FEDERAL STATE BUDGETARY EDUCATIONAL INSTITUTION OF HIGHER EDUCATION "VORONEZH STATE UNIVERSITY" OF THE MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION

CHARACTERISTICS OF FRAGMENTS AT BINARY NUCLEAR FISSION

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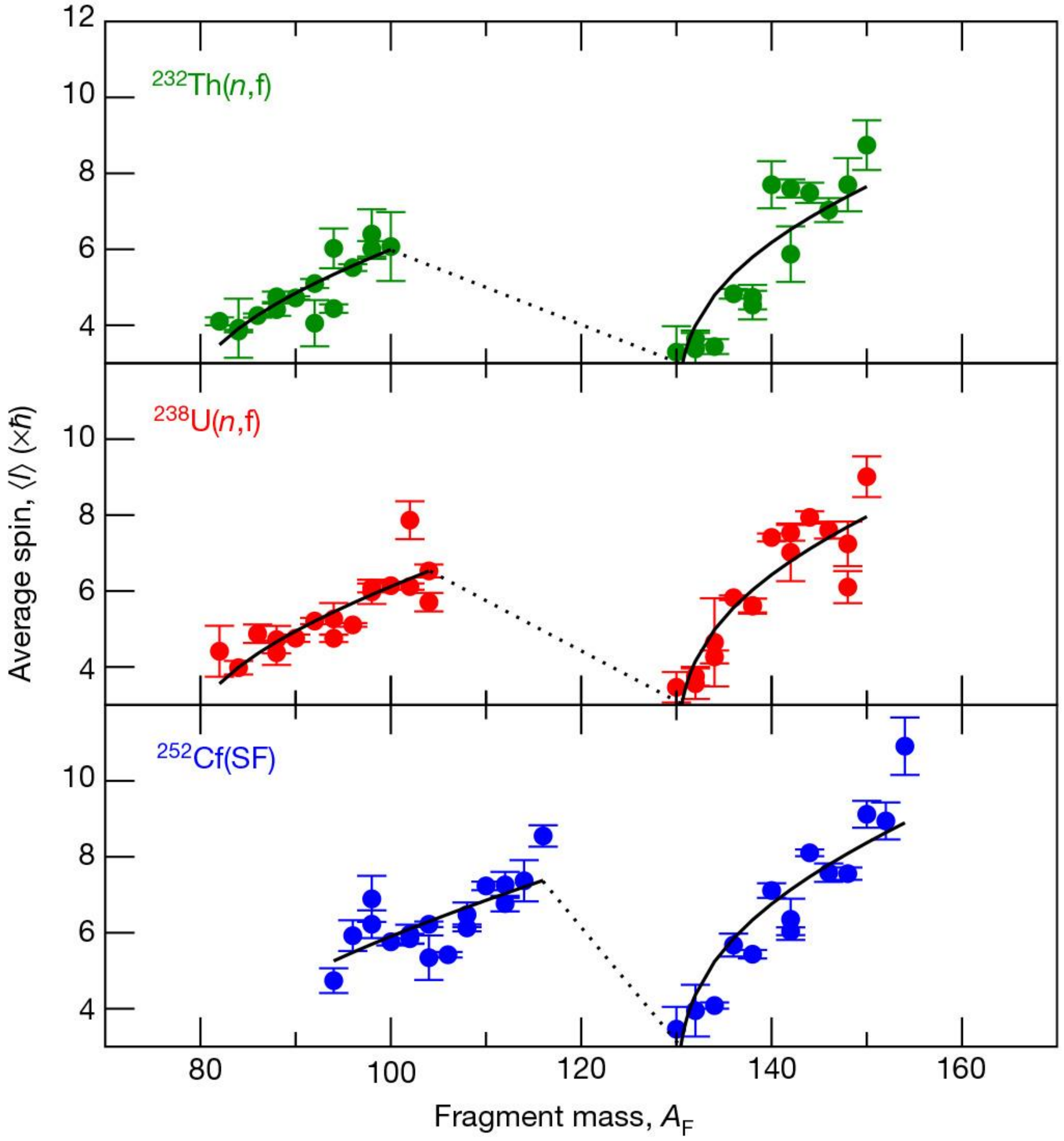
3. Novovoronezh NPP



Dependences of spins on mass numbers for the fission fragments 232Th, 238U and 252Cf.

1. Wilson, J.N., Thisse, D., Lebois, M. *et al.* Angular momentum generation in nuclear fission. *Nature* **590**, 566–570 (2021).

Description of the nuclear double fission process within the framework of the 'cold' fissile nucleus model

$$\Psi_K^{JM} = b_0 \Psi_{0_K}^{JM}\left(\beta_\lambda\right) + \sum_{i \neq 0} b_i \Psi_{i_K}^{JM},$$

where the function Ψ_{ik}^{JM} describes the quasiparticle excited state of the nucleus, and $\Psi_{0k}^{JM}(\beta_\lambda)$ describes the collective deformation motion of the nucleus with excitation energy $|B_n|$.

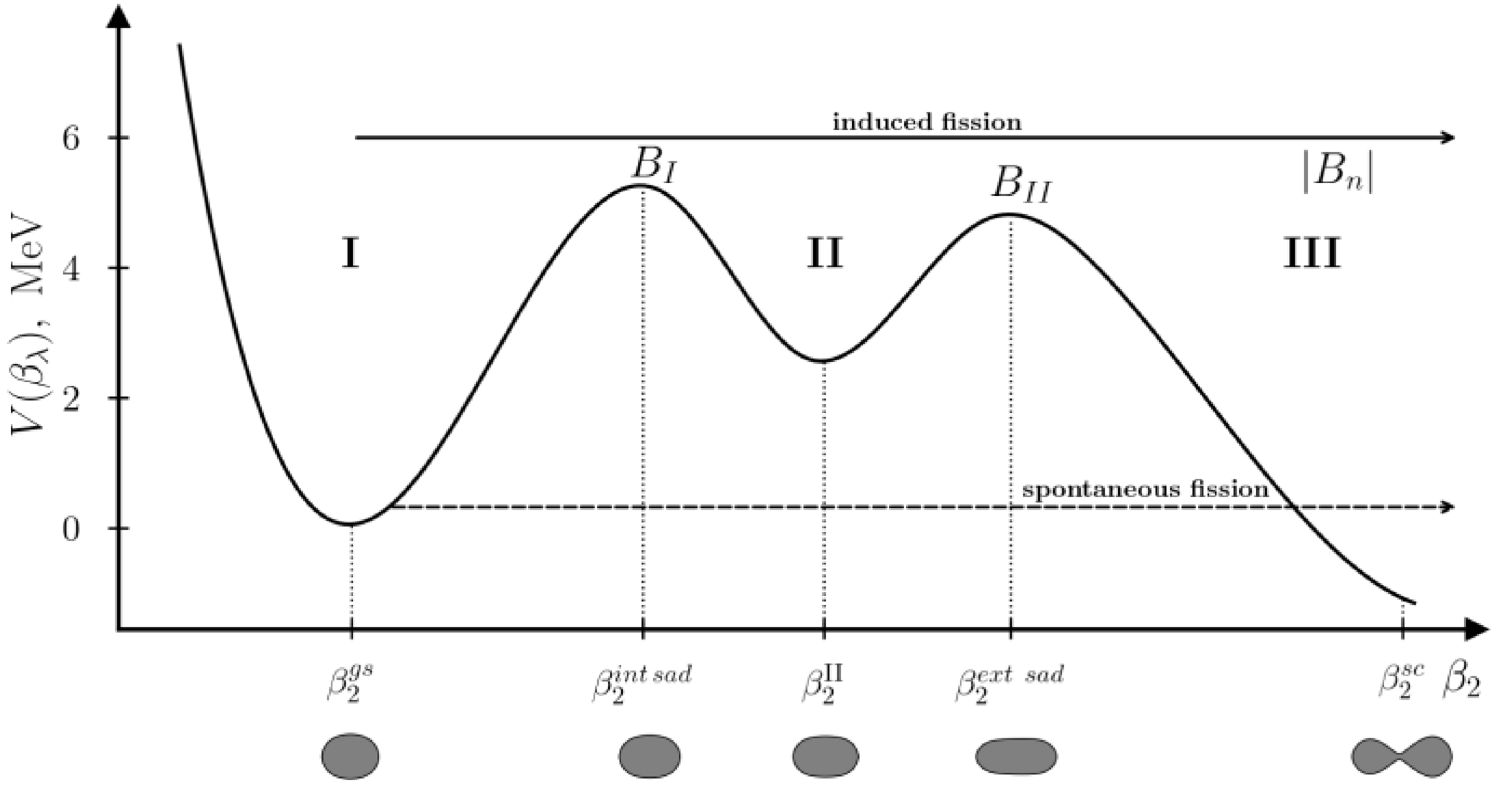
1. Kadmensky S.G. // Phys. Atom. Nucl. 2002. V. 65. P. 1390.

2. E. P. Wigner, Ann. Math. 62, 548 (1955).

3. E. P. Wigner, Ann. Math. 65, 203 (1957).

4. E. P. Wigner, Ann. Math. 67, 325 (1958)

Description of the nuclear double fission process within the framework of the ‘cold’ fissile nucleus model



Principal scheme of the potential V depending on the quadrupole deformation of the nucleus β_2. Region I corresponds to the ground state of the nucleus with β_2^{gs}. II - isomeric states, and III - the out-of-barrier region where the nucleus decays into fission fragments.

1. Kadmensky S.G. // Phys. Atom. Nucl. 2002. V. 65. P. 1390.
2. Kadmensky S.G., Titova L.V., *Physics of Atomic Nuclei*, 2009, **72**(10), 1738–1744
3. G. Danilyan, *Phys. At. Nucl.* **82**, 235 (2019);
4. A. Gagarski et al., *Phys. Rev. C* **93**, 054619 (2016)

Description of the nuclear double fission process within the framework of the ‘cold’ fissile nucleus model

$$C_{b(w)} = I_{b(w)}\hbar\omega_{b(w)} \coth\left(\frac{\hbar\omega_{b(w)}}{2T}\right) \rightarrow \begin{cases} 2I_{b(w)}T, & T \gg \hbar\omega_{b(w)} \\ I_{b(w)}\hbar\omega_{b(w)}, & T \ll \hbar\omega_{b(w)} \end{cases},$$

where $C_{b(w)}$, $I_{b(w)}$ and $\hbar\omega_{b(w)}$ are coefficient, moment of inertia and energy of pointing oscillations.

1. J.R. Nix, W.J. Swiatecki, Nucl. Phys. **71,** 1 (1965).
2. J. Randrup and R. Vogt, Phys. Rev. Lett. 127, 062502 (2021).

Spin distribution of double fission fragments

$$P\left(J_{k_x}, J_{k_y}\right) \equiv P\left(J_{k_x}\right) P\left(J_{k_y}\right) = \frac{1}{\pi I_k \hbar\omega_k} \exp\left[-\frac{J_{k_x}^2 + J_{k_y}^2}{I_k \hbar\omega_k}\right]$$

where the index k = w, b corresponds to the type of oscillations (wriggling or bending), I_k is the moment of inertia of these oscillations, the frequencies ω_k of the oscillations are determined by the classical formulae $\omega_k = \sqrt{\frac{K_k}{M_k}}$.

1. Nix J.R. and Swiatecki W.J. // Nucl. Phys. A. 1965. V. 71. P. 1.
2. S. G. Kadmensky et al., Phys. At. Nucl. 87, 359 (2024).

3. D.E. Lyubashevsky et al., arXiv preprint arXiv:2412.04410 (2024).

Spin distribution of double fission fragments

$$I_w = \frac{(I_1 + I_2)I_0}{I} \qquad I_b = \mu R^2 I_H \Big/ \left(\mu R^2 + I_H\right) \qquad I_0 = \mu\left(R_1 + R_2 + d\right)^2$$

$$I = I_0 + I_1 + I_2$$

$$J_{i\,x(y)} = \frac{I_i}{I_1 + I_2} J_{w_{x(y)}} + (-1)^{i+1} J_{b_{x(y)}}$$

$$I_{1,2} \equiv I_{i,\mathrm{rigit}} = \frac{M_i}{5} \sum R_i^2$$

$$J_{w_{x(y)}} = J_{1x(y)} + J_{2x(y)}$$

$$R_i = r_0 A^{1/3} \left[1 - \beta_i^2 / 4\pi + \beta_i \sqrt{5/4\pi}\,\right]$$

$$\mu = M_1 M_2 \Big/ \left(M_1 + M_2\right)$$

1. J. Randrup and R. Vogt, Phys. Rev. Lett. 127, 062502 (2021).
2. R. Vogt and J. Randrup, Phys. Rev. C 103, 014610 (2021).
3. T. Shneidman, G. Adamian, N. Antonenko, S. Ivanova, R. Jolos, and W. Scheid, Phys. Rev. C 65, 064302 (2002)
4. D. E. Lyubashevsky *et al* 2025 *Chinese Phys. C* **49** 034104

Spin distribution of double fission fragments

$$J_{b_{x(y)}} = J_{1x(y)} - \frac{I_1}{I_1 + I_2} J_{w_{x(y)}} = \frac{I_2 J_{1x(y)} - I_1 J_{2x(y)}}{I_1 + I_2}$$

$$P(J_{w_{x(y)}}, J_{b_{x(y)}}) = P(J_{w_{x(y)}}) P(J_{b_{x(y)}})$$

$$P(J_{1x.}, J_{2x}, J_{1y}, J_{2y}) =$$

$$\frac{1}{\pi^2 I_w \hbar\omega_w I_b \hbar\omega_b} \exp\left[-\left\{ \frac{J_{\omega_x}^2 + J_{\omega_y}^2}{I_w \hbar\omega_w} + \frac{J_{b_x}^2 + J_{b_y}^2}{I_b \hbar\omega_b} \right\} \right] \left| \frac{\partial(J_{w_x}, J_{b_x}, J_{w_y}, J_{b_y}}{\partial(J_{1x}, J_{2x}, J_{1y}, J_{2y})} \right| =$$

$$= \frac{1}{\pi^2 I_w \hbar\omega_w I_b \hbar\omega_b} \exp\left[\begin{array}{l} -\frac{1}{I_w \hbar\omega_w} \left\{ (J_{1x} + J_{2x})^2 + (J_{1y} + J_{2y})^2 \right\} - \\ -\frac{1}{I_b \hbar\omega_b (I_1 + I_2)^2} \left\{ (I_2 J_{1x} - I_1 J_{2x})^2 + (I_2 J_{1y} - I_1 J_{2y})^2 \right\} \end{array} \right].$$

