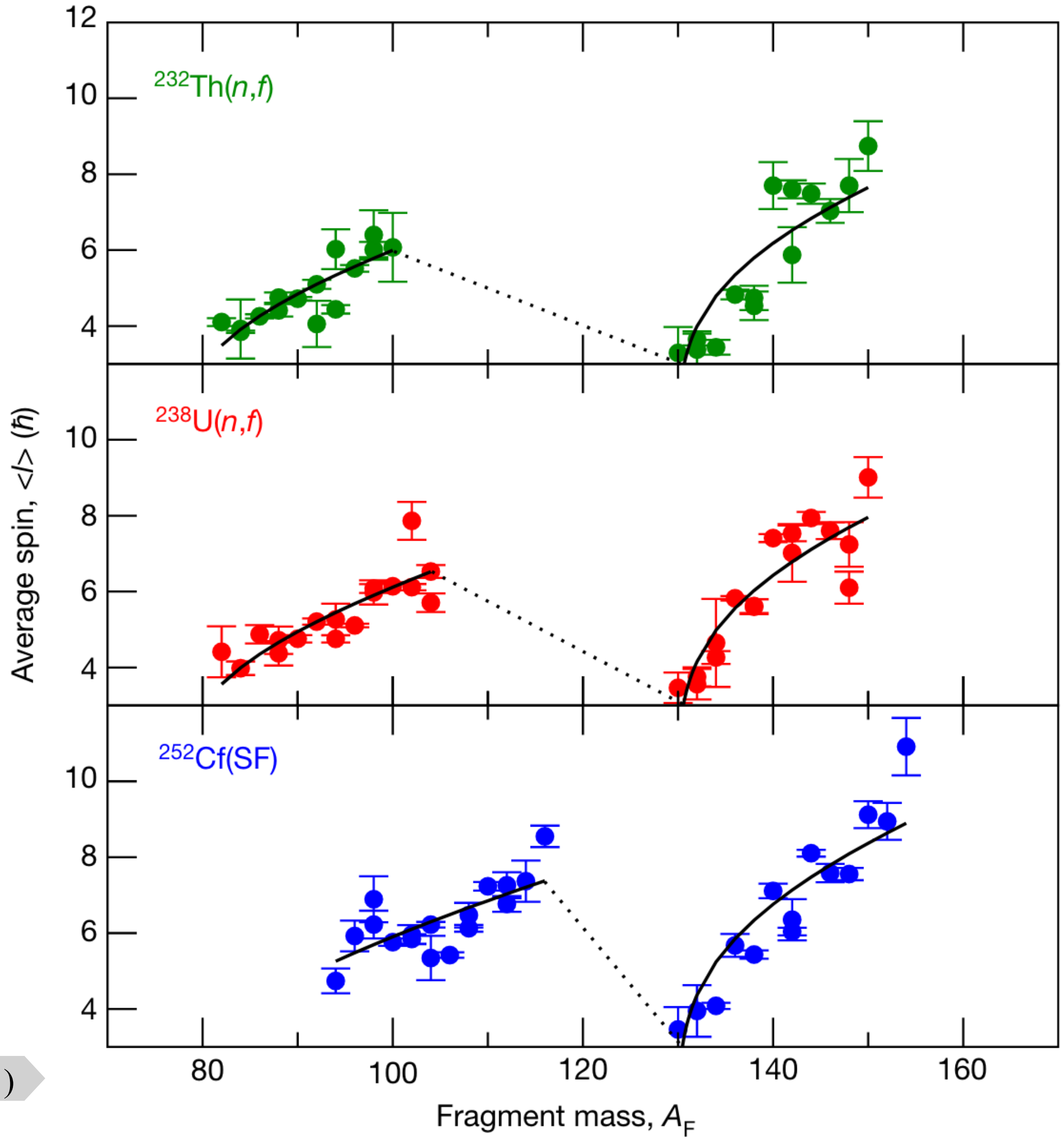
$q_3 = a_3$ *right-left asymmetry*

$q_4 = a_4 + \sqrt{(q_2/9)^2 + (a_4^0)^2}$ *neck thickness*

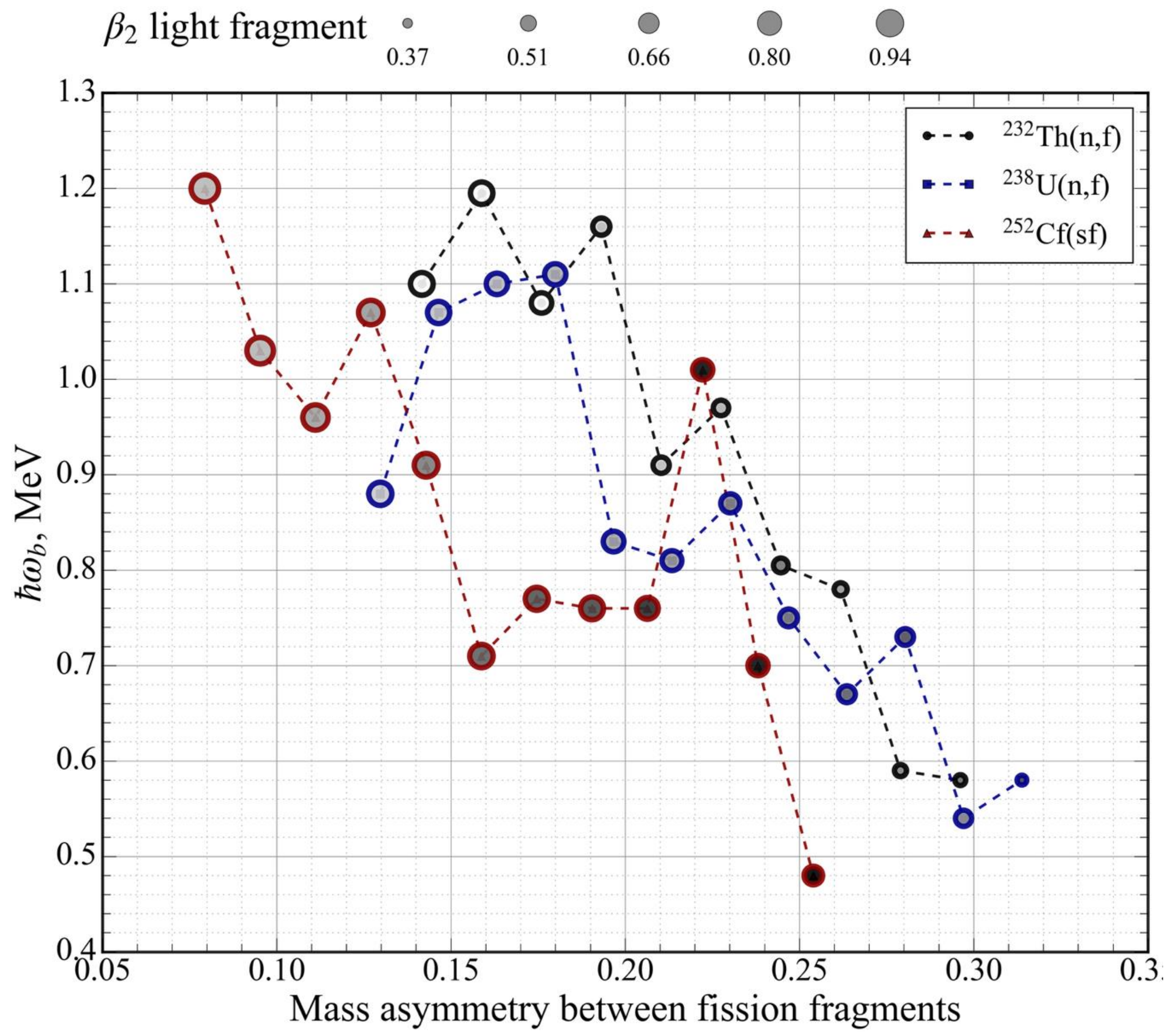
$q_5 = a_5 - \frac{a_3}{10}(q_2 - 2)$ *multipole moments*

