



National Research Center  
“Kurchatov Institute”, Moscow, Russia



# Study of the $^{11}\text{B}$ nucleus states in the transfer reaction $^{10}\text{B}(^7\text{Li}, ^6\text{Li})^{11}\text{B}$

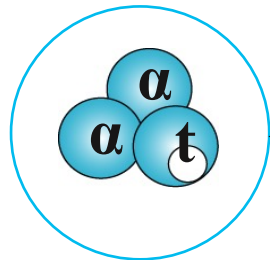
**S.K. Raidun**, S.S. Stukalov, Yu.G. Sobolev, Yu.E. Penionzhkevich, N. Burtebayev, S.A. Goncharov,  
Yu.B. Gurov, A.N. Danilov, A.S. Demyanova, S.V. Dmitriev, M.Nassurlla, V I. Starastsin, A.V. Shakhov

LXXV International Conference «NUCLEUS – 2025. Nuclear physics, elementary particle physics  
and nuclear technologies»



# Motivation

An analog of the Hoyle state:  
8.56 MeV  $3/2^-$  of  $^{11}\text{B}$  nucleus



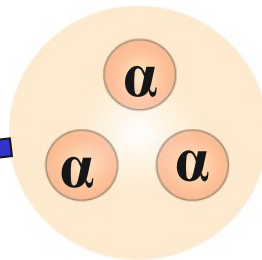
$^{11}\text{B}$   
 $\alpha + \alpha + t$

$E^* = 8.56 \text{ MeV}, 3/2^-$

$R_{\text{RMS}}^{8.56} = 2.87 \pm 0.12 \text{ fm [1]}$

$R_{\text{RMS}}^{\text{g.s.}} = 2.29 \pm 0.02 \text{ fm [3]}$

The Hoyle state 7.65 MeV  $0^+$   
of  $^{12}\text{C}$  nucleus



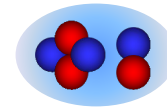
$^{12}\text{C}$   
 $\alpha + \alpha + \alpha$

$E^* = 7.65 \text{ MeV}, 0^+$

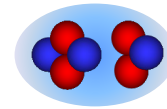
$R_{\text{RMS}}^{8.56} = 2.89 \pm 0.04 \text{ fm [2]}$

$R_{\text{RMS}}^{\text{g.s.}} = 2.31 \pm 0.02 \text{ fm [3]}$

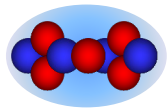
Examples of cluster structures:



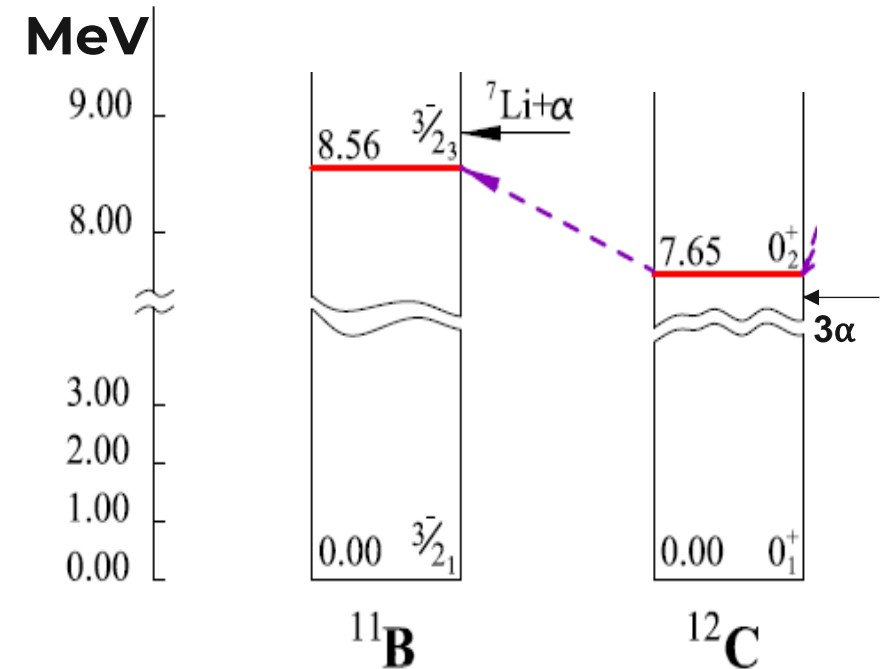
$^6\text{Li} (\alpha + d)$   
 $S_\alpha = 1.47 \text{ MeV}$