1. D.E. Lyubashevsky et al., arXiv preprint arXiv:2412.04410 (2024).

Spin distribution of double fission fragments

$$P(J_1, J_2, \varphi) = \frac{2J_1J_2}{\pi I_w \hbar\omega_w I_b \hbar\omega_b} \exp\begin{bmatrix} -J_1^2(\alpha I_2^2 + \beta) - J_2^2(\alpha I_1^2 + \beta) + \\ + 2J_1J_2 \cos\varphi(\alpha J_1 J_2 - \beta) \end{bmatrix},$$

$$\alpha = \frac{1}{I_b \hbar\omega_b (I_1 + I_2)^2}, \beta = \frac{1}{I_w \hbar\omega_w}$$

1. D.E. Lyubashevsky et al., arXiv preprint arXiv:2412.04410 (2024).

Spin distribution of double fission fragments

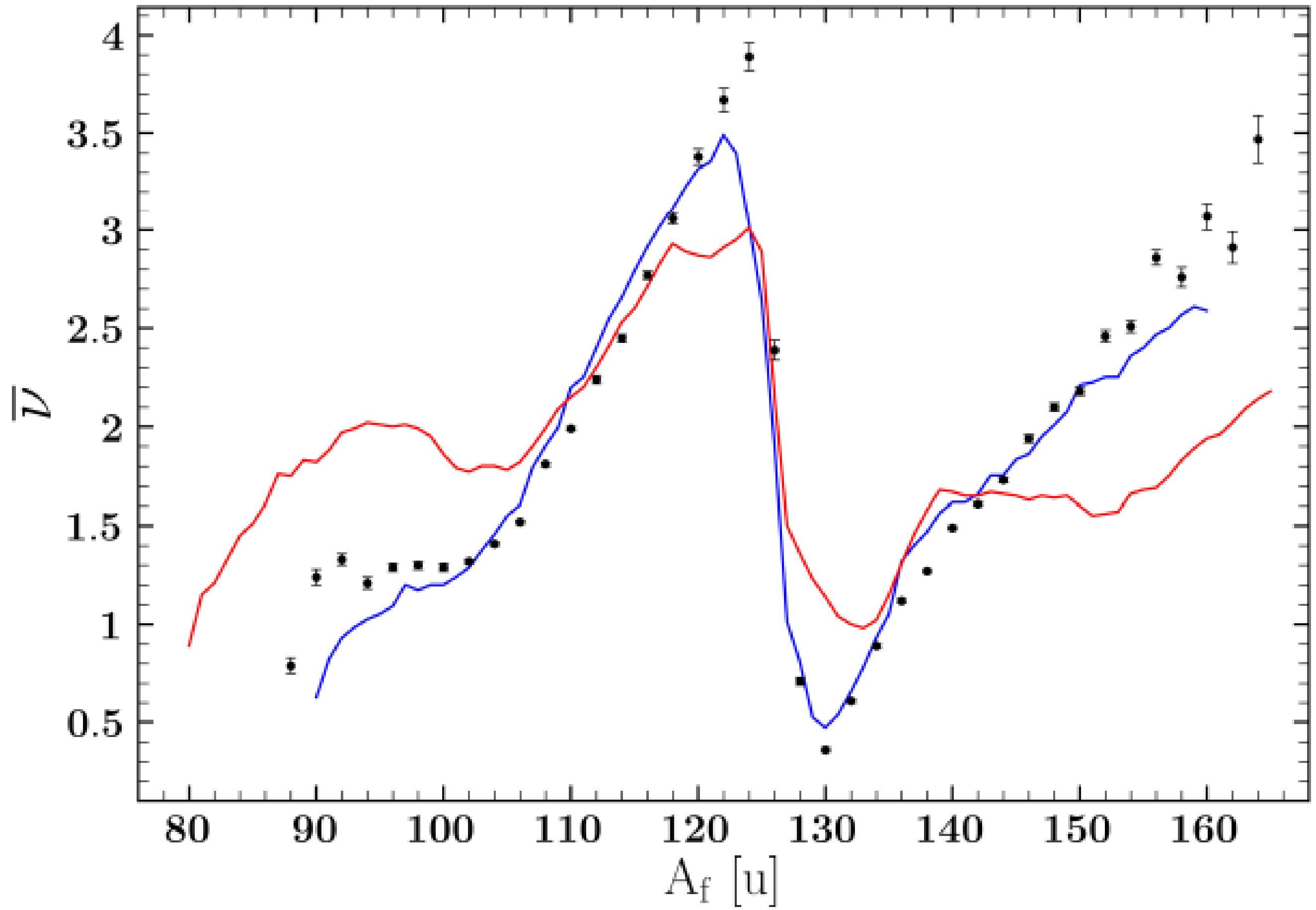
$$P(J_i) = \frac{2J_i}{d_i} \exp\left[-\frac{J_i^2}{d_i} \right]$$

$$d_i = \frac{I_i^2 I_w \hbar\omega_w}{(I_1 + I_2)^2} + I_b \hbar\omega_b$$

$$\bar{J}_i = \int_0^{\infty} P(J_i) J_i \, dJ_i = \int_0^{\infty} \frac{2J_i^2}{d_i} \exp\left[-\frac{J_i^2}{d_i} \right] dJ_i = \frac{1}{2}\sqrt{\pi d_i}$$

1. S. G. Kadmensky et al., Phys. At. Nucl. 87, 359 (2024).
2. D.E. Lyubashevsky et al., arXiv preprint arXiv:2412.04410 (2024).

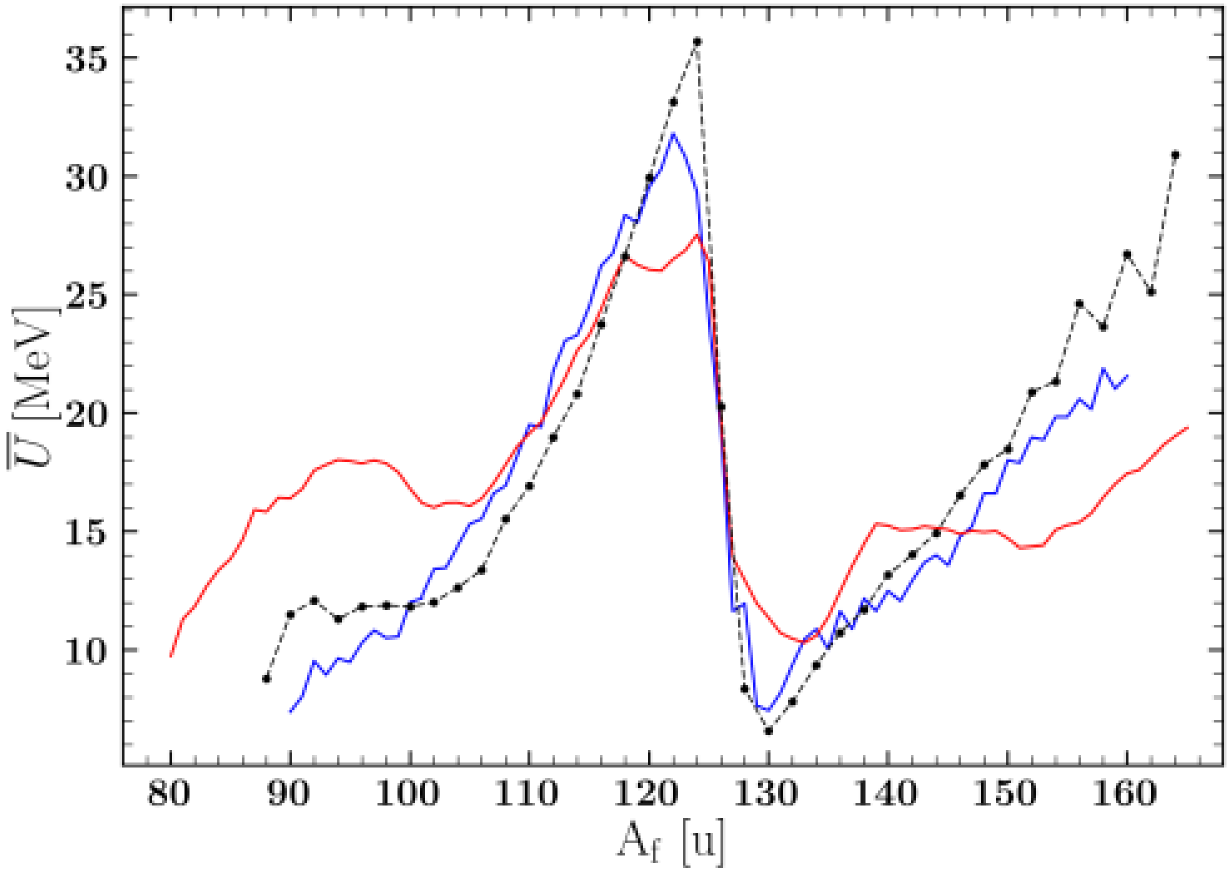
Approaches to calculating moments of inertia of nuclei



Average neutron multiplicities as a function of the mass of 252Cf spontaneous fission fragments. The black dots indicate the experimental data from [1], the red line indicates the estimate according to the FREYA model [2], and the blue line indicates the estimate according to the Grudzevich method [3].

1. R. Walsh and J. Boldeman, Nucl. Phys. A 276, 189 (1977).
2. J. Randrup and R. Vogt, Phys. Rev. C 80, 024601 (2009).
3. O. T. Grudzevich, Probl. At. Sci. Technol. Ser: Nucl. Const. 1, 39 (2000).

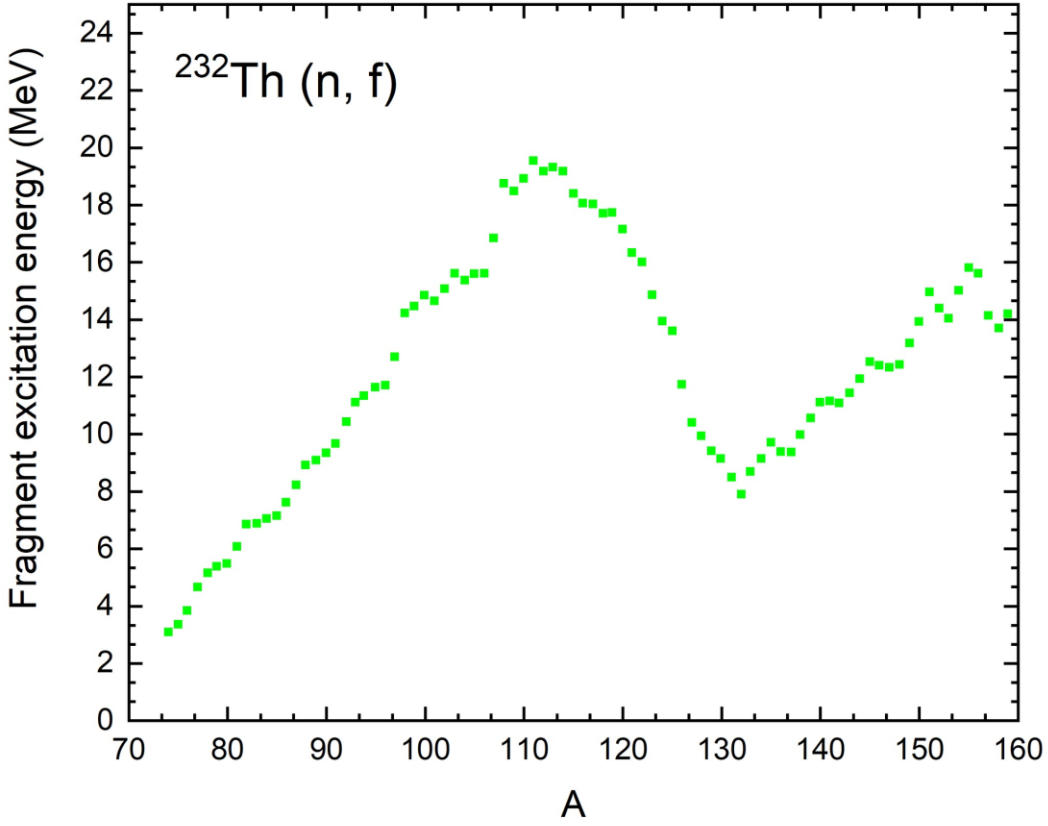
Approaches to calculating moments of inertia of nuclei



Average excitation energies as a function of the mass of a 252Cf spontaneous fission fragment. Black dots with dashed lines indicate the reconstructed excitation energies from the data of [1], red line – FREYA [2], blue line - the estimate by the method of Grudzevich [3].

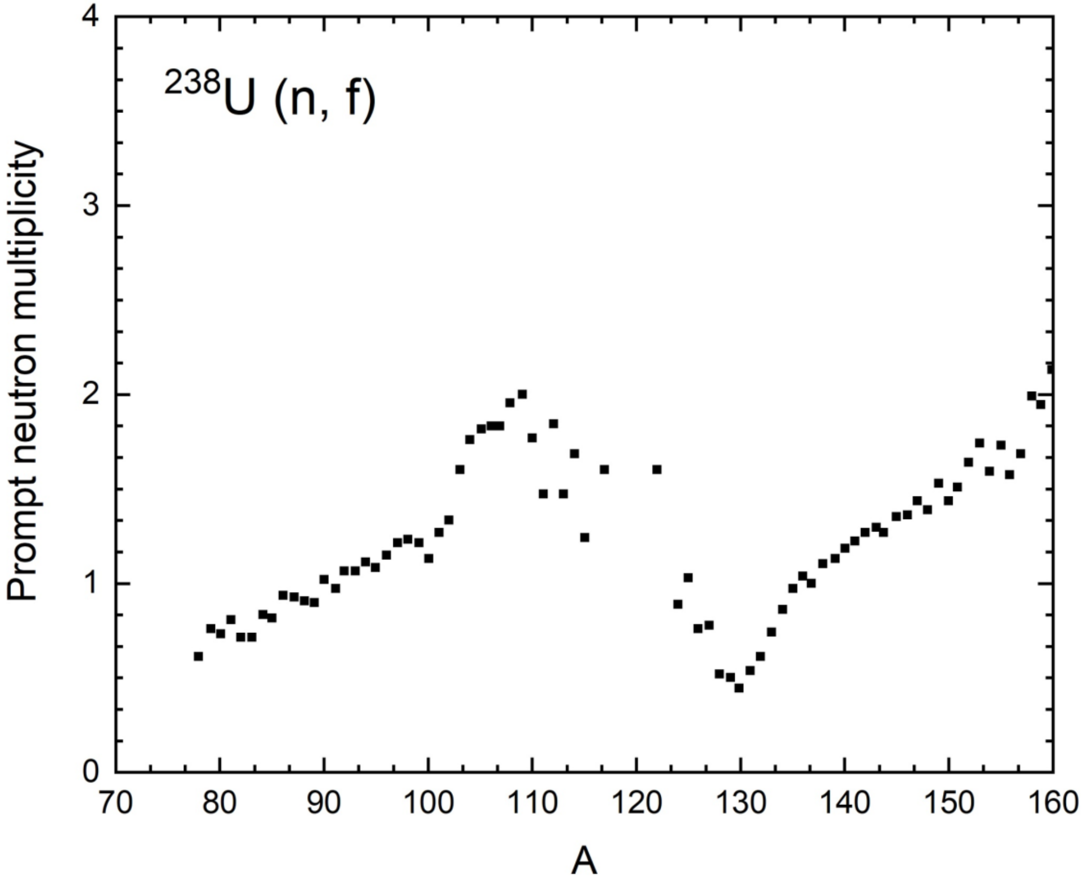
1. R. Walsh and J. Boldeman, Nucl. Phys. A 276, 189 (1977).
2. J. Randrup and R. Vogt, Phys. Rev. C 80, 024601 (2009).
3. O. T. Grudzevich, Probl. At. Sci. Technol. Ser: Nucl. Const. 1, 39 (2000).
4. T. Døssing, S. Åberg, M. Albertsson, B. G. Carlsson, J. Randrup, PHYSICAL REVIEW C 109, 034615 (2024)].