$q_6 = a_6 - \sqrt{(q_6/100)^2 + (a_6^0)^2}$

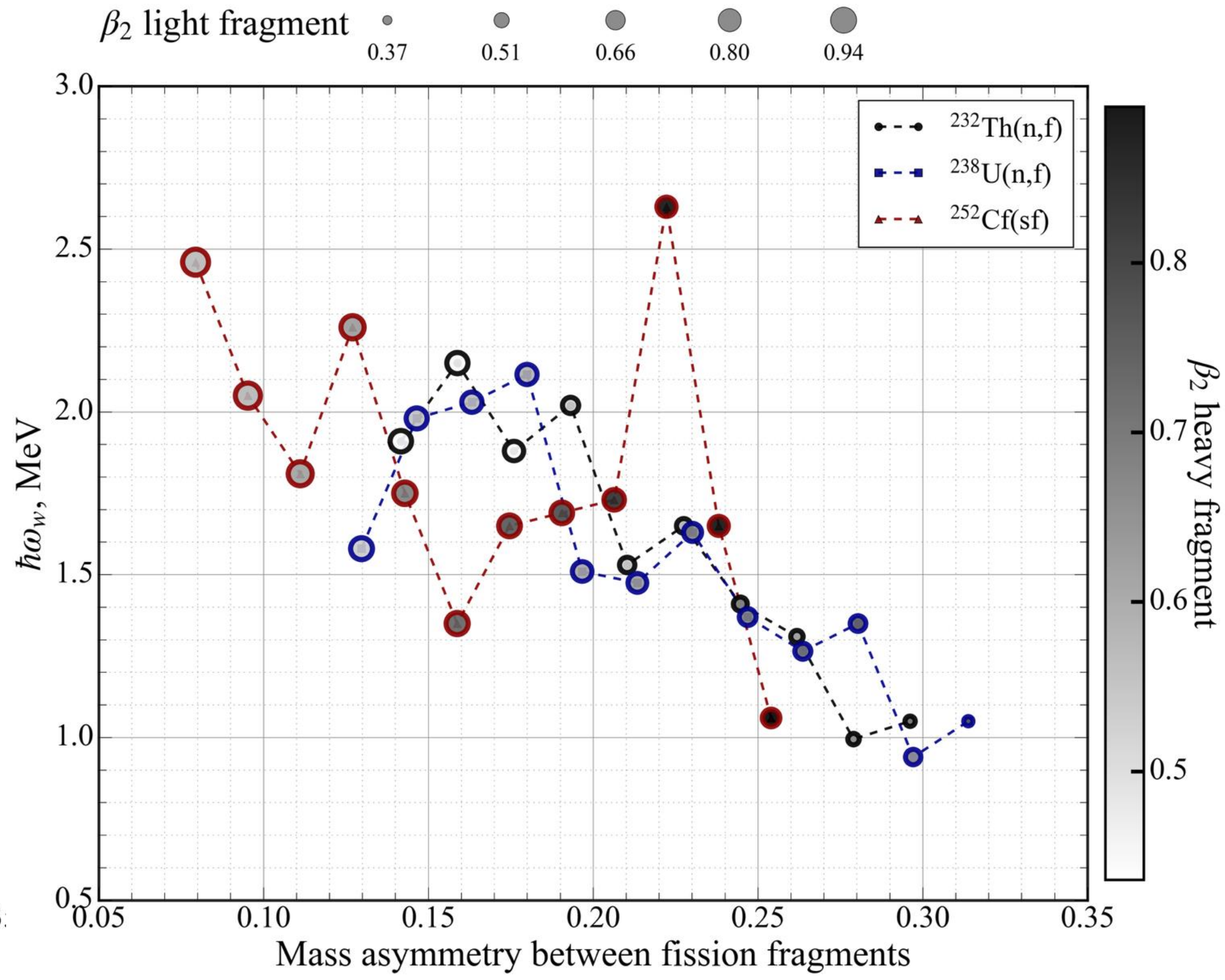




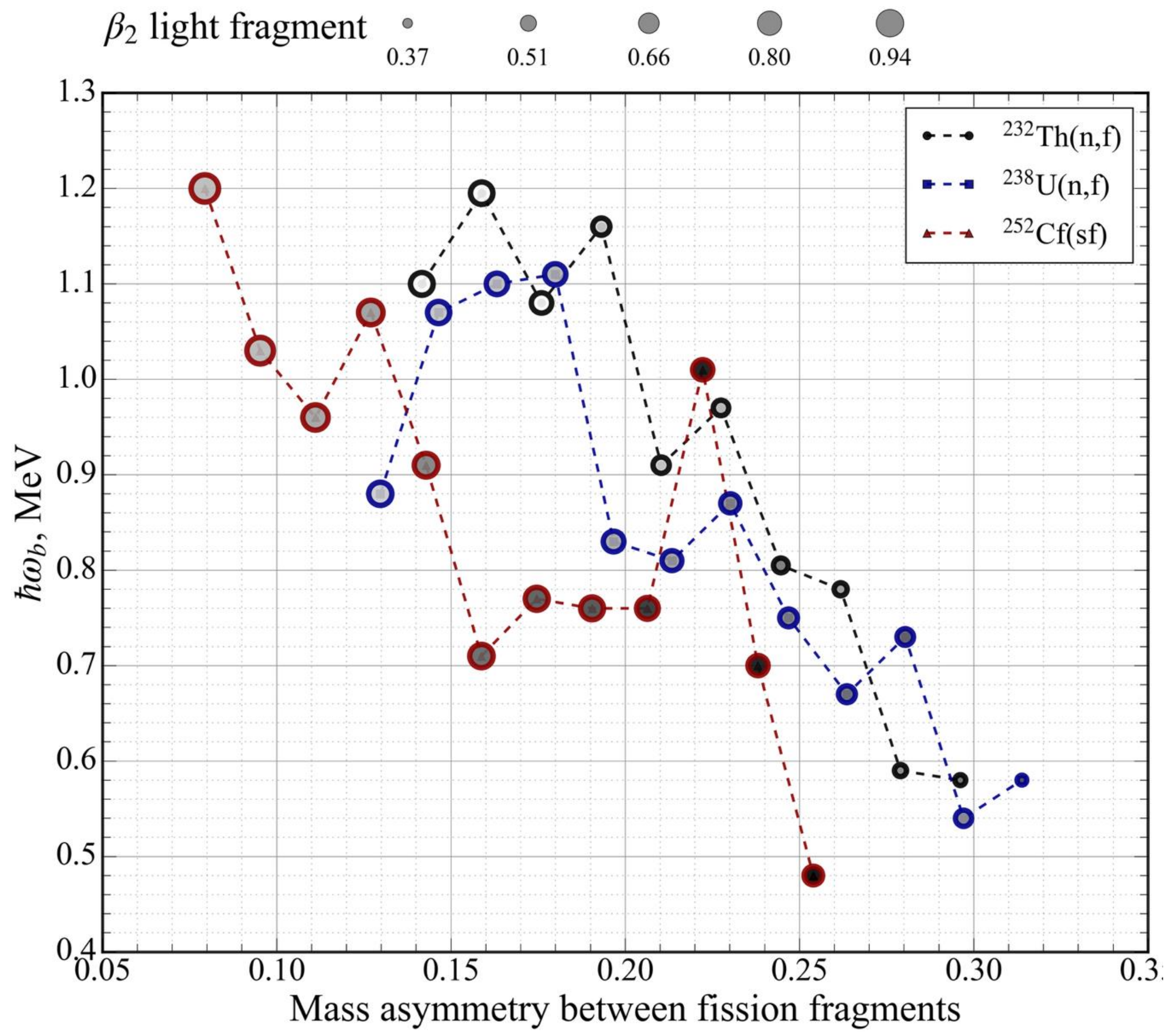
J. Wilson et al., Nature 590, 566 (2021)