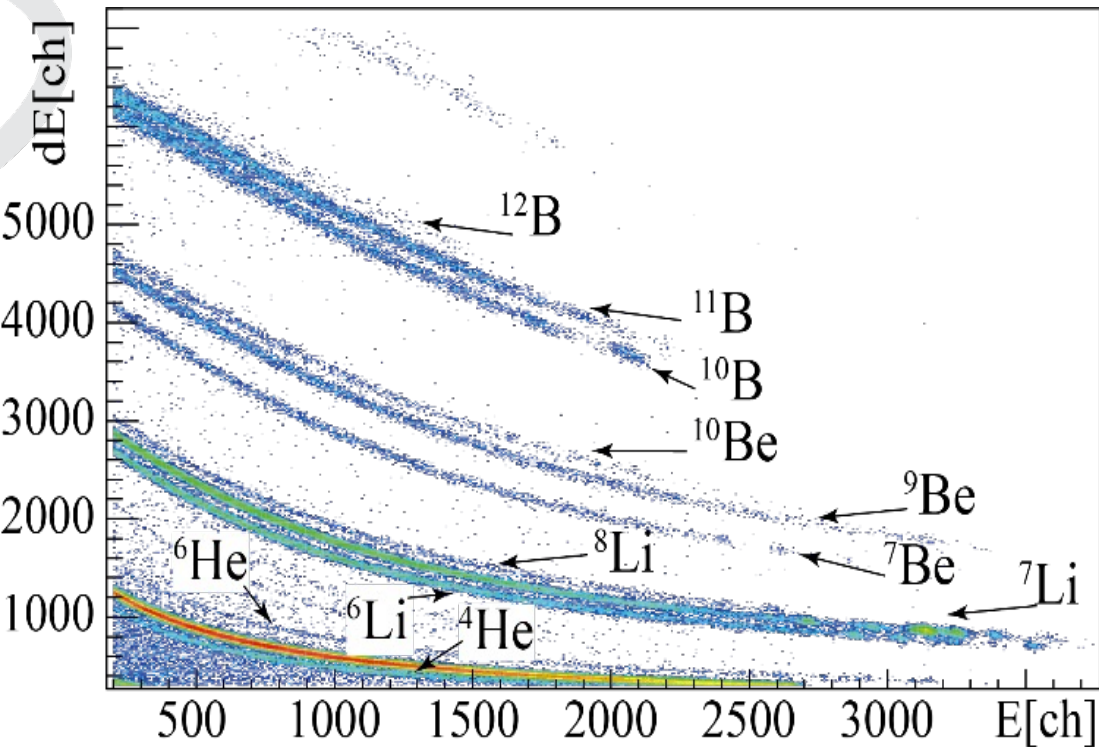
$^7\text{Li} (\alpha + t)$   
 $S_\alpha = 2.47 \text{ MeV}$



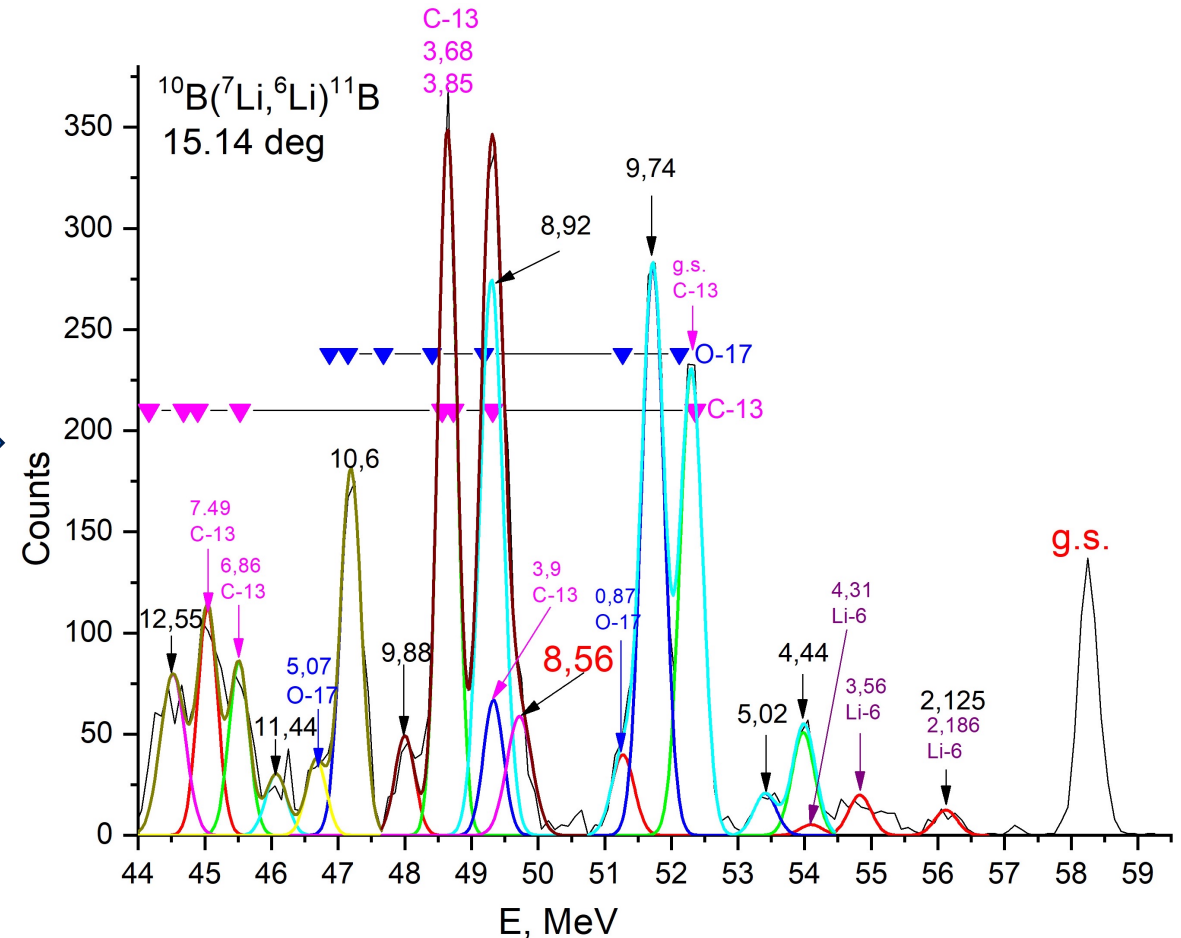
$^9\text{Be} (\alpha + n + \alpha)$   
 $S_n = 1.66 \text{ MeV}$