Approaches to calculating moments of inertia of nuclei



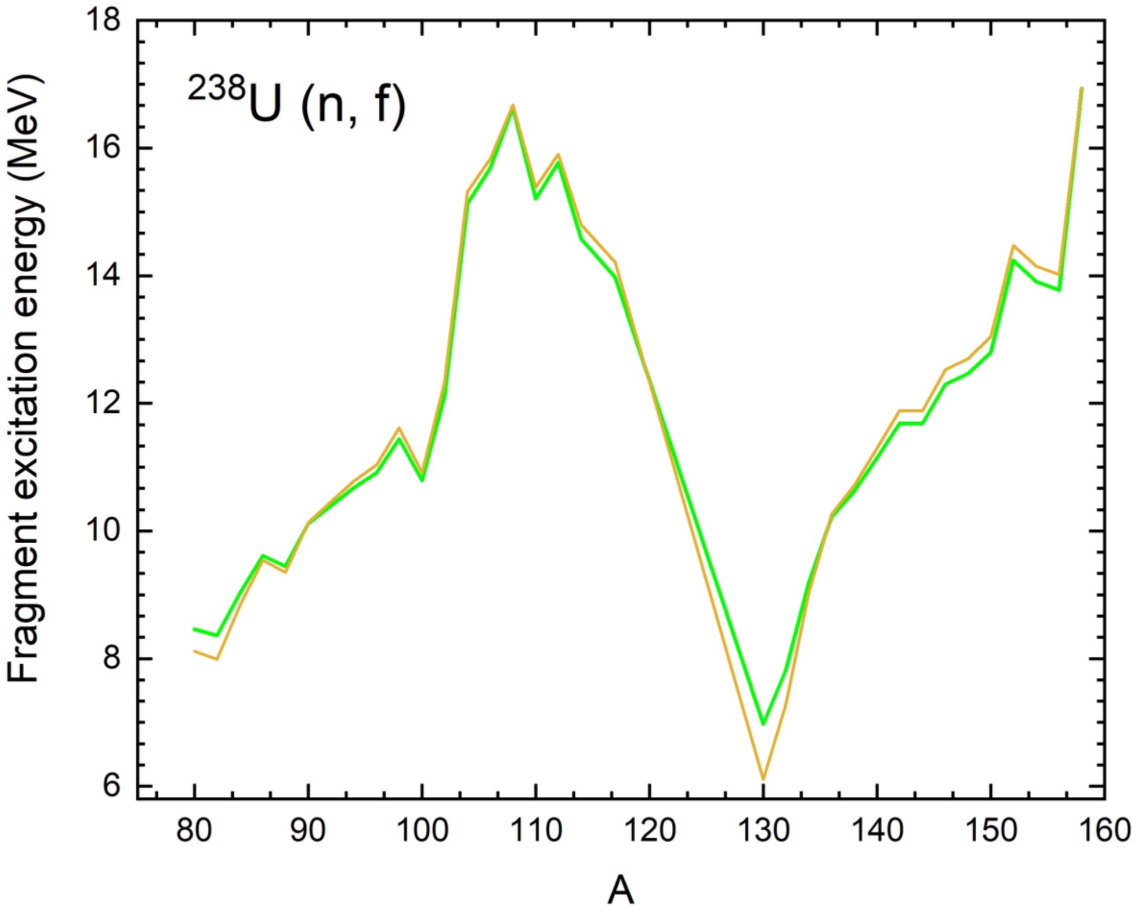
1. A. Tudora Nuclear Physics A 916 (2013) 79–101.

Approaches to calculating moments of inertia of nuclei



1. A. Tudora, F.-J. Hambsch, and V. Tobosaru PHYSICAL REVIEWC94,044601 (2016).

Approaches to calculating moments of inertia of nuclei



1. O. T. Grudzevich, Probl. At. Sci. Technol. Ser: Nucl. Const. 1, 39 (2000).
2. T. Døssing, S. Åberg, M. Albertsson, B. G. Carlsson, J. Randrup, PHYSICAL REVIEW C 109, 034615 (2024).

Approaches to calculating moments of inertia of nuclei

$$U = 5 + 4\nu + \nu^2$$

$$U = 7\left(\nu + \frac{3}{7}\right)$$

$$U = \sigma A^{\frac{2}{3}}\left[0{,}4(1 - x)\alpha^2 - 0{,}0381(1 - 2x)\alpha^3\right]$$

1. O. T. Grudzevich, Probl. At. Sci. Technol. Ser: Nucl. Const. 1, 39 (2000).
2. T. Døssing, S. Åberg, M. Albertsson, B. G. Carlsson, J. Randrup, PHYSICAL REVIEW C 109, 034615 (2024)].
3. V. Strutinsky, Nucl. Phys. A 95, 420 (1967).

Approaches to calculating moments of inertia of nuclei

- hydrodynamic model

$$J = \frac{9mR^2}{4\pi}\frac{\beta^2\left(1+\frac{1}{4}\sqrt{\frac{5}{4\pi}}\beta\right)^2}{2+\sqrt{\frac{5}{4\pi}}\beta+\frac{25}{16\pi}\beta^2}.$$

$$\frac{J}{J_0} = \frac{45\beta^2\left(1+\frac{\beta}{4}\sqrt{\frac{5}{4\pi}}\right)}{8\pi\left(2+\sqrt{\frac{5}{4\pi}}\beta+\frac{25}{16\pi}\beta^2\right)\left(1+\sqrt{\frac{5}{16\pi}}\beta+\frac{25}{32\pi}\beta^2\right)}.$$

The hydrodynamic approach describes the nucleus as a drop of nuclear fluid in which the moments of inertia are determined on the basis of the mass distribution and shape of the fragments. This method takes into account the contribution of the collective motion of nuclear matter.

1. D. E. Lyubashevsky *et al* 2025 *Chinese Phys. C* **49** 034104 .

Approaches to calculating moments of inertia of nuclei

- oscillatory potential

$$\frac{J}{J_0} = \frac{N}{A}\Phi(\chi_n) + \frac{Z}{A}\Phi(\chi_p),$$

$$J = J_0 \left\{ 1 - g_1 + \frac{g_1^2 \chi^2}{v_1^2 g_1 + g_2^2 \nu_2} \right\} = J_0 \Phi_1 \left(\chi \right).$$

The oscillatory model is based on the representation of nuclear levels in the framework of the oscillatory potential. This approach takes into account quantum effects and the distribution of nucleons in the field approximated to a harmonic oscillator.

1. Migdal A. B. Superfluidity and the moments of inertia of nuclei //Sov. Phys. JETP. – 1960. – T. 10. – №. 1. – C. 176.
2. D. E. Lyubashevsky *et al* 2025 *Chinese Phys. C* **49** 034104 .

Approaches to calculating moments of inertia of nuclei

- rectangular potential well

$$\frac{J}{J_0} = \frac{N}{A}\Phi(\chi_n) + \frac{Z}{A}\Phi(\chi_p),$$

$$J_1 = J_0\left\{1 - \frac{45}{4}\int_0^1 d\xi\xi^3\sqrt{1-\xi^2}\int_0^1 d\eta\,(1-\eta^2)g\left(\aleph\frac{\eta}{\xi}\right)\right\}$$

$$= J_0\Phi_2(\chi).$$

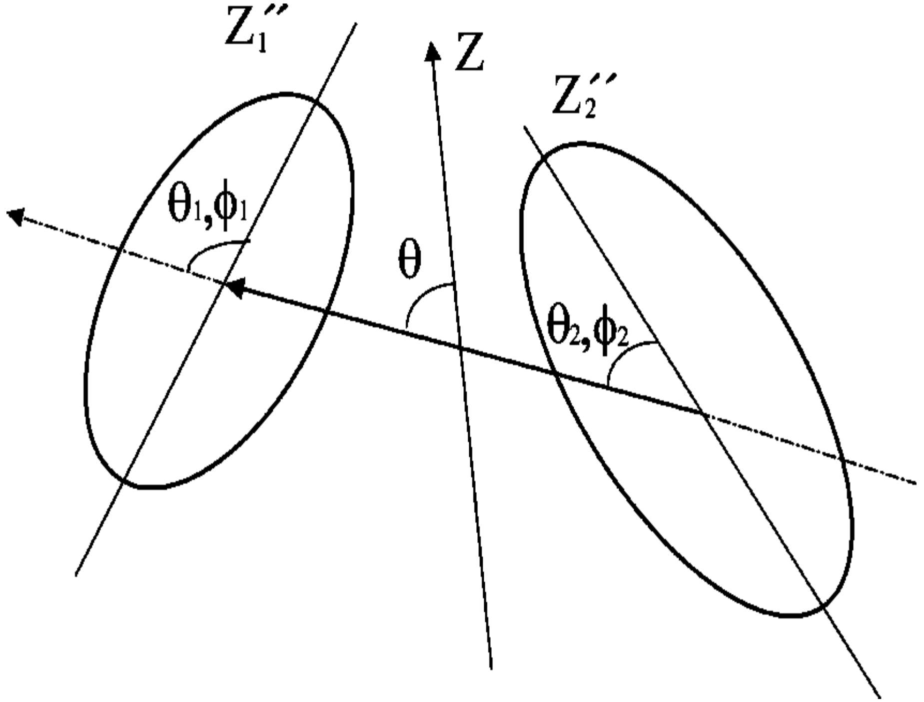
The rectangular potential model uses the approximation of a spherical or deformed potential with rigid walls. This method considers the motion of nucleons in a potential of finite depth, which allows us to estimate the contribution of shell effects.

1. Migdal A. B. Superfluidity and the moments of inertia of nuclei //Sov. Phys. JETP. – 1960. – T. 10. – №. 1. – C. 176.
2. D. E. Lyubashevsky *et al* 2025 *Chinese Phys. C* **49** 034104 .

Axially symmetric compound fissile systems

$$R = R_m \approx R_1\left(1 - \frac{\beta_1^2}{4\pi} + \sqrt{\frac{5}{4\pi}}\beta_1\right) + R_2\left(1 - \frac{\beta_2^2}{4\pi} + \sqrt{\frac{5}{4\pi}}\beta_2\right) + d$$

$$U(R,\beta_i,\ \Omega_i)=U_C(R,\beta_i\ ,\Omega_i)+U_N(R,\beta_i\ ,\Omega_i).$$



Schematic representation and definitions of various configuration coordinates of constituent fissile systems.

1. T. Shneidman, G. Adamian, N. Antonenko, S. Ivanova, R. Jolos, and W. Scheid, Phys. Rev. C 65, 064302 (2002);
2. G. G. Adamian et al., Int. J. Mod. Phys. E 5, 191 (1996)

Axially symmetric compound fissile systems

$$R_1\left(\pi - \theta_1\right) = -R_2\theta_2$$

$$\epsilon = \pi - \theta_1$$

$$H = T_{\text{rot}} + T_\epsilon + U_\epsilon$$

$$T_\epsilon = -\frac{\hbar^2}{2J_\epsilon}\frac{1}{\epsilon}\frac{\mathrm{d}}{\mathrm{d}\epsilon}\left(\epsilon\frac{\mathrm{d}}{\mathrm{d}\epsilon}\right)$$

1. S. Misicu, A. Săndulescu, G. M. Ter-Akopian, W. Greiner, *Phys. Rev. C*, **60**, 034613

Axially symmetric compound fissile systems

$$\theta = \frac{J_1 \tilde{R}_2 - J_2 \tilde{R}_1}{\tilde{R}_2 (\mu R_m^2 + J_1 + J_2)} \epsilon$$

$$U_\epsilon = \frac{1}{2} C_{11} (\pi - \theta_1)^2 + C_{12} (\pi - \theta_1) \theta_2 + \frac{1}{2} C_{22} \theta_2^2$$

1. G. G. Adamian, N.V.Antonenko, R. V. Jolos, Yu.V.Palchikov, T.M.Shneidman, W.Scheid Physics of Atomic Nuclei, 2007, Vol. 70, No. 8, pp. 1350–1356
2. P. O. Hess и W. Greiner, Nuovo Cimento A 83, 76 (1984); P. O. Hess, W. Greiner и W. T. Pinkston, Phys. Rev. Lett. 53, 1535 (1984)
3. T. Shneidman, G. Adamian, N. Antonenko, S. Ivanova, R. Jolos, and W. Scheid, Phys. Rev. C 65, 064302 (2002);
4. G. G. Adamian et al., Int. J. Mod. Phys. E 5, 191 (1996)

Axially symmetric compound fissile systems

$$U_\epsilon = \frac{1}{2} C_\epsilon \epsilon^2$$

$$C_b = C_{11} - 2\frac{R_1}{R_2} C_{12} + \left(\frac{R_1}{R_2}\right)^2 C_{22}$$

Schrödinger equation for bending - oscillations:

$$-\frac{\hbar^2}{2J_\epsilon} \frac{1}{\epsilon} \frac{\mathrm{d}}{\mathrm{d}\epsilon}\left(\epsilon \frac{\mathrm{d}}{\mathrm{d}\epsilon}\right)\psi_n + \frac{1}{2} C_\epsilon \epsilon^2 \psi_n = E_n \psi_n$$