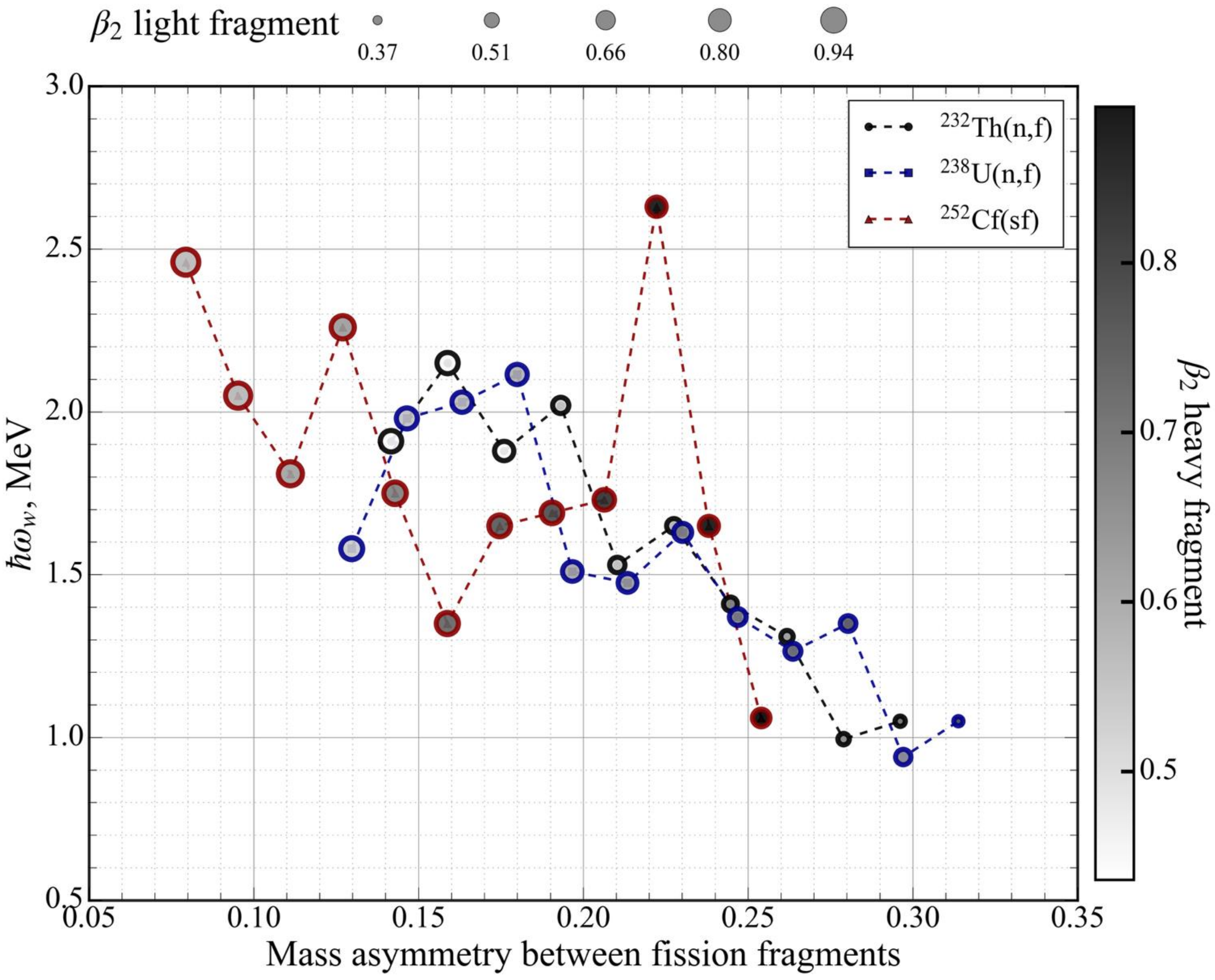


β_2 light fragment
0.37
0.51
0.66
0.80
0.94
^{232}Th(n,f)
^{238}U(n,f)
^{252}Cf(sf)
$\hbar\omega_b$, MeV
Mass asymmetry between fission fragments