# Experimental data



An example of a two-dimensional spectrum obtained using the  $\Delta E$ - $E$  method from the reaction  $^7\text{Li} + ^{10}\text{B}$  at  $\Theta_{\text{LAB}} = 10, 20$ . The X-axis is the energy loss of the particles in the  $E$  detector (in channels), the Y-axis is the energy loss of the particles in the  $\Delta E$  detector (in channels). Each hyperbole is responsible for a specific reaction channel.



An example of the obtained one-dimensional energy spectrum when processing hyperbole  $^6\text{Li}$ . X-axis – energy loss of the particles ( $\Delta E + E$ ) in MeV, the Y-axis is the number of events.

# A program for processing energy spectra

**The objective is to reduce the time needed to analyze experimental data. The possibility of rapid data analysis on an experiment.**

One of the methods for analyzing energy spectra is "manual" processing using the Origin Pro program.

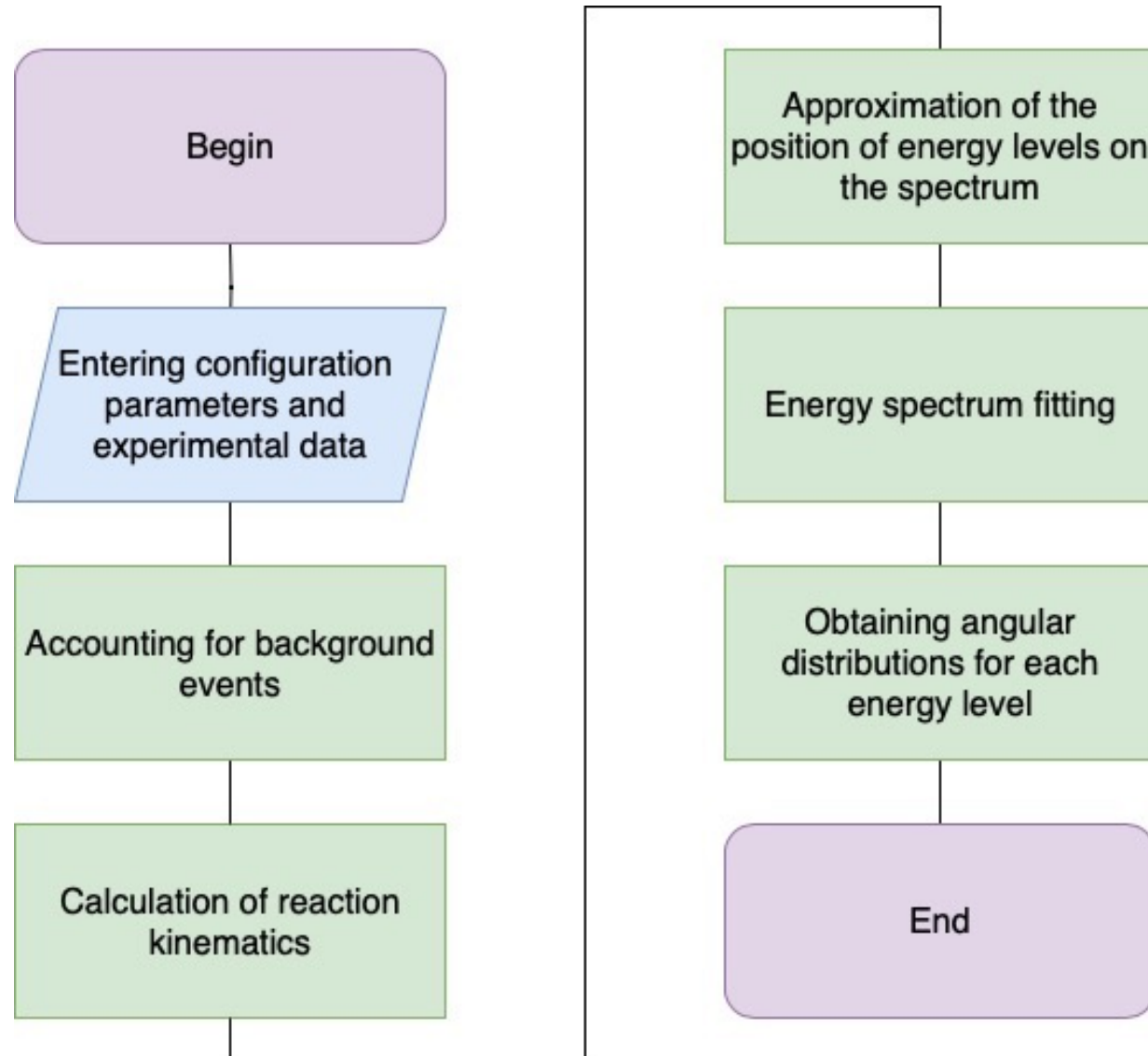
The solution is automation of energy spectrum analysis.