Its solutions:

$$\Psi\left(J_{b_x}, J_{b_y}\right) \equiv \Psi\left(J_{b_x}\right)\Psi\left(J_{b_y}\right) = \frac{1}{\pi I_b \hbar\omega_b} \exp\left[-\frac{J_{b_x}^2 + J_{b_y}^2}{I_b \hbar\omega_b}\right],$$

$$E_b = \hbar w_b\left(n + 1/2\right), \ n = 0,1,2,...$$

$$w_b = \sqrt{C_b / I_b}$$

Axially symmetric compound fissile systems

$$C_w = C_{11} + 2\frac{R_1}{R_2}C_{12} + \left(\frac{R_1}{R_2}\right)^2 C_{22}$$

$$\Psi\left(J_{w_x}, J_{w_y}\right) \equiv \Psi\left(J_{w_x}\right)\Psi\left(J_{w_y}\right) = \frac{1}{\pi I_w \hbar\omega_w}\exp\left[-\frac{J_{w_x}^2 + J_{w_y}^2}{I_w \hbar\omega_w}\right],$$

$$E_w = \hbar w_w \left(n + 1/2\right), \;\; n = 0,1,2,...$$

$$w_w = \sqrt{C_w / I_w}$$

1. S. Misicu, A. Săndulescu, G. M. Ter-Akopian, W. Greiner, *Phys. Rev. C*, **60**, 034613

Axially symmetric compound fissile systems

$$C^n_{11} = -3\left[\sqrt{\frac{5}{4\pi}} U_{22} + \frac{5}{4\pi} c^2_{11}\left(2U_{33} + U_{32} + \sqrt{\frac{5}{4\pi}}(2U_{42} + U_{41}) + \frac{5}{2\pi} U_{51} \right) \right.$$
$$\left. + \frac{5}{4\pi} c^2_{21}\left(2U_{31} + U_{32} + \sqrt{\frac{5}{4\pi}}(2U_{41} + U_{42}) + \frac{5}{2\pi} U_{51} \right) \right],$$

$$C^n_{12} = -\frac{15}{4\pi}\left[c_{12} c_{11}\left(2U_{33} + U_{32} + + \sqrt{\frac{5}{4\pi}}(2U_{42} + U_{41}) + \frac{5}{2\pi} U_{51} \right) \right.$$
$$\left. + c_{21} c_{22}\left(2U_{31} + U_{32} + + \sqrt{\frac{5}{4\pi}}(2U_{41} + U_{42}) + \frac{5}{2\pi} U_{51} \right) \right],$$

$$C^n_{22} = -3\left[\sqrt{\frac{5}{4\pi}} U_{21} + \frac{5}{4\pi} c^2_{22}\left(2U_{31} + U_{32} + \sqrt{\frac{5}{4\pi}}(2U_{41} + U_{42}) + \frac{5}{2\pi} U_{51} \right) \right.$$
$$\left. + \frac{5}{4\pi} c^2_{12}\left(2U_{33} + U_{32} + \sqrt{\frac{5}{4\pi}}(2U_{42} + U_{41}) + \frac{5}{2\pi} U_{51} \right) \right].$$

1. T. Shneidman, G. Adamian, N. Antonenko, S. Ivanova, R. Jolos, and W. Scheid, Phys. Rev. C 65, 064302 (2002);

Axially symmetric compound fissile systems

$$U_N(R,\beta_i,\theta_i)$$
$$\approx U_{11}+U_{21}Y_{20}(\theta_2)+U_{22}Y_{20}(\theta_1)+U_{31}Y_{20}^2(\alpha_2)$$
$$+U_{32}Y_{20}(\alpha_1)Y_{20}(\alpha_2)+U_{33}Y_{20}^2(\alpha_1)$$
$$+U_{41}Y_{20}(\alpha_1)Y_{20}^2(\alpha_2)+U_{42}Y_{20}^2(\alpha_1)Y_{20}$$
$$\times(\alpha_2)+U_{51}Y_{20}^2(\alpha_1)Y_{20}^2(\alpha_2),$$

$$U_{11}=C_0[\Lambda_1(a_1)+\Lambda_2(a_2)+F_{ex}]I_0(a_1,a_2),$$

$$U_{21}=C_0\beta_1R_1\frac{d}{dR_1}[\xi_1\Lambda_2(a_2)I_1(b_1,a_2)$$
$$+\xi_1'\Lambda_1(b_1')I_1(b_1',a_2)+F_{ex}\xi_1I_1(b_1,a_2)],$$

$$U_{22}=C_0\beta_2R_2\frac{d}{dR_2}[\xi_2\Lambda_1(a_1)I_2(a_1,b_2)$$
$$+\xi_2'\Lambda_2(b_2')I_2(a_1,b_2')+F_{ex}\xi_2I_2(a_1,b_2)],$$

$$U_{22}=C_0\beta_2R_2\frac{d}{dR_2}[\xi_2\Lambda_1(a_1)I_2(a_1,b_2)$$
$$+\xi_2'\Lambda_2(b_2')I_2(a_1,b_2')+F_{ex}\xi_2I_2(a_1,b_2)],$$

$$U_{31}=C_0\beta_2^2\frac{R_2^2}{2}\frac{d^2}{dR_2^2}[\xi_2\Lambda_1(a_1)I_0(a_1,b_2)$$
$$+\xi_2'\Lambda_2(b_2')I_0(a_1,b_2')+F_{ex}\xi_2I_0(a_1,b_2)],$$

$$U_{33}=C_0\beta_1^2\frac{R_1^2}{2}\frac{d^2}{dR_1^2}[\xi_1'\Lambda_1(b_1')I_0(b_1',a_2)$$
$$+\xi_1\Lambda_2(a_2)I_0(b_1,a_2)+F_{ex}\xi_1I_0(b_1,a_2)],$$

$$U_{32}=C_0\beta_1\beta_2R_1R_2\frac{d^2}{dR_1dR_2}[\xi_1'\xi_2\Lambda_1(b_1')I_0(b_1',b_2)$$
$$+\xi_1\xi_2'\Lambda_2(b_2)I_0(b_1,b_2')+\xi_1\xi_2I_0(b_1,b_2)],$$

$$U_{41}=C_0\beta_1\beta_2^2\frac{R_1R_2^2}{2}\frac{d^3}{dR_1dR_2^2}[\xi_1'\xi_2\Lambda_1(b_1')I_0(b_1',b_2)$$
$$+\xi_1\xi_2'\Lambda_2(b_2)I_0(b_1,b_2')+\xi_1\xi_2I_0(b_1,b_2)],$$

1. T. Shneidman, G. Adamian, N. Antonenko, S. Ivanova, R. Jolos, and W. Scheid, Phys. Rev. C 65, 064302 (2002);

Axially symmetric compound fissile systems

$$U_{42}=C_0\beta_1^2\beta_2\frac{R_1^2R_2}{2}\frac{d^3}{dR_1^2dR_2}[\xi_1'\xi_2\Lambda_1(b_1')I_0(b_1',b_2)$$
$$+\xi_1\xi_2'\Lambda_2(b_2)I_0(b_1,b_2')+\xi_1\xi_2I_0(b_1,b_2)],$$

$$U_{51}=C_0\beta_1^2\beta_2^2\frac{R_1^2R_2^2}{4}\frac{d^4}{dR_1^2dR_2^2}[\xi_1'\xi_2\Lambda_1(b_1')I_0(b_1',b_2)$$
$$+\xi_1\xi_2'\Lambda_2(b_2)I_0(b_1,b_2')+\xi_1\xi_2I_0(b_1,b_2)]$$

$$I_0(a,b)=-4\pi\int_0^\infty \rho_1(p,a)\rho_2(p,b)j_0(pR)p^2dp,$$

$$I_1(a,b)=-(4\pi)^2\int_0^\infty dpp^2j_2(pR)\rho_2(p,b)$$
$$\times\int_0^\infty drr^2j_2(pr)\rho_1(r,a),$$

$$I_2(a,b)=-(4\pi)^2\int_0^\infty dpp^2j_2(pR)\rho_1(p,a)$$
$$\times\int_0^\infty drr^2j_2(pr)\rho_2(r,b).$$

1. T. Shneidman, G. Adamian, N. Antonenko, S. Ivanova, R. Jolos, and W. Scheid, Phys. Rev. C 65, 064302 (2002);

Axially symmetric compound fissile systems

$$U_C = \frac{Z_1 Z_2 e^2}{R} + \frac{3}{5}\frac{Z_1 Z_2 e^2}{R^3}\sum_{i=1}^{2} R_i^2 \beta_i Y_2(\theta_i)$$

$$+ \frac{12}{35}\frac{Z_1 Z_2 e^2}{R^3}\sum_{i=1}^{2} R_i^2 [\beta_i Y_2(\theta_i)]^2.$$

$$U_{C0} = \frac{Z_1 Z_2 e^2}{R}\left\{1 + \frac{1}{R^2}\sum_{i=1}^{2} R_{0i}^2\left[\left(\frac{9}{20\pi}\right)^{1/2}\beta_i + \frac{3}{7\pi}\beta_i^2\right]\right\},$$

$$C_{ii}^c = -3\frac{Z_1 Z_2 e^2}{R^3} R_{0i}^2\left[\left(\frac{9}{20\pi}\right)^{1/2}\beta_i + \frac{6}{7\pi}\beta_i^2\right] \quad (i=1,2).$$

$$U_C = U_{C0} + \frac{1}{2}C_{11}^c \tilde{\theta}_1^2 + \frac{1}{2}C_{22}^c \tilde{\theta}_2^2,$$

$$C_{ij} = C_{ij}^n + C_{ij}^c$$

1. T. Shneidman, G. Adamian, N. Antonenko, S. Ivanova, R. Jolos, and W. Scheid, Phys. Rev. C 65, 064302 (2002);

Axially symmetric compound fissile systems

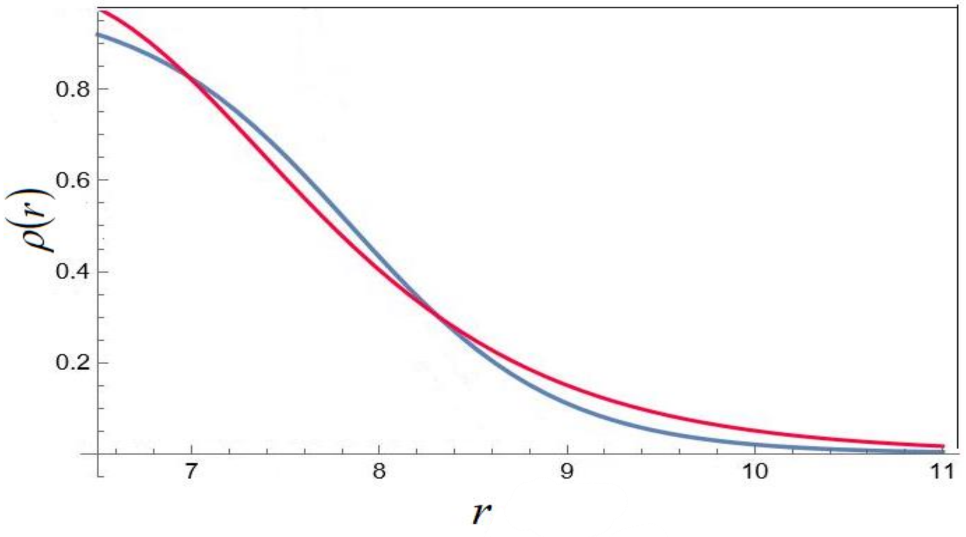
$$\rho_i(\mathbf{r},a_i)=\frac{\rho_0 \sinh[R_i(\theta',\phi')/a_i]}{\cosh[R_i(\theta',\phi')/a_i]+\cosh(r/a_i)},$$

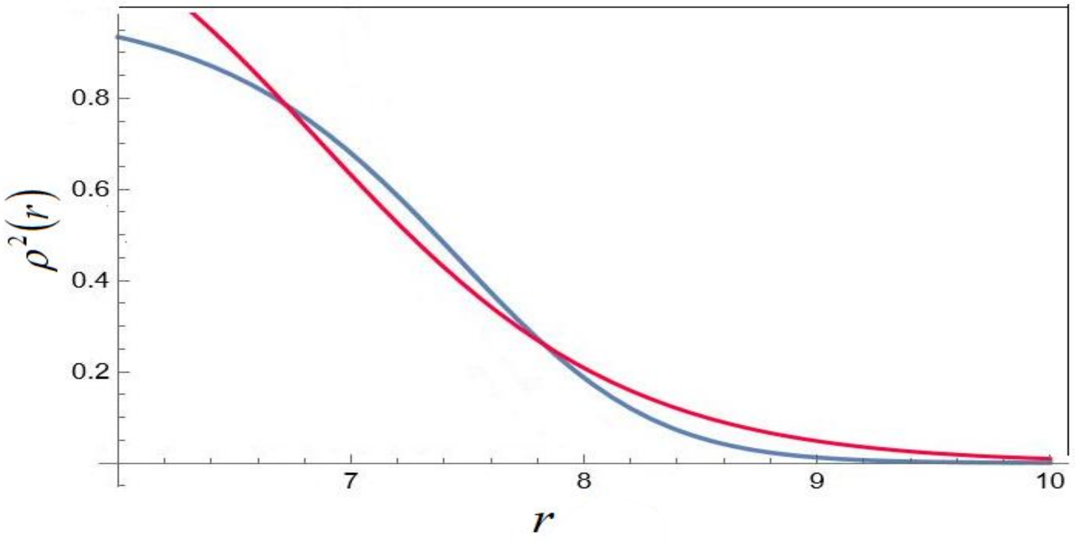
$$\rho_i(\mathbf{r},a_i)=\rho_i(r,a_i)+\xi\left[R_i\frac{d\rho_i(r,b_i)}{dR_i}\beta_i Y_{20}(\theta',\phi')\right.$$
$$\left.+\frac{R_i^2}{2}\frac{d^2\rho_i(r,b_i)}{dR_i^2}\beta_i^2 Y_{20}^2(\theta',\phi')\right],$$

$$\rho_i^2(\mathbf{r},a_i)=\rho_i^2(r,a_i)+\xi'\left[R_i\frac{d\rho_i^2(r,b_i')}{dR_i}\beta_i Y_{20}(\theta',\phi')\right.$$
$$\left.+\frac{R_i^2}{2}\frac{d^2\rho_i^2(r,b_i')}{dR_i^2}\beta_i^2 Y_{20}^2(\theta',\phi')\right].$$