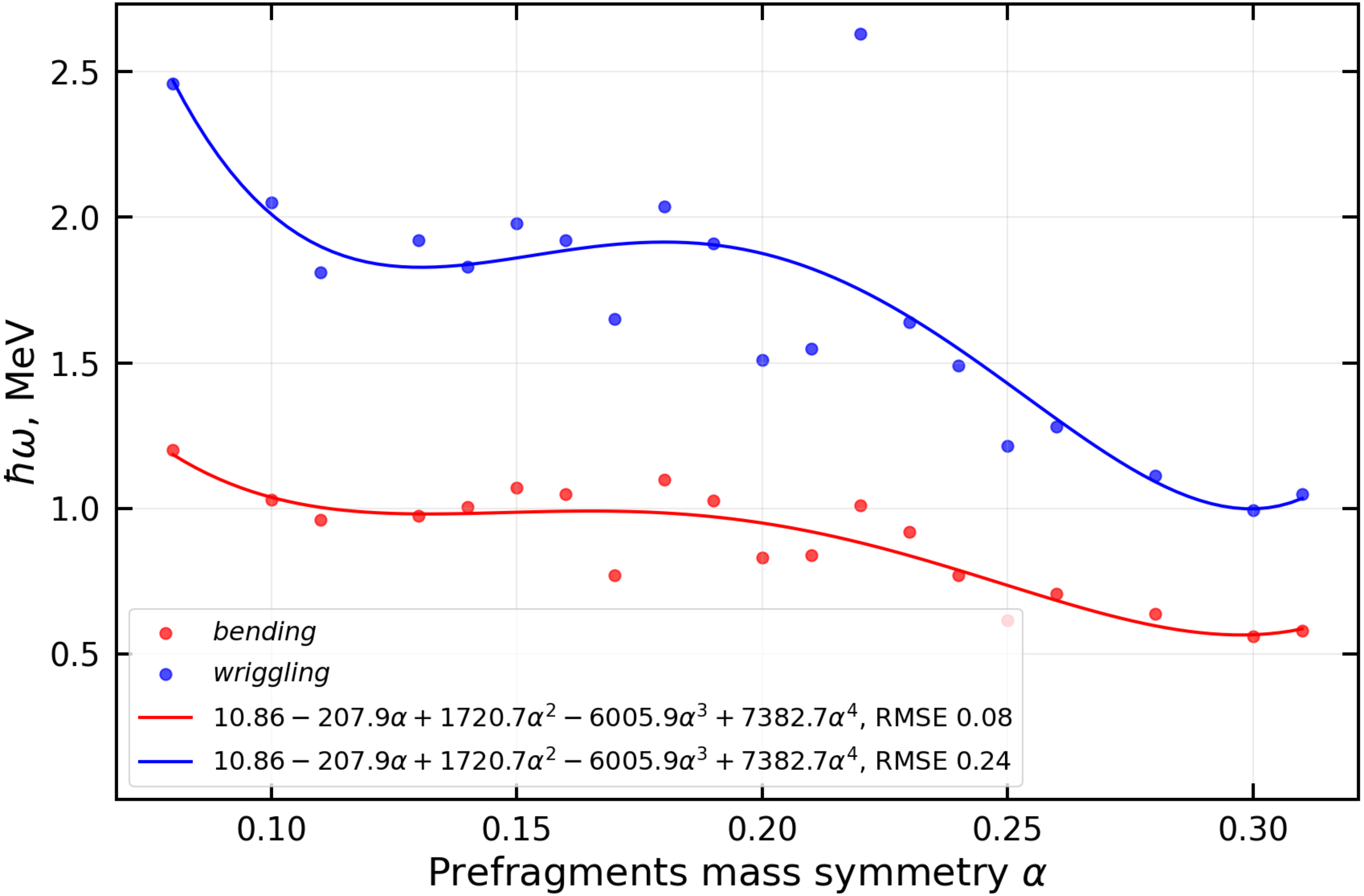
β_2 light fragment
0.37
0.51
0.66
0.80
0.94
^{232}Th(n,f)
^{238}U(n,f)
^{252}Cf(sf)
$\hbar\omega_w$, MeV
Mass asymmetry between fission fragments
β_2 heavy fragment