# The main functionality of the program

- Automatic calculation of the positions of the studied energy states on the spectrum.
- Automatic fitting of energy states.
- Automatic calculation of the differential cross section.

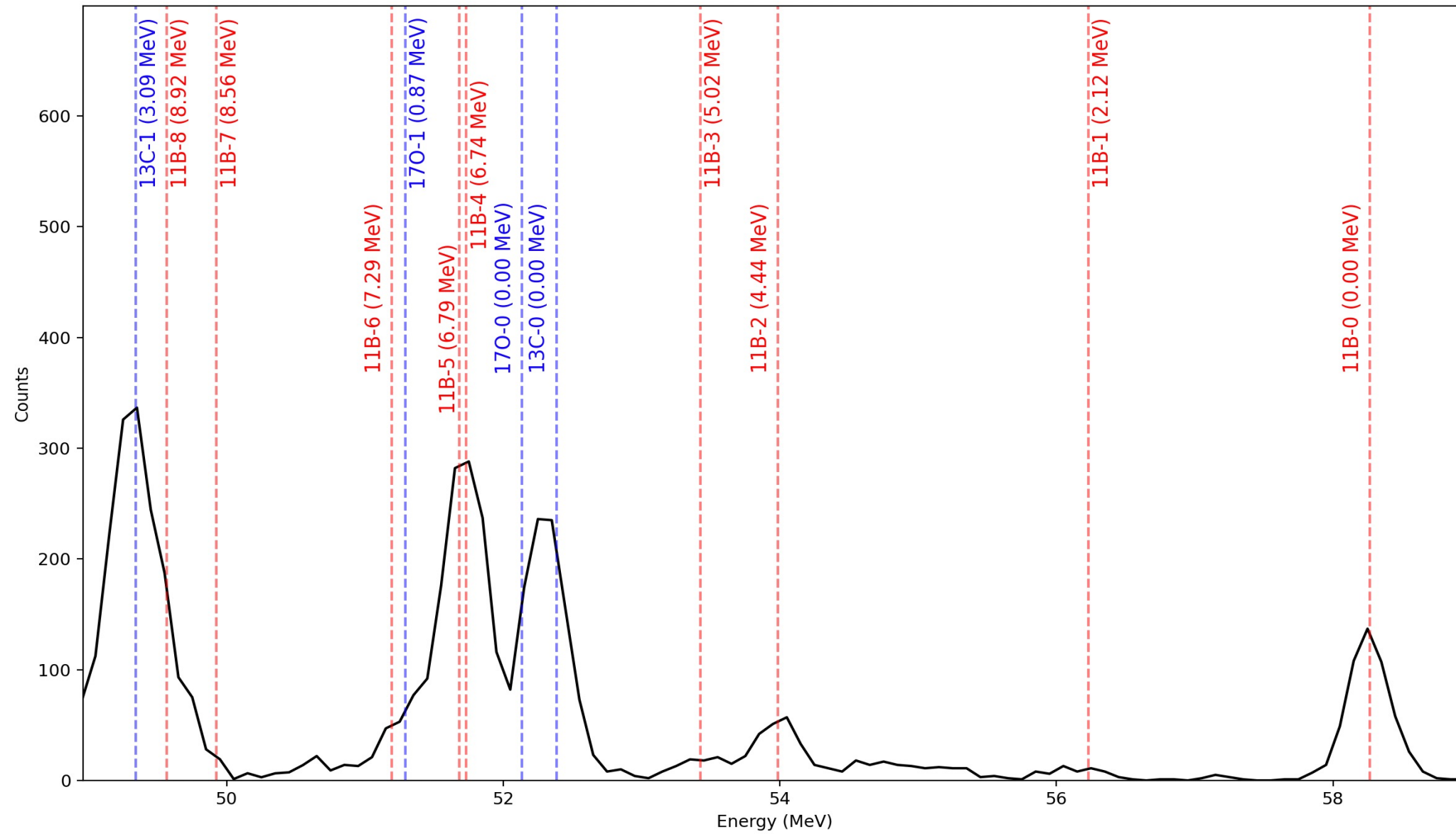


# The algorithm of the program



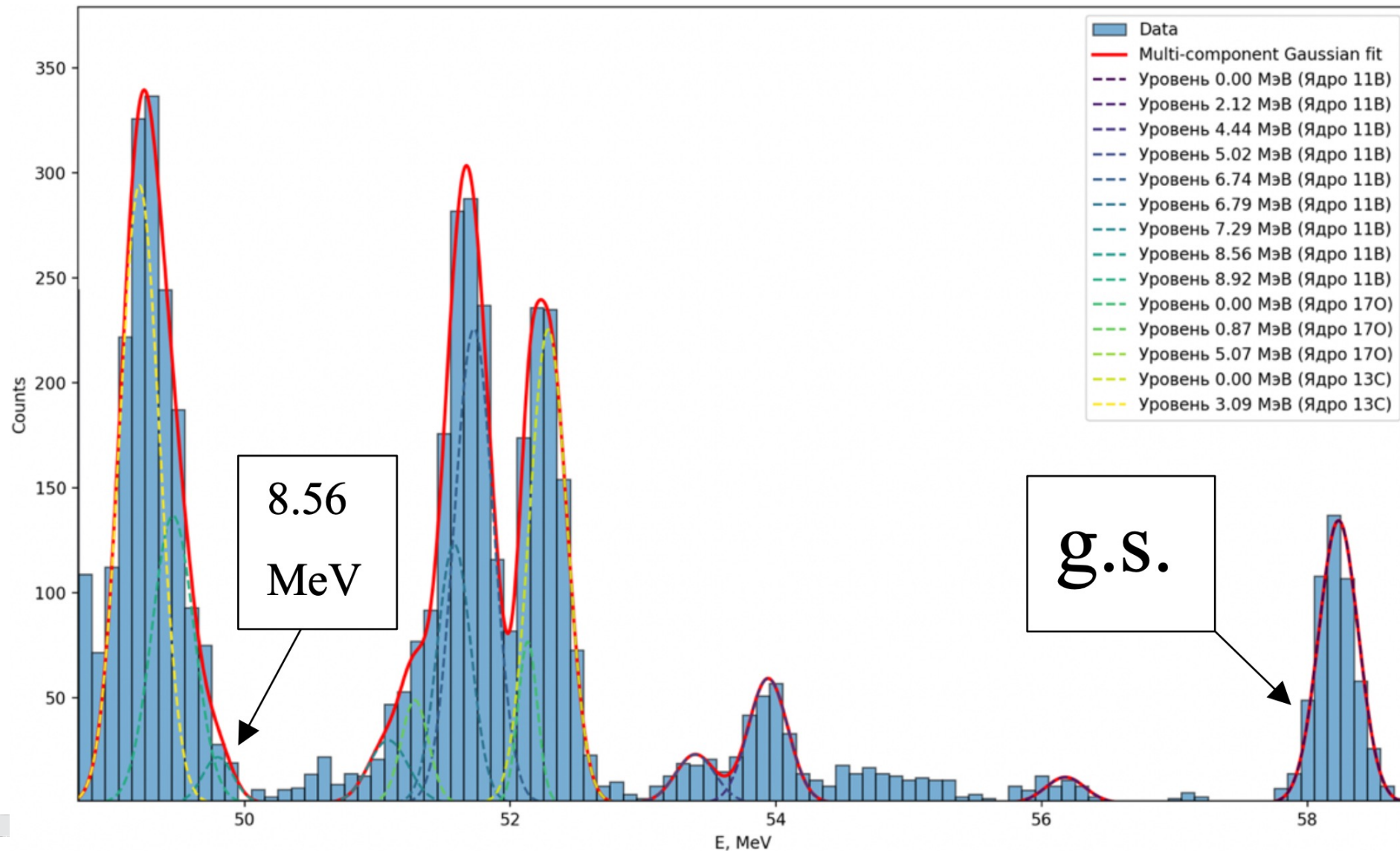


# Calculation of the positions of the studied energy states on the spectrum



**The stage of the program.** One-dimensional energy spectrum with calculated positions of energy states. The X-axis is the energy of the particles, the Y-axis is the number of events. Red is the states of the  $^{11}\text{B}$  core. Blue – the states of the impurity nuclei