1. T. Shneidman, G. Adamian, N. Antonenko, S. Ivanova, R. Jolos, and W. Scheid, Phys. Rev. C 65, 064302 (2002);

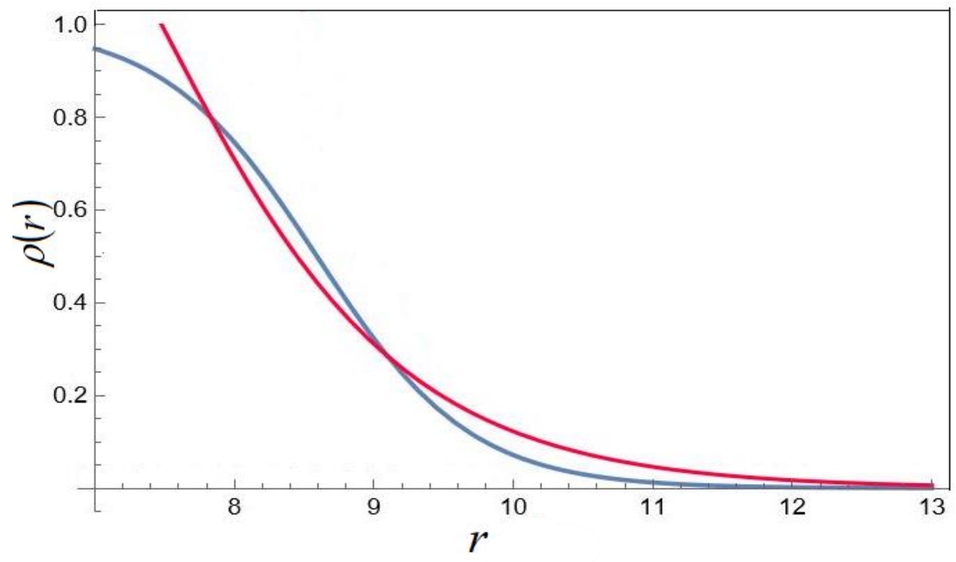
Axially symmetric compound fissile systems

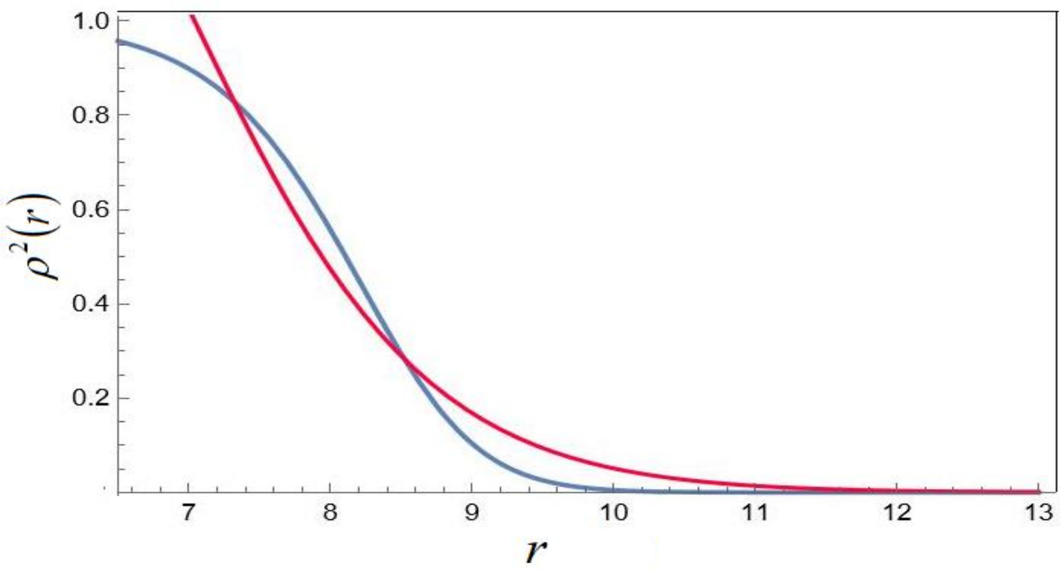




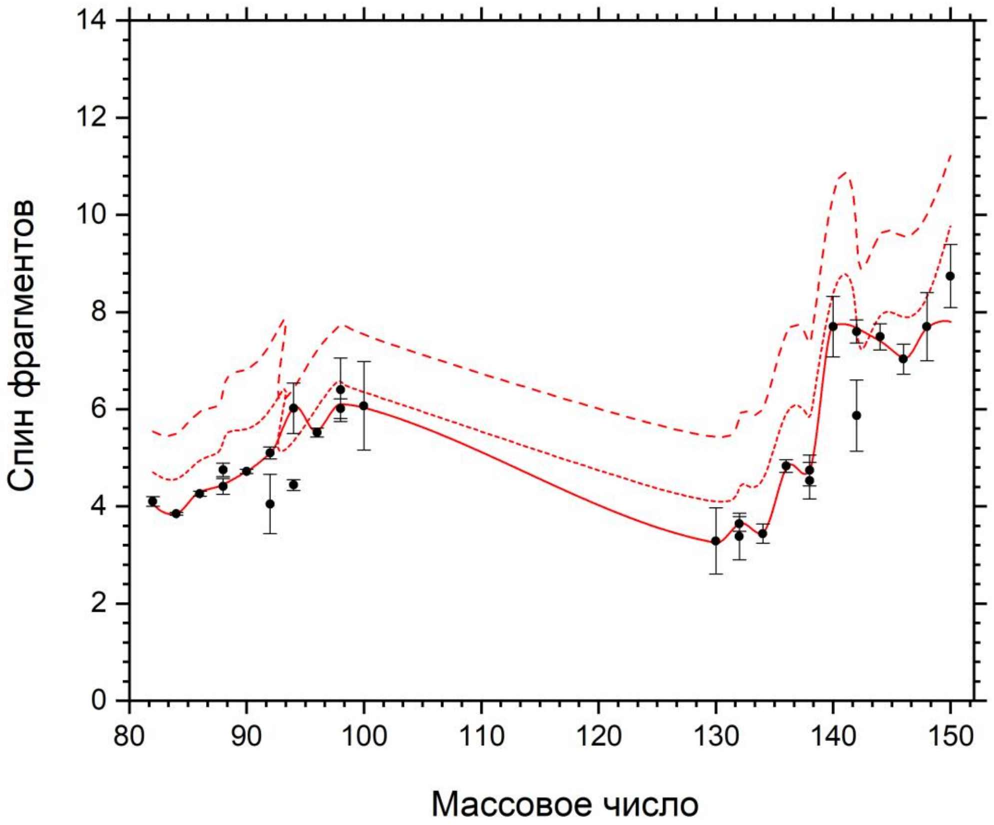
(On the left) The dependence of the nucleon density on r, calculated for Te-134 at β=0.549, $\theta' = \alpha_1$, and $\varphi' = \alpha_i$. The blue line represents the density function. Red is the result of approximating the density function at ξ= 0.87 and b = 0.87 fm. (On the right) The dependence of the square of the nucleon density on r. The blue line represents the density squared function. Red is the result of approximating the square of the density function at $\xi\,'$= 1.30 and b' = 1.12 fm.

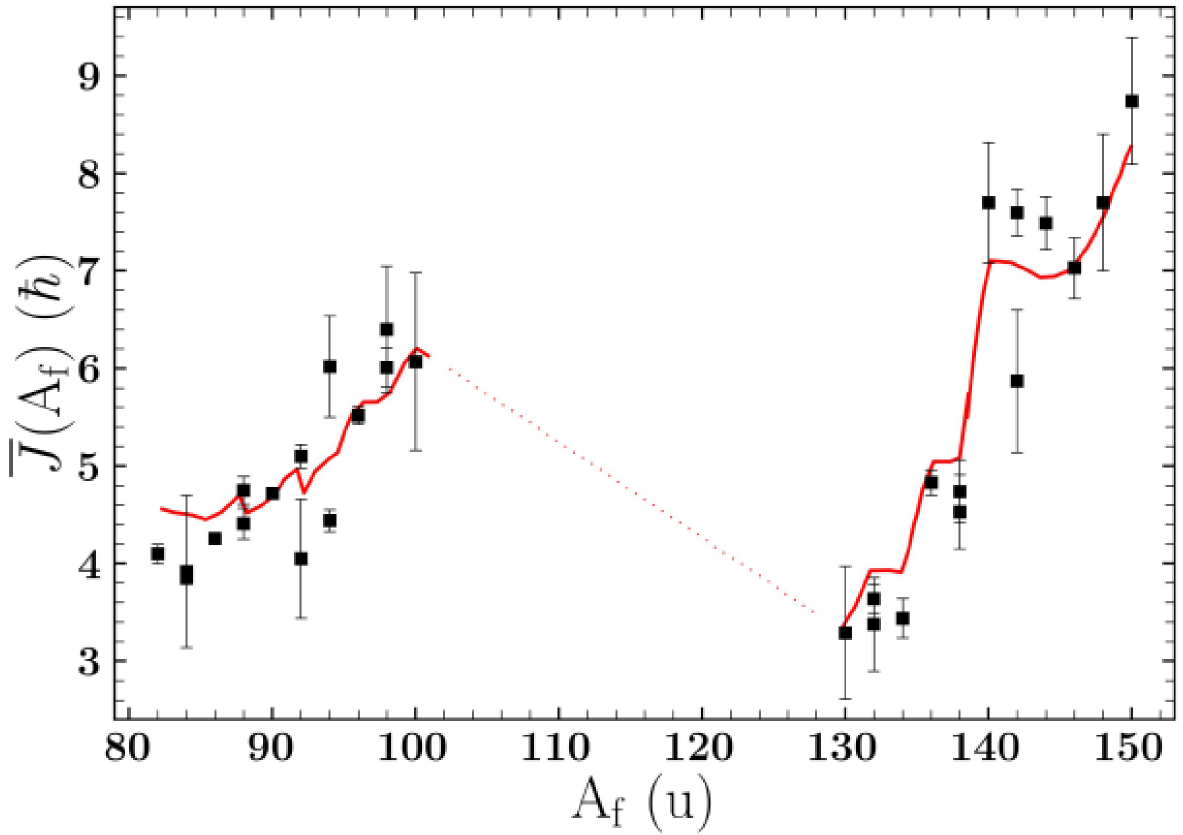
Axially symmetric compound fissile systems



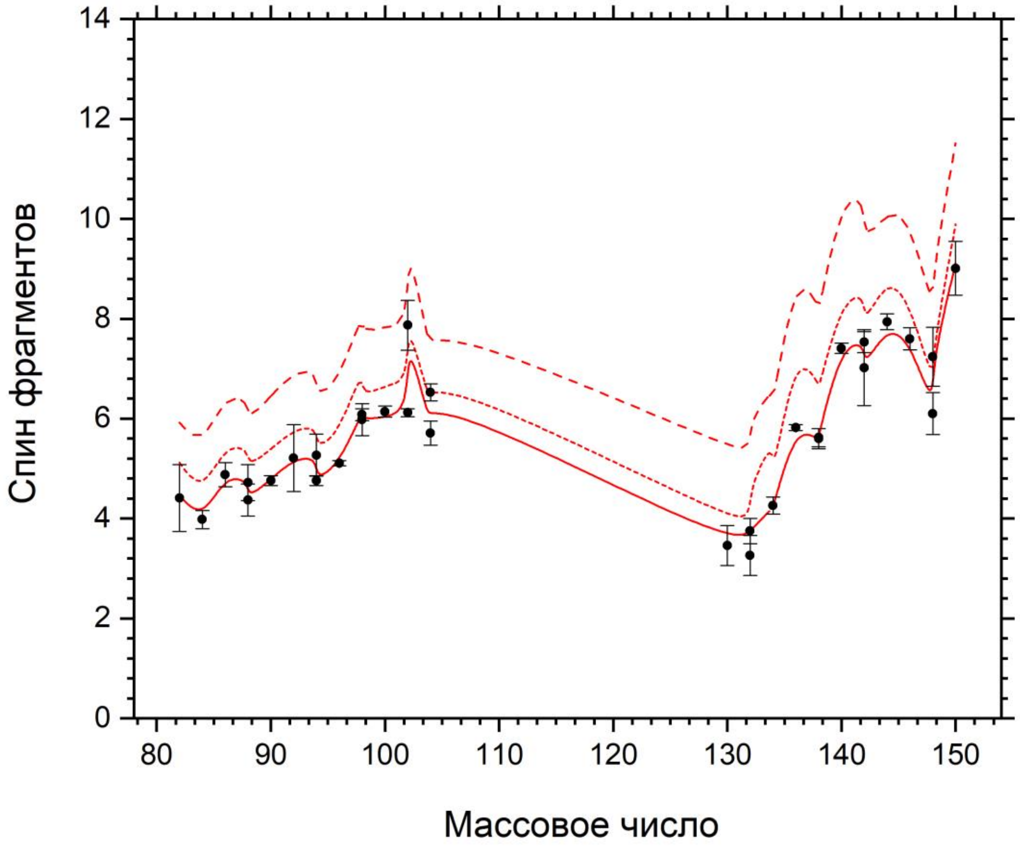


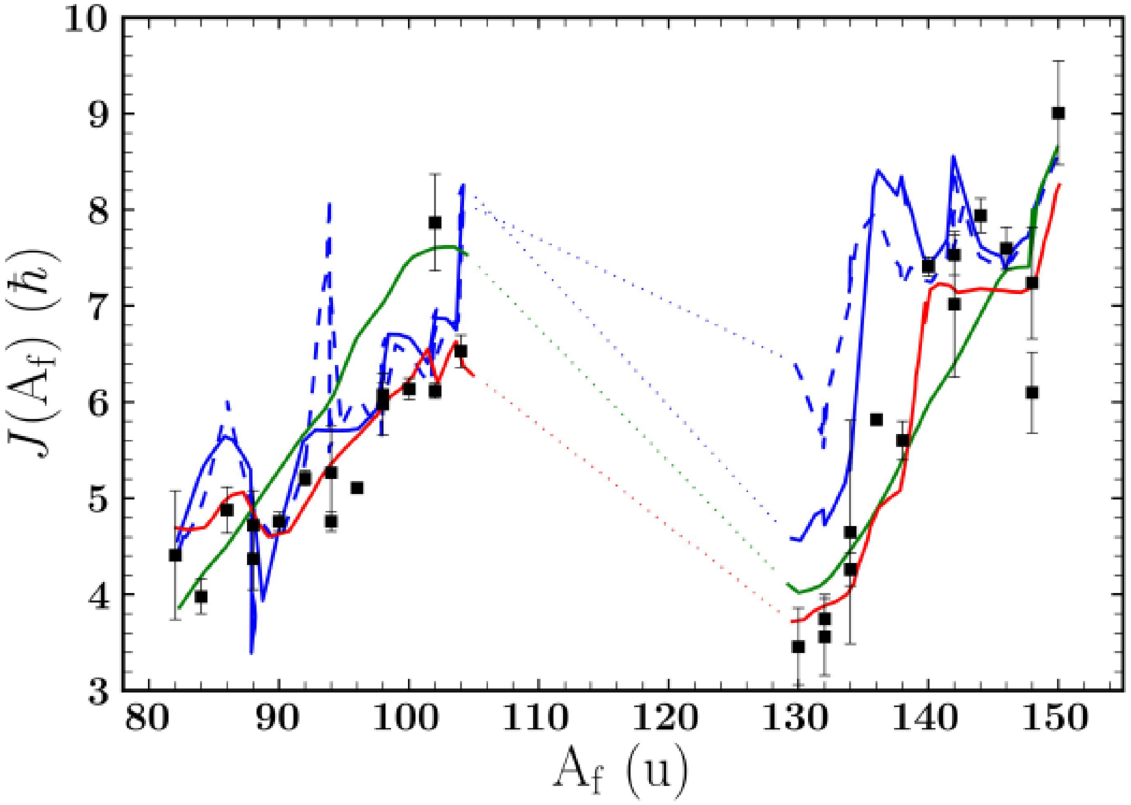
(On the left) The dependence of the nucleon density on r, calculated for Pd-118 at β=0.914, $\theta' = \alpha_1$, and $\varphi' = \alpha_i$. The blue line represents the density function. Red is the result of approximating the density function at ξ= 0.98 and b = 0.98 fm. (On the right) The dependence of the square of the nucleon density on r. The blue line represents the density squared function. Red is the result of approximating the square of the density function at $\xi\,'$= 1.54 and b' = 1.45 fm.