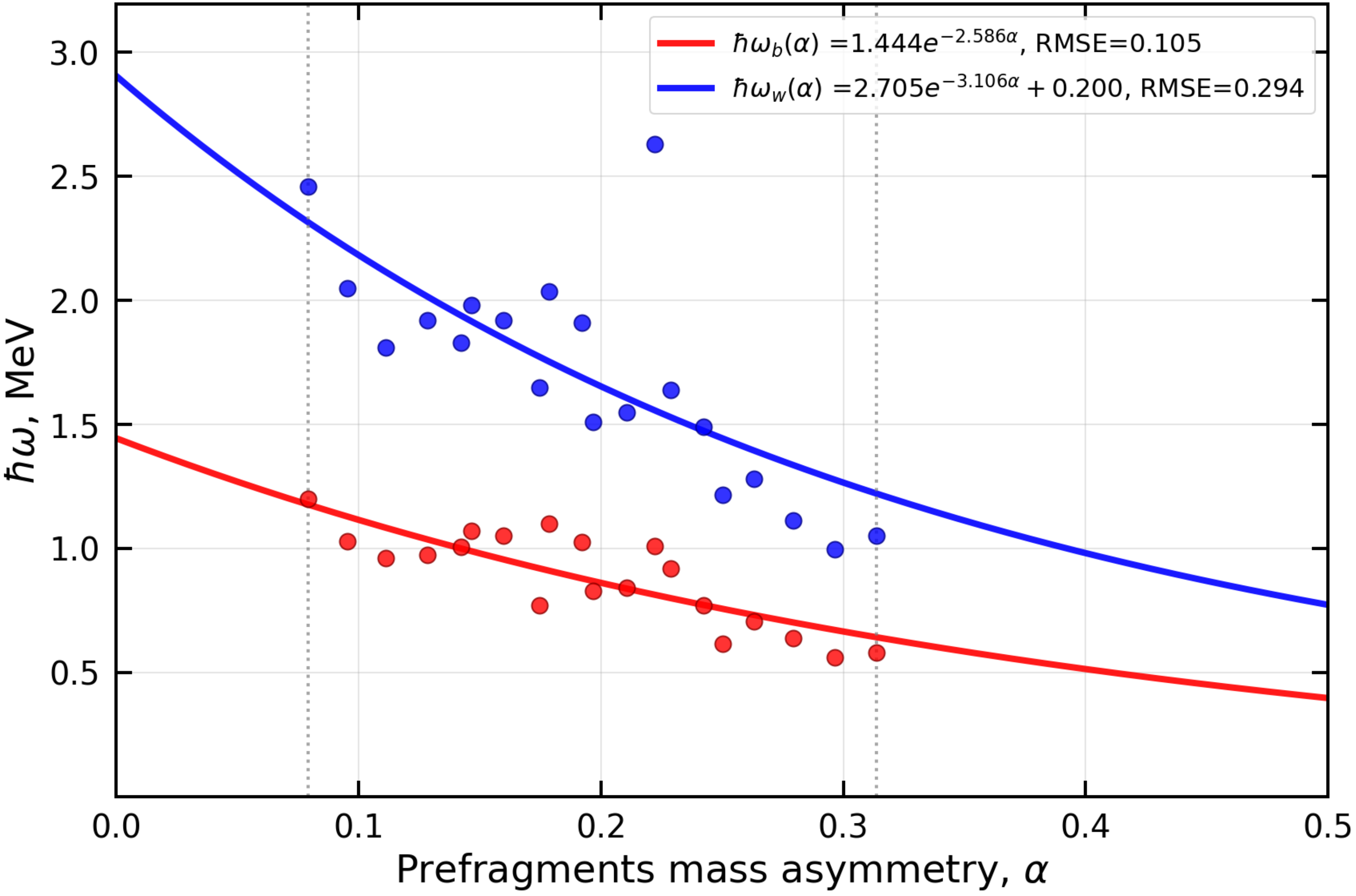
PVK, D.E. **Luybashevsky**, N.N. **Volokhin**,
INWT (Borovetz, Bulgaria) talk 04.07.25