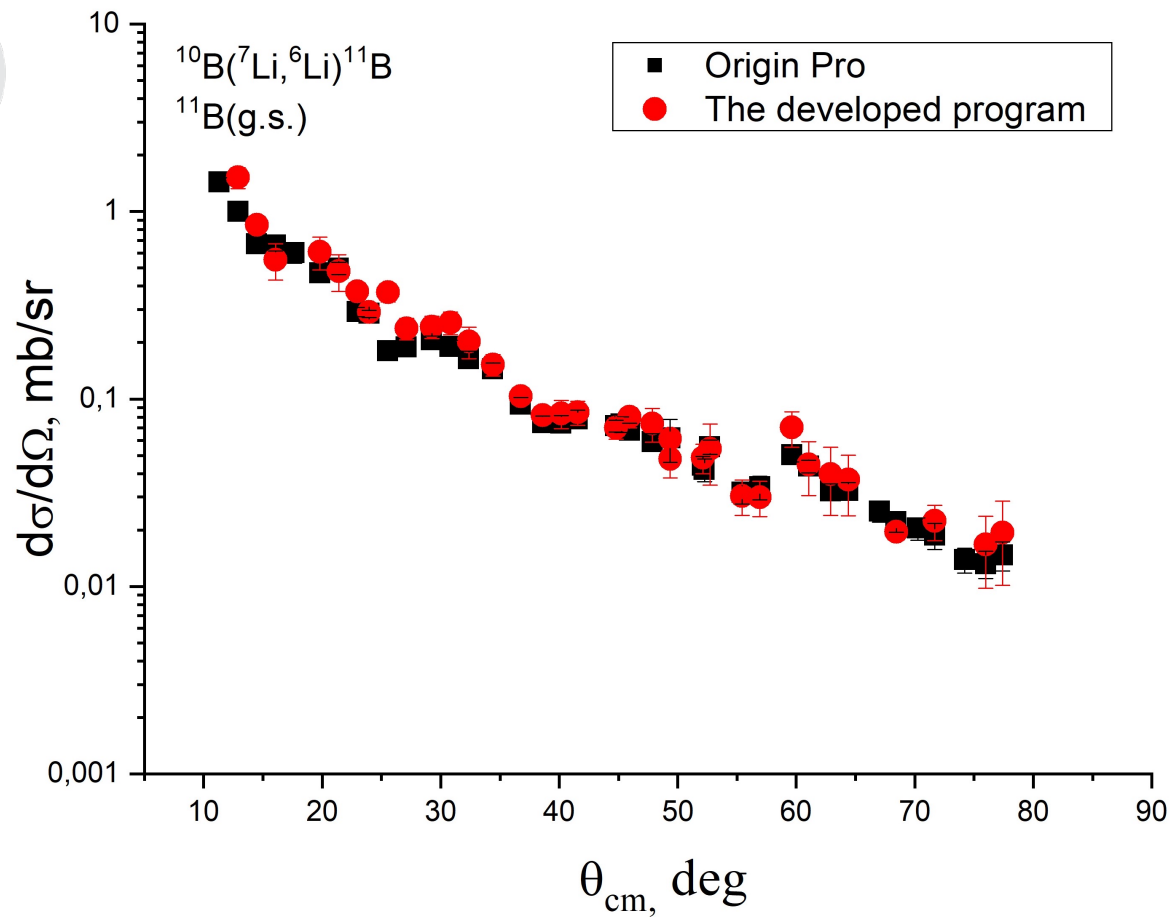
# Energy spectrum fitting



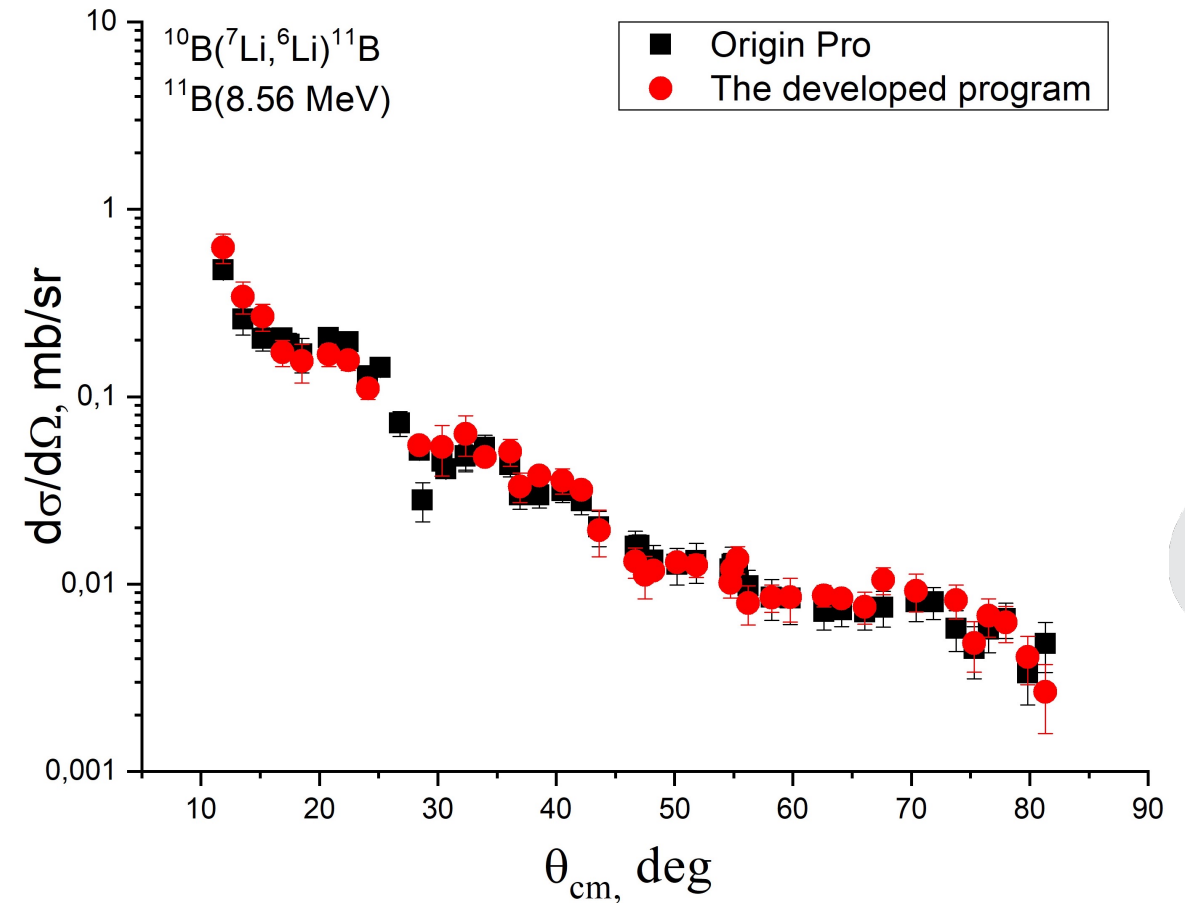
**The stage of the program.** An example of fitting the energy spectrum using Gaussian functions. The X-axis is the energy of the particles, the Y-axis is the number of events.



# Calculation of the differential cross sections



Differential cross section for the  
ground state of the  $^{11}\text{B}$  nucleus



Differential cross section for the 8.56 MeV of  
the  $^{11}\text{B}$  nucleus

# Theoretical analysis of the obtained angular distributions

$\frac{d\sigma_{\alpha\beta}}{d\Omega} = \frac{\mu_{\alpha}\mu_{\beta}}{(2\pi\hbar^2)^2} \frac{K_{\beta}}{K_{\alpha}} |T_{\beta\alpha}|^2$ ,  $\mu_i$  - reduced mass of channel  $i$ ,  $T_{\beta\alpha}$  - the transition amplitude from channel  $\alpha$  to channel  $\beta$



## Distorted Wave Born Approximation (DWBA)

$$T_{\text{DWBA}} = \int u_b^{(-)*}(\mathbf{K}_b, \mathbf{r}_b) \langle \Psi_b \Psi_B | V | \Psi_a \Psi_A \rangle u_a^{(+)}(\mathbf{K}_a, \mathbf{r}_a) d\mathbf{r}_a d\mathbf{r}_b,$$

$u_a^{(+)}$  и  $u_b^{(-)}$  - wave functions that describe elastic scattering in the input and output channels of the reaction,  $\Psi_i$  - the wave functions of the corresponding states of the nuclei in the input and output channels of the reaction,  $\mathbf{K}_i$  - is the relative momentum in the channel.

$\langle \Psi_b \Psi_B | V | \Psi_a \Psi_A \rangle$  - the overlap integral of the interaction  $V$  and the wave functions of the interacting nuclei ( $a, A$ ) and product nuclei ( $b, B$ ) that contains all the structural information.