1. Wilson, J.N., Thisse, D., Lebois, M. *et al.* Angular momentum generation in nuclear fission. *Nature* **590**, 566–570 (2021).





1. Wilson, J.N., Thisse, D., Lebois, M. *et al.* Angular momentum generation in nuclear fission. *Nature* **590**, 566–570 (2021).
2. J. Randrup and R. Vogt PHYSICAL REVIEW LETTERS 127, 062502 (2021).
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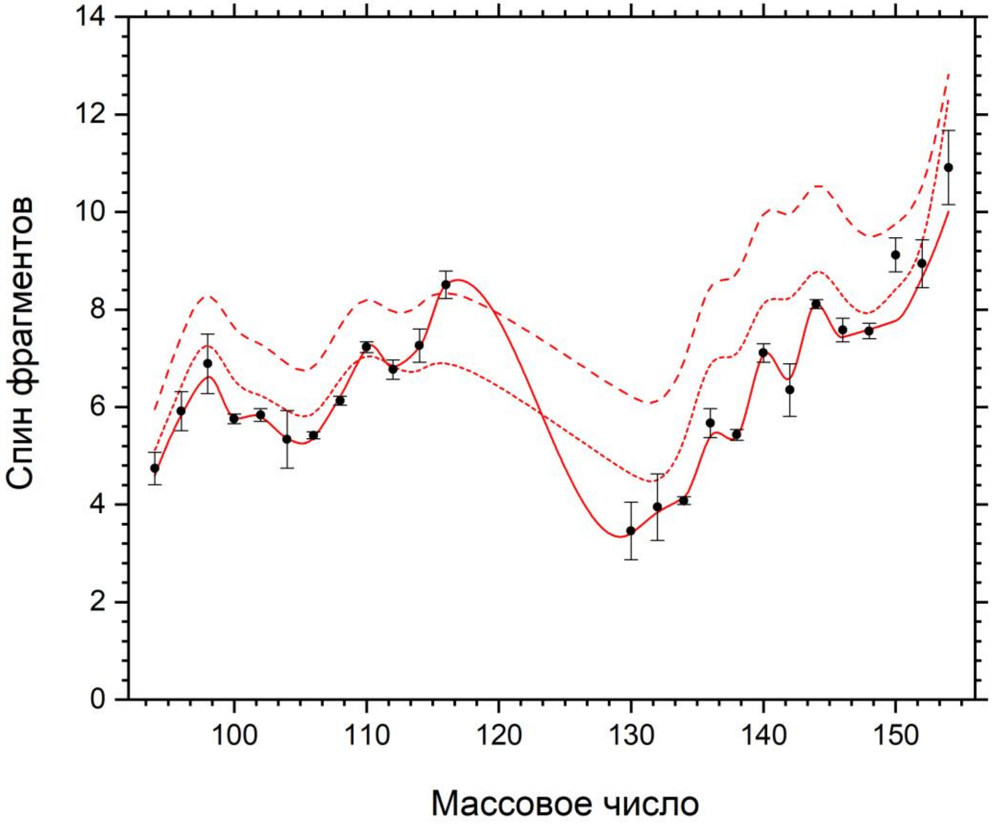

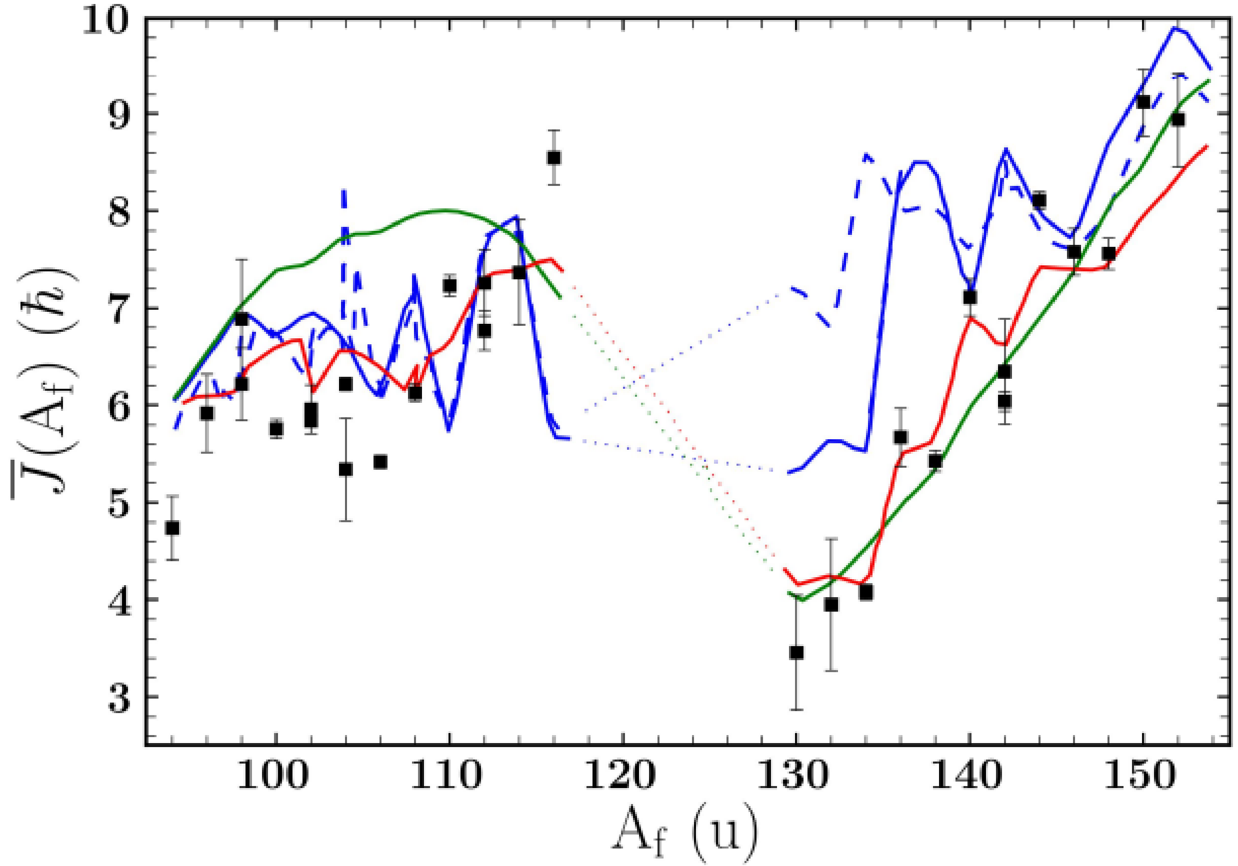

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Correlation moment and correlation coefficient

$$\mu_{J_1 J_2} = \int_0^{\infty}\int_0^{\infty} \left(J_1 - \langle J_1 \rangle\right)\left(J_2 - \langle J_2 \rangle\right) \times P\left(J_1, J_2\right) dJ_1 dJ_2,$$

$$\int_0^{\infty} e^{x\cos\varphi} d\varphi = \pi \tilde{j}_0\left(t\right) = \pi j\left(it\right) = \pi \sum_{k=0}^{\infty} \frac{x^{2k}}{\left(2^k k!\right)^2},$$

$$P\left(J_1, J_2\right) = \frac{2J_1 J_2}{C_w C_b} \sum_{n=0}^{\infty} \left[\frac{J_1 J_2}{n!}\left(\alpha I_1 I_2 - \beta\right)^n\right]^2 \times \left[-J_1^2\left(\alpha I_2^2 + \beta\right) - J_2^2\left(\alpha I_1^2 + \beta\right)\right].$$

$$c_{J_1 J_2}\left(A_1, A_2\right) = \frac{\mu J_1 J_2}{\sigma_{J_1} \sigma_{J_2}},$$

$$\sigma_{J_i} = \sqrt{\left\langle J_i^2 \right\rangle - \left\langle J_i \right\rangle^2}$$

$$\tilde{c}_{J_1 J_2} = \frac{\sum c_{J_1 J_2}\left(A_1, A_2\right) Y\left(A_1, A_2\right)}{\sum Y\left(A_1, A_2\right)},$$

1. D. E. Lyubashevsky, A. A. Pisklyukov, S. V. Klyuchnikov, and P. V. Kostryukov, Phys. Rev. C 111. 054601. (2025).

Correlation moment and correlation coefficient

$$c(S_L, S_H) = \frac{\sigma(S_L, S_H)}{\sigma(S_L)\sigma(S_H)},$$

$$c(S_L, S_H) = \frac{\langle S_L \cdot S_H \rangle - \langle S_L \rangle \langle S_H \rangle}{\sigma_L \sigma_H} = -\sqrt{\frac{I_L I_H}{(I_R + I_L)(I_R + I_H)}},$$

	^{235}U (n, f)	^{238}U (n, f)	^{239}Pu (n, f)	^{252}Cf (sf)
$\langle S_L \rangle$	4.27	4.43	4.58	5.08
$\langle S_H \rangle$	5.66	5.80	5.93	6.33
$c(S_L, S_H)$	0.002	0.002	0.001	0.001

Mean values of the primary spins of the fission fragments $\langle S_L \rangle$ and $\langle S_H \rangle$ and the corresponding correlation coefficients $c(s_L, s_H)$

Coefficients $C_{J_1 J_2}$ and $\tilde{c}_{J_1 J_2}$, and yields $Y(A_f)$ for the studied reactions

Fragment	^{232}Th (n, f)		^{238}U (n, f)		^{252}Cf (sf)	
	$c_{J_1 J_2}$	$Y(A_f)$	$c_{J_1 J_2}$	$Y(A_f)$	$c_{J_1 J_2}$	$Y(A_f)$
^{82}Ge	0.203	0.64	0.207	0.12		
^{84}Ge	0.196	0.32				
^{84}Se	0.198	1.09	0.202	0.17		
^{86}Se	0.085	4.68	0.121	0.84		
^{88}Se	0.042	2.21	0.064	0.54		
^{88}Kr	0.114	0.85	0.133	0.37		
^{90}Kr	0.027	5.34	0.017	1.85		
^{92}Kr	0.012	3.92	0.005	2.50		
^{94}Kr	0.001	0.60				
^{92}Sr	0.062	0.20	0.006	0.73		
^{94}Sr	0.007	2.04	0.006	1.51	0.055	0.55
^{96}Sr	0.020	3.54	0.002	4.13	0.018	0.89
^{98}Sr	0.019	1.32	0	2.27	0.015	0.37
^{98}Zr	0.020	0.37	0	0.49	0.014	0.59
^{100}Zr	0.047	0.88	0.016	3.30	0.001	2.06
^{102}Zr			0.033	4.09	0	1.45
^{104}Zr			0.037	1.01	0.002	0.22
^{102}Mo			0.025	0.08	0.002	0.46
^{104}Mo			0.031	1.08	0.001	2.83
^{106}Mo					0.001	3.47
^{108}Mo					0	0.67
^{108}Ru					0	1.98
^{110}Ru					0.011	3.62
^{112}Ru					0.014	0.94
^{112}Pd					0.014	0.75
^{114}Pd					0.035	1.82
^{116}Pd					0.038	0.82
^{130}Sn	0.031	0.84	0.028	1.65	0.050	0.36
^{132}Sn	0.023	1.54	0.027	1.88	0.037	0.14
^{134}Sn			0.026	0.18		
^{132}Te	0.008	0.35	0.029	0.47		
^{134}Te	0.023	3.11	0.024	3.95	0.059	2.35
^{136}Te	0.006	3.44	0.013	3.53	0.050	0.91
^{138}Te	0.005	0.76	0.007	0.55	0.047	3.63
^{138}Xe	0.006	2.08	0.007	2.04	0.026	2.55
^{140}Xe	0.011	5.73	0.004	4.04	0.023	0.37
^{142}Xe	0.014	2.25	0.009	1.53	0.019	2.70
^{142}Ba	0.014	0.64	0.011	0.69	0.001	3.37
^{144}Ba	0.035	4.49	0.012	2.46	0	0.98
^{146}Ba	0.040	2.76	0.014	1.98		
^{148}Ba			0.035	0.25		
^{148}Ce	0.056	0.56	0.036	0.75	0	2.35
^{150}Ce	0.079	0.41	0.067	0.86	0.004	0.94
^{152}Nd					0.009	0.83
^{154}Nd					0.020	0.42
$\tilde{c}_{J_1 J_2}$	0.034		0.020		0.017	

1. J. Randrup and R. Vogt, Phys. Rev. Lett. 127, 062502 (2021).
2. D. E. Lyubashevsky, A. A. Pisklyukov, S. V. Klyuchnikov, and P. V. Kostryukov, Phys. Rev. C 111. 054601. (2025).