Searching an approximation

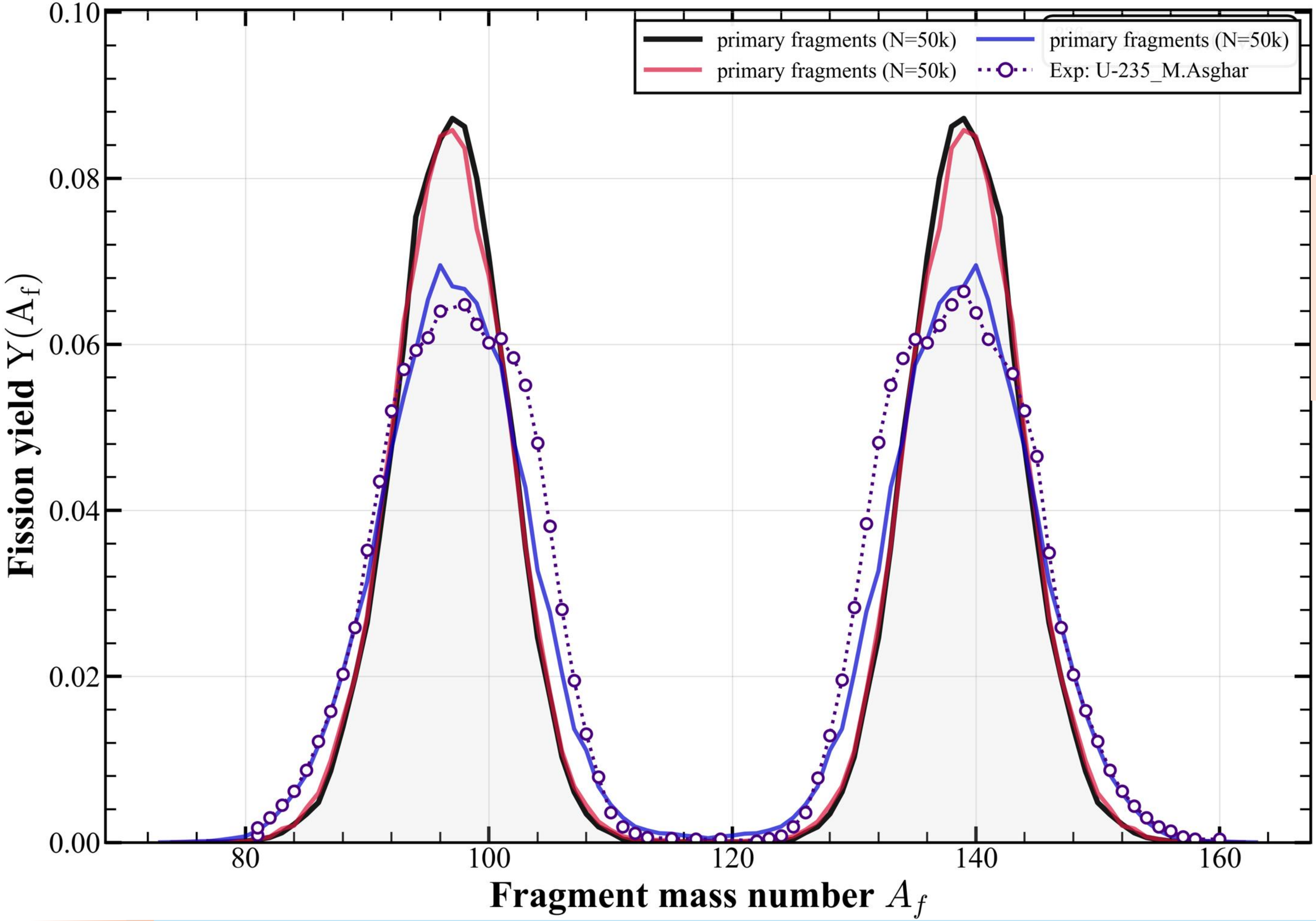


$$E_0 = \frac{1}{2}(\hbar\omega_b + \hbar\omega_w)$$

Searching an approximation



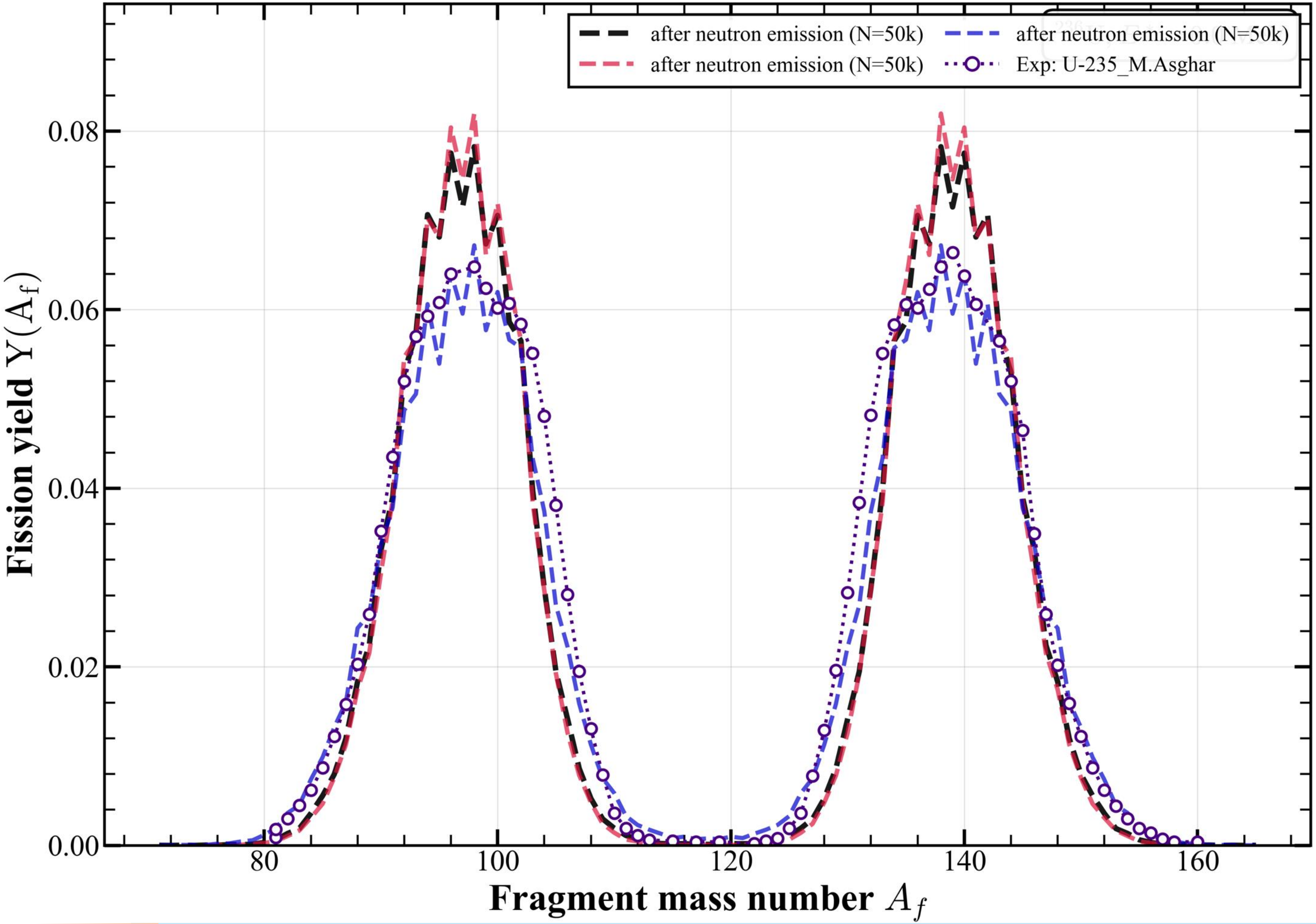
$$E_0 = \frac{1}{2}(\hbar\omega_b + \hbar\omega_w)$$