Correlation moment and correlation coefficient

$$\bar{P}_{12}(\varphi) \approx 1 + \bar{f}_1 \cos\varphi$$

$$\widetilde{f}_1 = -0{,}264$$

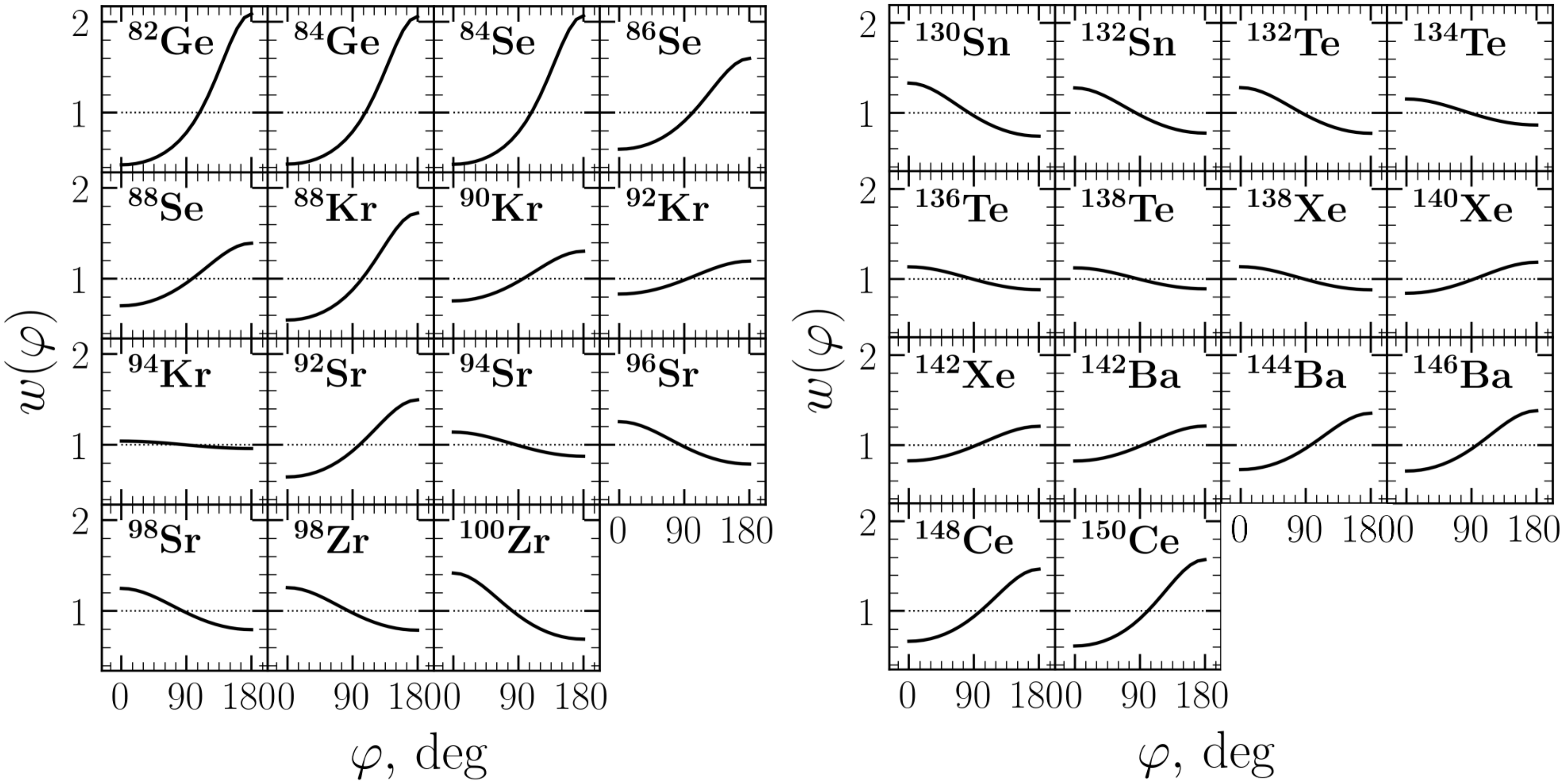
$$\tilde{P}_{12}(\varphi) \approx 1 + \tilde{f}_1 \cos\varphi + \widetilde{f}_2 \cos 2\varphi$$

$$\widetilde{f}_1 = -0{,}086 \qquad \widetilde{f}_2 = 0{,}028$$

1. J. Randrup and R. Vogt, Phys. Rev. Lett. 127, 062502 (2021).

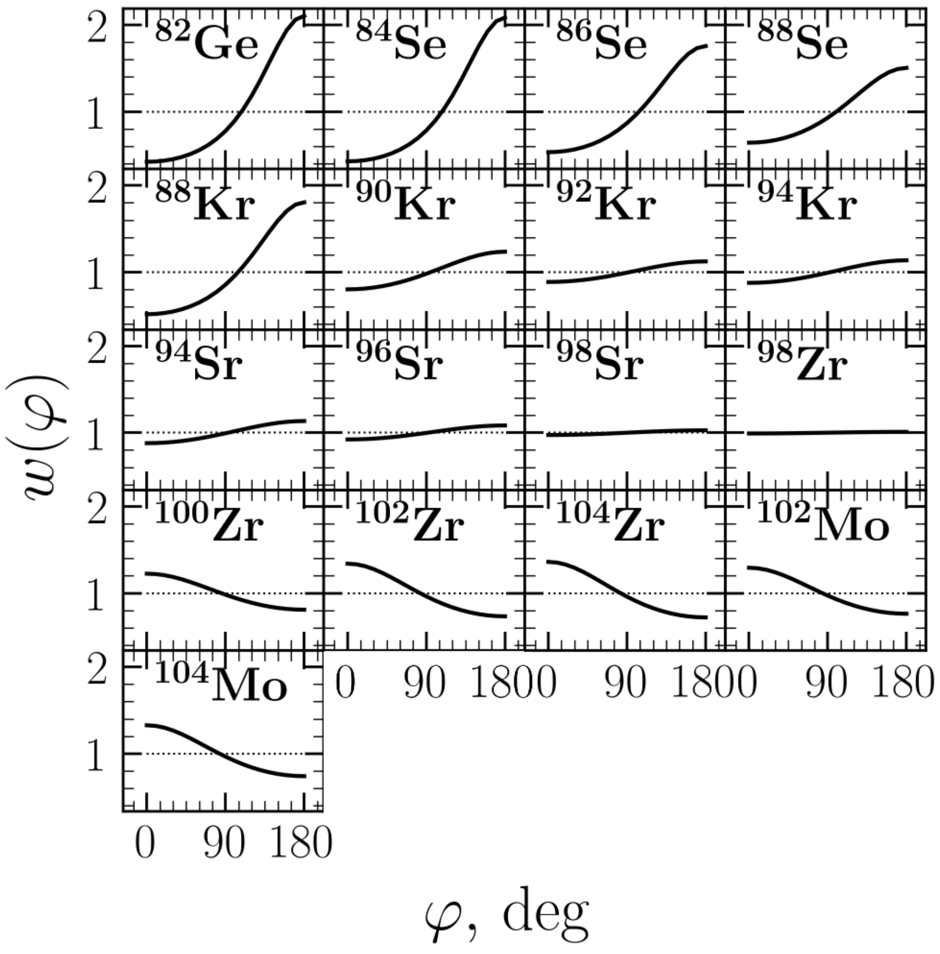
Correlation moment and correlation coefficient

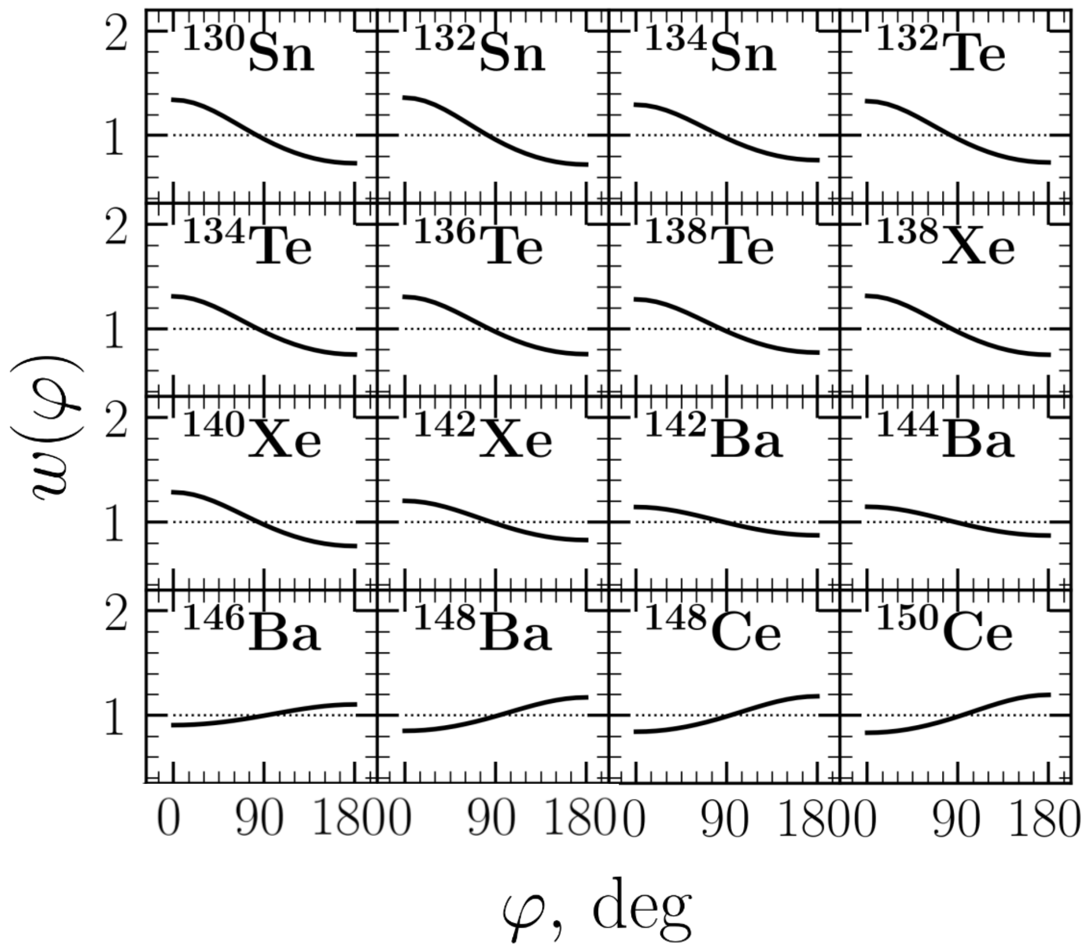
Угловые распределения фрагментов деления для $^{232}\mathrm{Th}(n,f)$



Correlation moment and correlation coefficient

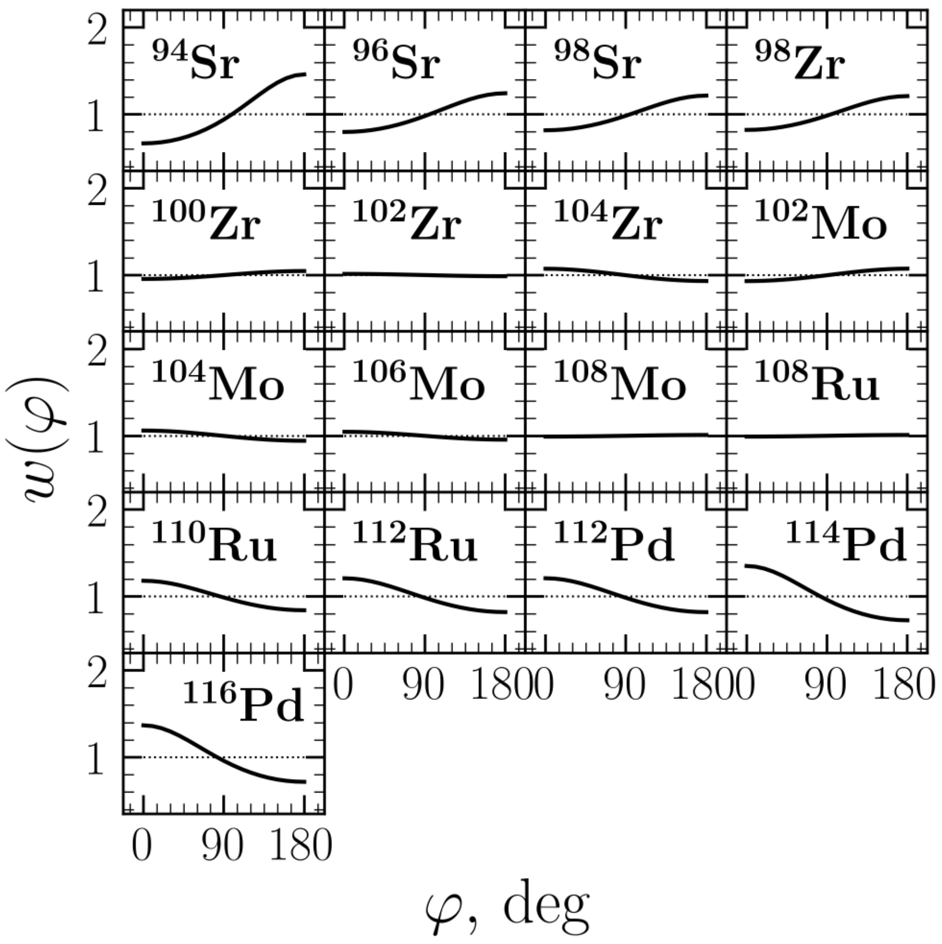
Угловые распределения фрагментов деления для $^{238}U(n,f)$