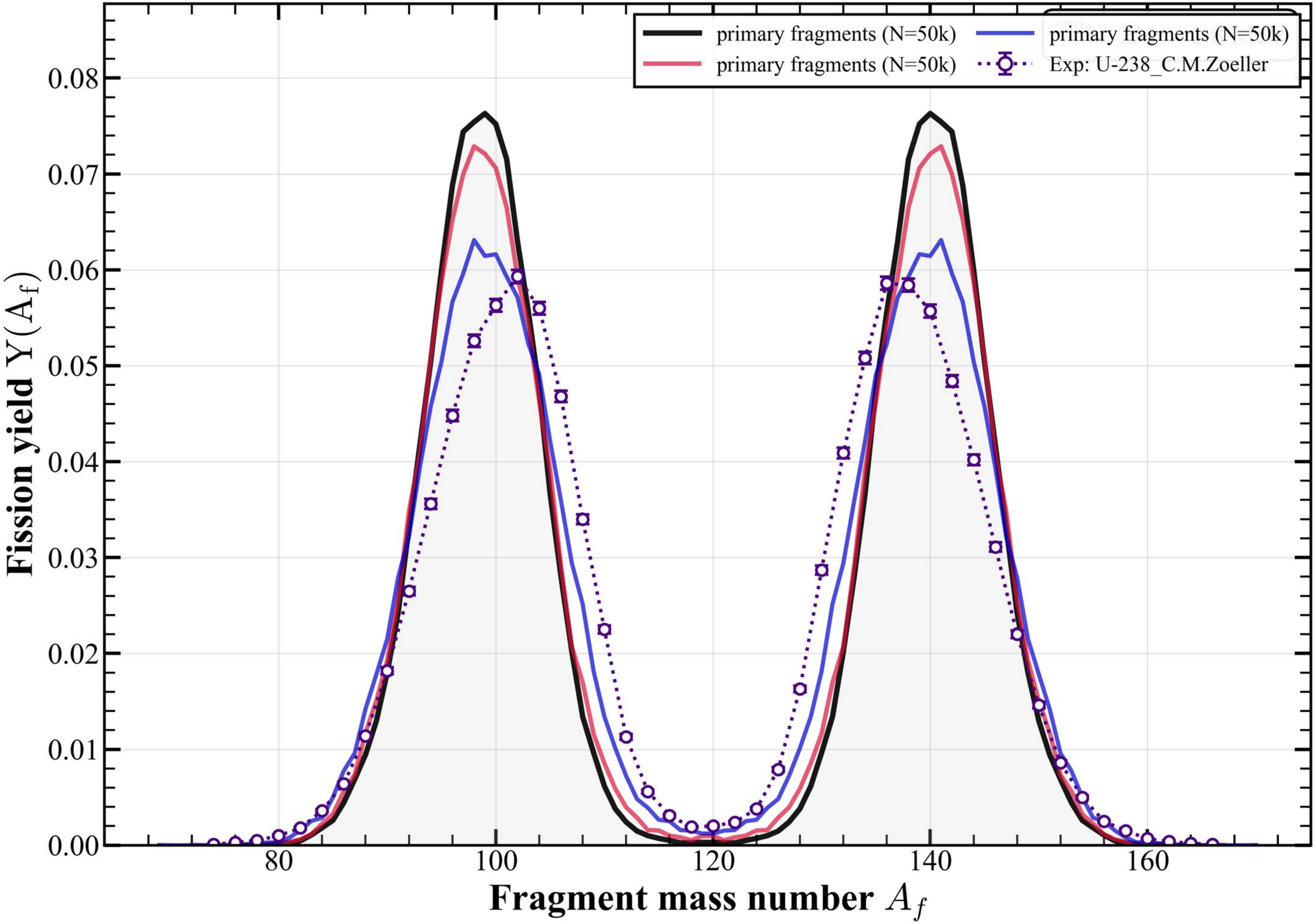
$^{235}U(n_{th}, f)$

- constant $\hbar\omega_0$
- dynamic $\hbar\omega_0$
- dynamic $\hbar\omega_1 = \frac{3}{2}\hbar\omega_0$, when $T > 0.5$ MeV

$^{235}U(n_{th}, f)$

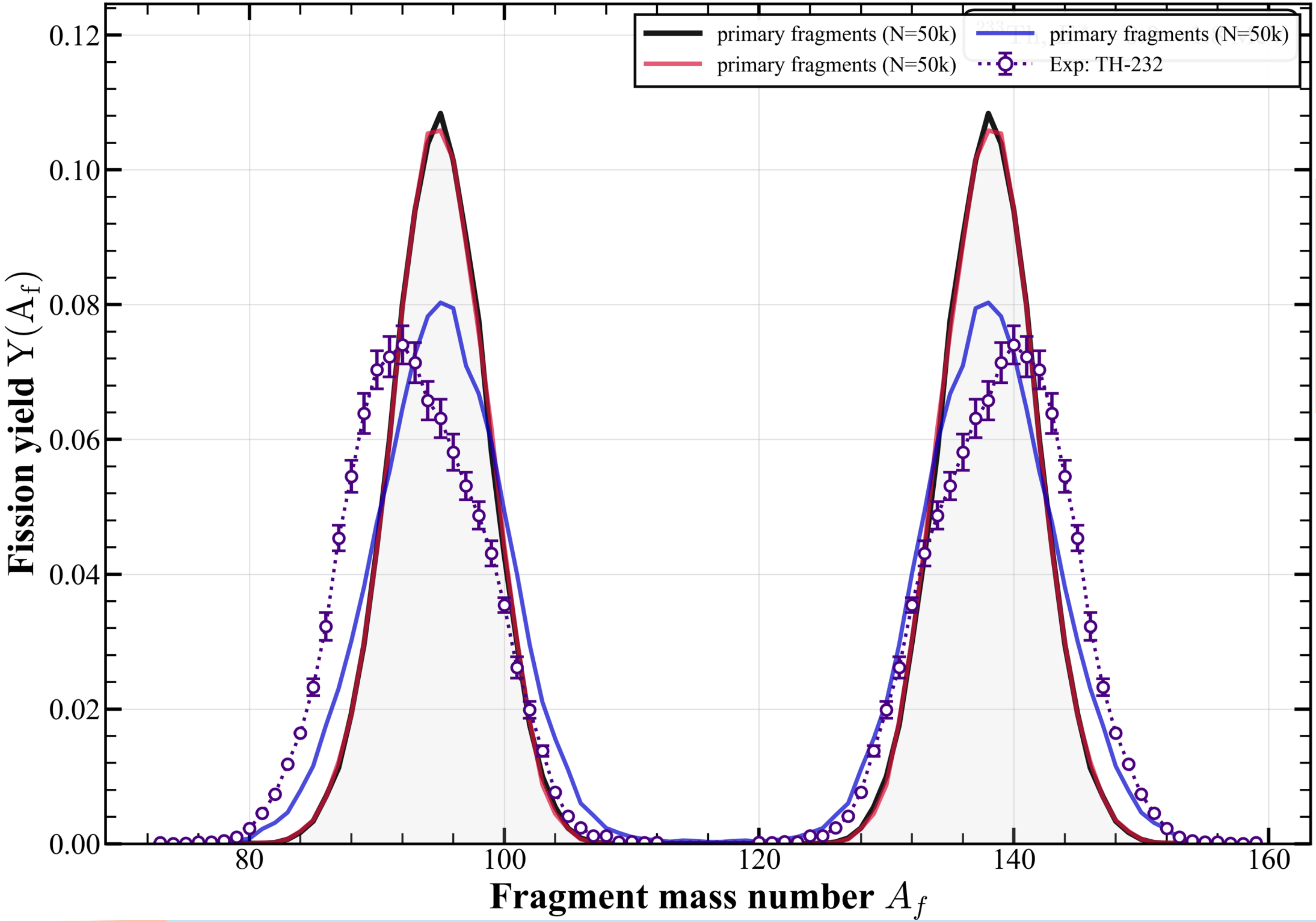


- constant $\hbar\omega_0$
- dynamic $\hbar\omega_0$
- dynamic $\hbar\omega_1 = \frac{3}{2}\hbar\omega_0$, when $T > 0.5$ MeV