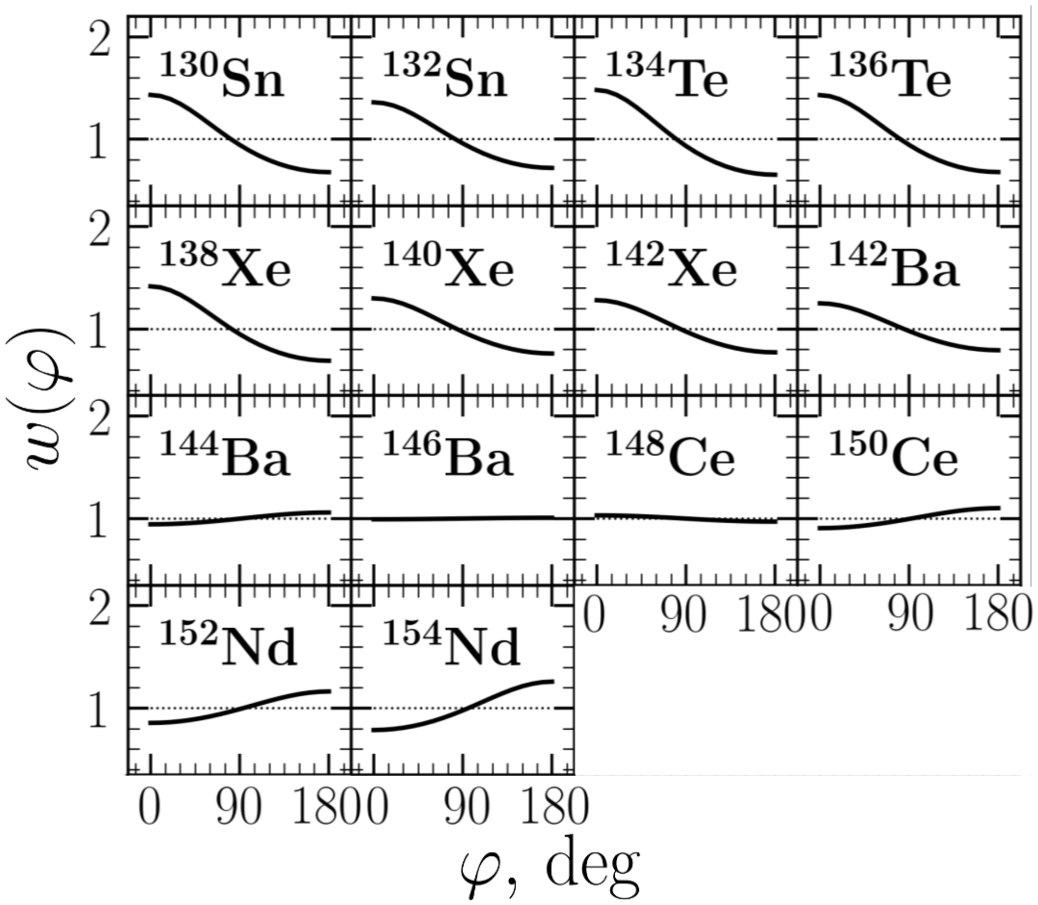




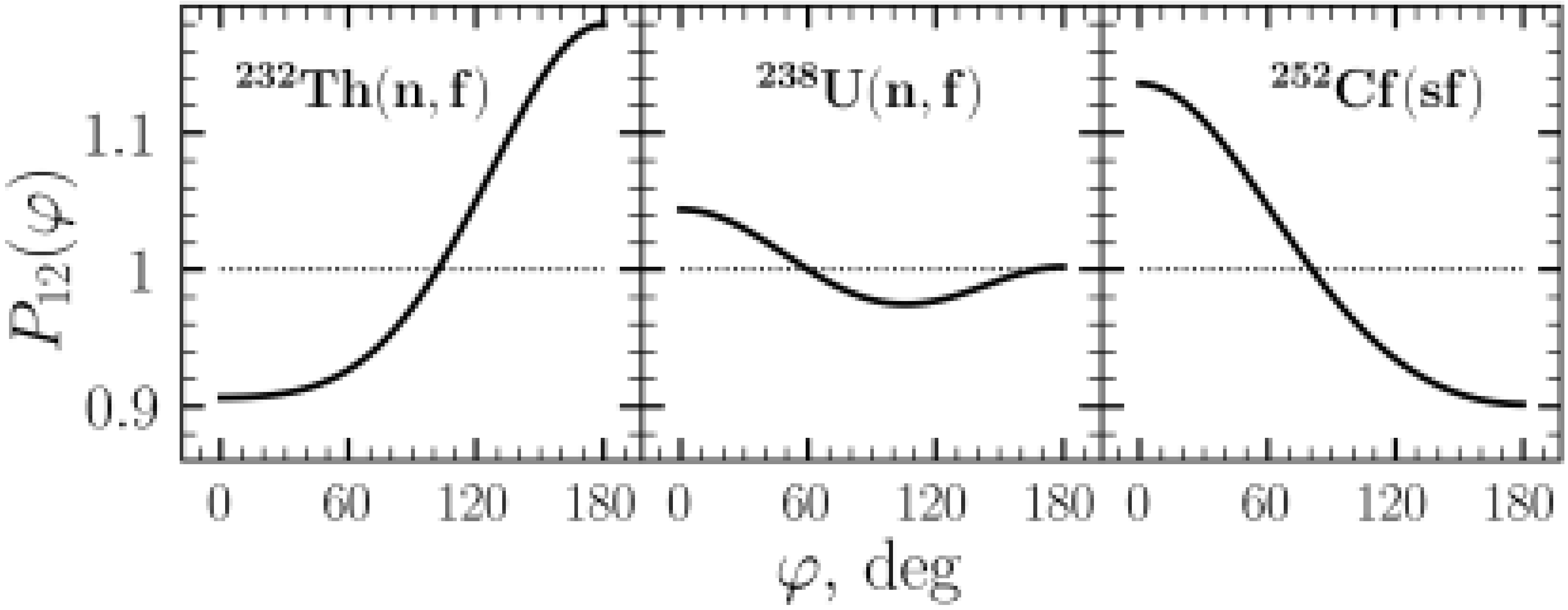
Correlation moment and correlation coefficient

Угловые распределения фрагментов деления для $^{252}\mathrm{Cf}(\mathrm{sf})$



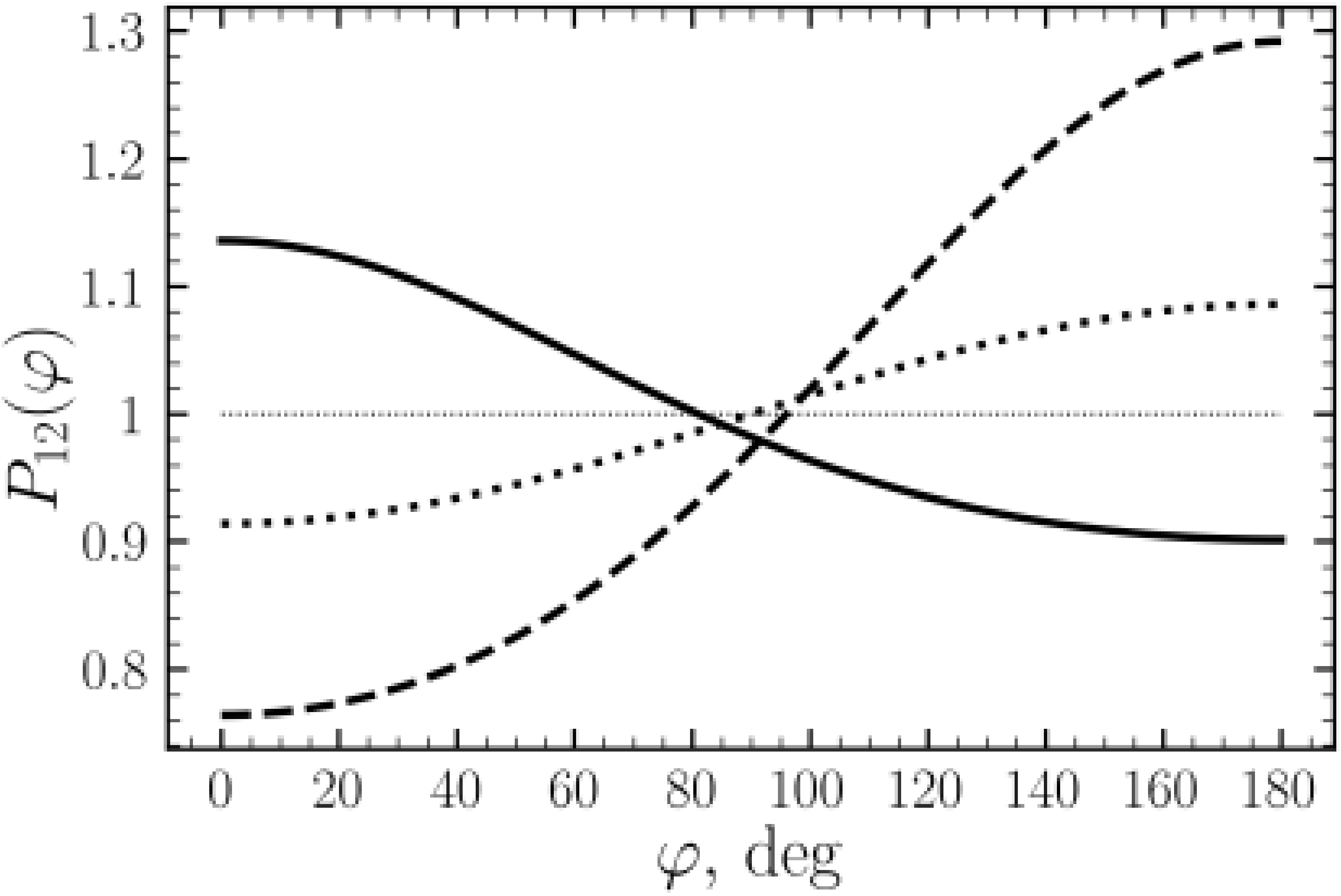


Correlation moment and correlation coefficient



Full angular distributions for the three reactions investigated

Correlation moment and correlation coefficient



Comparison of the angle dependence of the spin distribution for the 252Cf (sf) response. The solid line is the result of the present study, the short and long dashed lines are the first and second limiting cases of the Randrup [2] approach

1. G. Scamps; I. Abdurrahman; M. Kafker; A. Bulgac; I. Stetcu PhysRev C. 108L061602(2023).
2. J. Randrup, T. Døssing, and R. Vogt, Phys. Rev. C 106, 014609 (2022).

Orbital momentum of double fission fragments

$$T^J_{MK}(\Omega) = \frac{2J+1}{16\pi^2}\int d\omega\left[\left|D^J_{MK}(\omega)\right|^2 + \left|D^J_{M-K}(\omega)\right|^2\right]T(\Omega'),$$

$D^J_{MK}(\omega)$ is the Wigner function $\omega = (\alpha, \beta, \gamma)$ are Euler angles

Ω is solid angle

$$T(\Omega') = (1/4)\left|\Psi_L Y_{L0}(\Omega')\left(1 + \pi\pi_1\pi_2(-1)^L\right)\right|^2,$$

$\pi,\ \pi_1,\ \pi_2$ - parity of the parent nucleus, light and heavy fission fragment

Ψ_L - unit-normalized wave function

Orbital momentum of double fission fragments

$$\vec{L} = -\left(\vec{J}_1 + \vec{J}_2\right); \qquad \vec{G} = \left(\vec{J}_1 - \vec{J}_2\right);$$

$$\vec{J}_1 = -\vec{L}/2 + \vec{G}; \qquad \vec{J}_1 = -\vec{L}/2 - \vec{G},$$

$\vec{L}$ is relative orbital moment

$\vec{G}$ is relative spin

$$\vec{L}^2 = \left(\vec{J}_1 + \vec{J}_2\right)^2 = \left(J_{1x} + J_{2x}\right)^2 + \left(J_{1y} + J_{2y}\right)^2;$$

$$\vec{G}^2 = \left(\vec{J}_1 - \vec{J}_2\right)^2 = \left(J_{1x} - J_{2x}\right)^2 + \left(J_{1y} - J_{2y}\right)^2.$$

Orbital momentum of double fission fragments

$$P\left(J_{1x},J_{2x},J_{1y},J_{2y}\right)=\frac{1}{\pi I_w\hbar\omega_w}\exp\left[-\frac{1}{\pi I_w\hbar\omega_w}\left\{\left(J_{1x}+J_{2x}\right)^2+\left(J_{1y}+J_{2y}\right)^2\right\}\right]$$

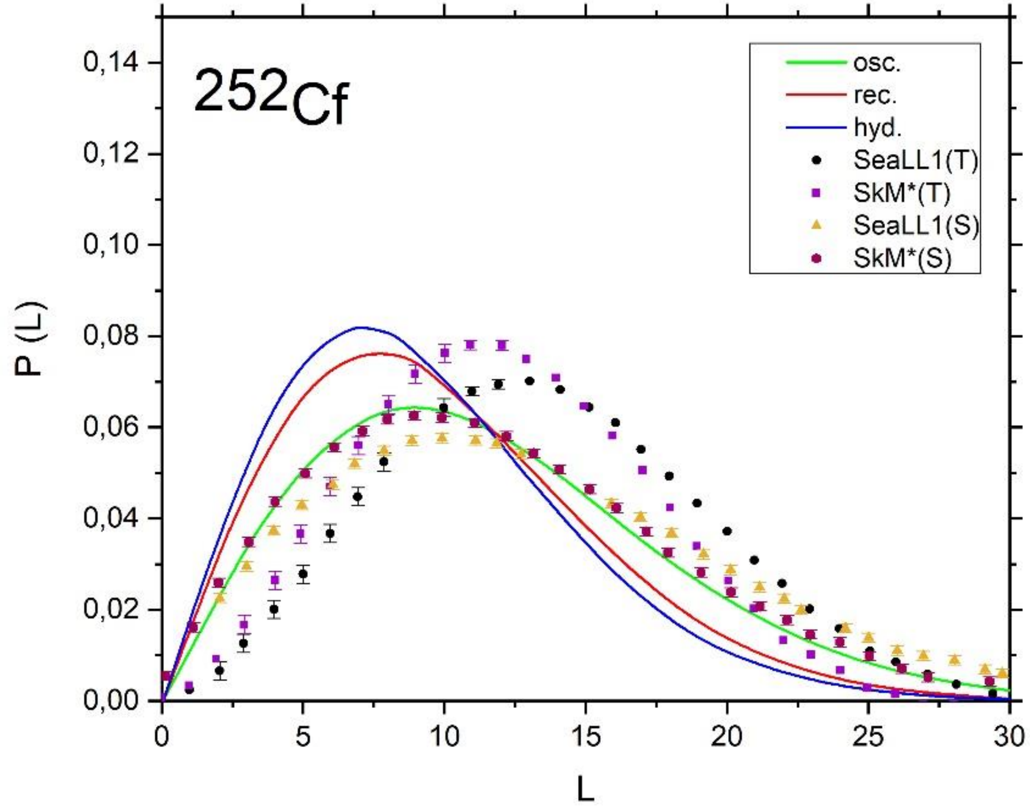
Substituting $\vec{L}^2$ and multiply by the Jacobian of the transition, we get:

$$P\left(\vec{L}\right)=\frac{1}{\pi I_w\hbar\omega_w}\exp\left[-\frac{L^2}{\pi I_w\hbar\omega_w}\right]$$

Orbital momentum of double fission fragments

$$P(L)=\int_0^{\pi}\frac{L}{\pi I_w\hbar\omega_w}\exp\left[-\frac{L^2}{I_w\hbar\omega_w}\right]d\varphi_L=\frac{2L}{I_w\hbar\omega_w}\exp\left[-\frac{L^2}{I_w\hbar\omega_w}\right].$$

$$\bar{L}=\int_0^{\infty}\frac{2L^2}{I_w\hbar\omega_w}\exp\left[-\frac{L^2}{I_w\hbar\omega_w}\right]dL=\frac{\sqrt{I_w\hbar\omega_w\pi}}{2}.$$



Comparison of the distribution of orbital momentum of nuclear double fission fragments calculated by formula (5) (solid line) with similar values obtained with the nuclear matter density functional (NEDF), SkM in the case of the 252Cf nucleus

1. Bulgac A., SnAbdurrahman SnI., Godbey K., and SnplaceStetcu SnI. // Phys. Rev. Lett. 2022. **128**, 022501

Conclusion

1. Within the framework of the «cold» nucleus model, the projection on the Z axis vanishes and a two-dimensional spin model is implemented.
2. Using the approach developed by our group, taking into account the moments of inertia of the fission prefragments obtained in the work of D. E. Lyubashevsky et al 2025 Chinese Phys. C49 034104 the stiffness coefficients of bending and wriggling vibrations, as well as the frequencies of these vibrations, were evaluated. There is a good agreement with the experimental data of Wilson, J.N., Thisse, D., et al. Nature 590, 566-570 (2021), surpassing the accuracy of descriptions of other theoretical groups.
3. The correlation coefficients of the spins of the fission fragments were calculated, and the estimates found reasonable agreement with experimental data from Wilson, J.N., and Thisse, D., et al. Nature 590, 566-570 (2021), as well as with the theoretical predictions of the theoretical group J. Randrup and R. Vogt, Phys. Rev. Lett. 127, 062502 (2021).

Thank you for your attention!