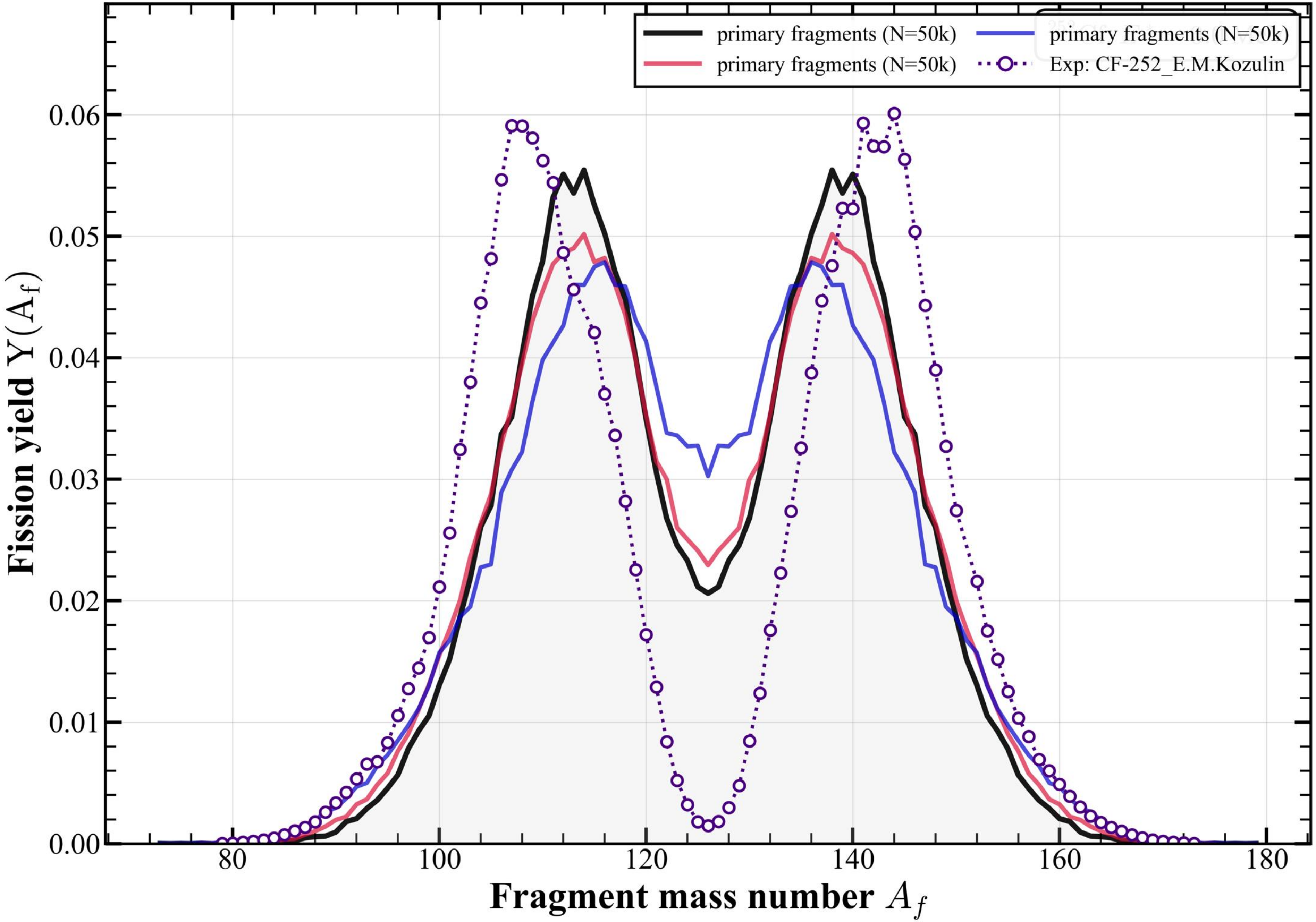
$^{239}U(n_{1.8}, f)$

- constant $\hbar\omega_0$
- dynamic $\hbar\omega_0$
- dynamic $\hbar\omega_1 = \frac{3}{2}\hbar\omega_0$, when $T > 0.5$ MeV



$^{232}Th(n_{1.8}, f)$

- constant $\hbar\omega_0$
- dynamic $\hbar\omega_0$
- dynamic $\hbar\omega_1 = \frac{3}{2}\hbar\omega_0$, when $T > 0.5$ MeV



^{252}Cf(sf)

- constant $\hbar\omega_0$
- dynamic $\hbar\omega_0$
- dynamic $\hbar\omega_1 = \frac{3}{2}\hbar\omega_0$, when $T > 0.5$ MeV

Summary

1.Limitations of Zero-Point Oscillations

- While zero-point oscillations (bending/wriggling) provide valuable insights:
- They prove **insufficient** for complete description of mass distributions in **induced fission**

2.Success in Spontaneous Fission Cases

- Good agreement observed for ^{252}Cf(sf) but with caveats:
- Strong dependence on **initial conditions**
- Requires validation across other spontaneously fissioning nuclei

3.Critical Improvements Needed

- Essential model extensions should include angular degrees of freedom between prefragments
- Expansion to broader range of nuclei
- Comprehensive analysis of fission fragment correlations (charges, TKE, multiplicities, etc)

THANK YOU FOR ATTENTION