# Theoretical analysis of the obtained angular distributions. Asymptotic normalization coefficient

$$\langle \Psi_b \Psi_B | V | \Psi_a \Psi_A \rangle \sim I_{lsj}^{ab}(r) * I_{lsj}^{BA}(r)$$

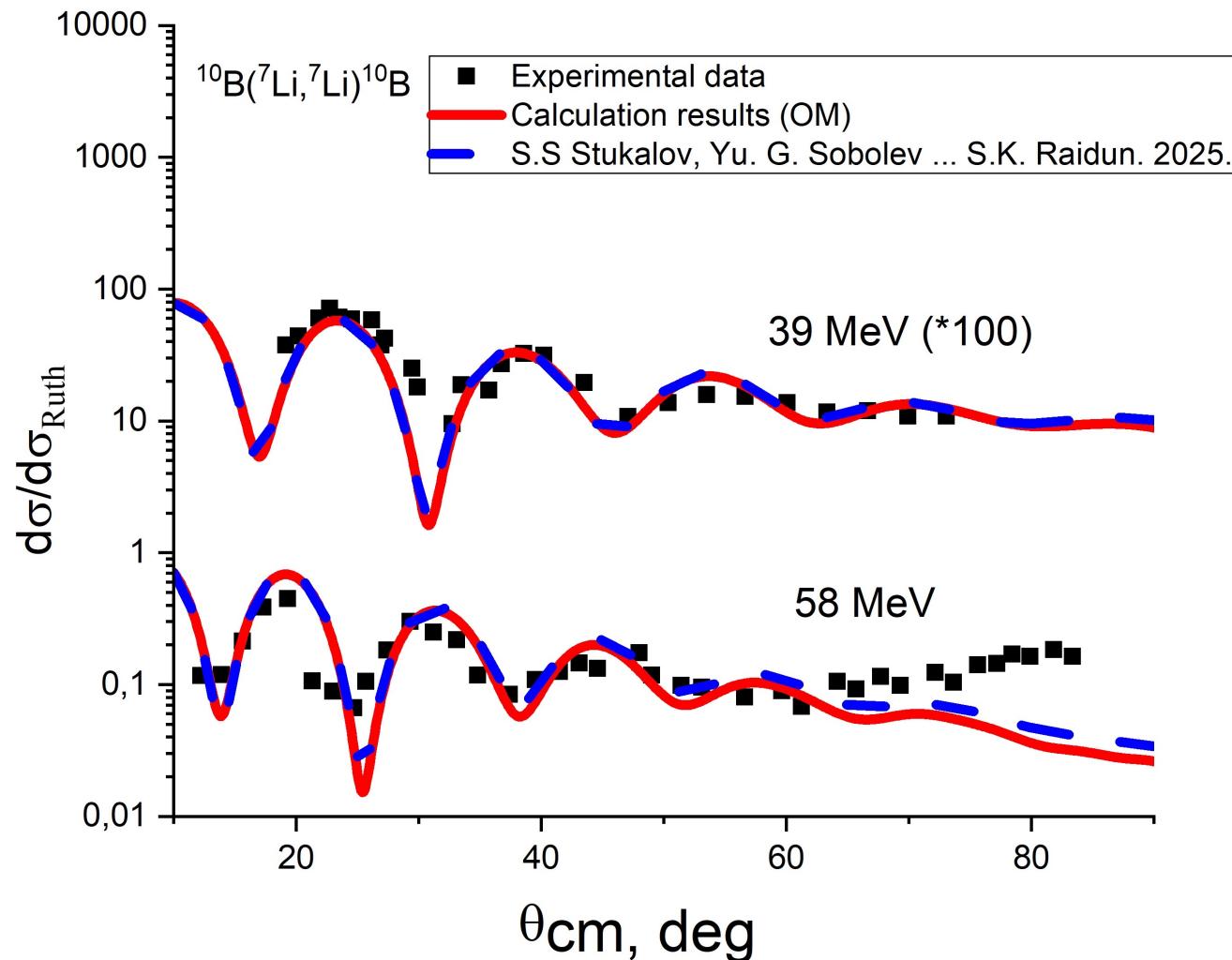
$I_{lsj}^{ab}(r)$ ,  $I_{lsj}^{BA}(r)$  – reaction form factors for the «**light**» ( $a = b + n$ ) and «**heavy**» ( $B = A + n$ ) systems.

$$I_{lsj}(r \rightarrow \infty) = N^{\frac{1}{2}} C_{lsj} k h_l(ikr),$$

где  $k^2 = \frac{2\mu e_{lsj}}{\hbar^2}$ ,  $\mu$  – reduced mass,  $e_{lsj}$  – is the binding energy of the transferred particle in a given state of the nucleus B(a). N – coefficient taking into account the antisymmetrization of wave functions.

$NC_{lsj}^2$  – **asymptotic normalization coefficient (ANC)**, the empirical value of which can be obtained from the description of the main peak of the experimental angular distribution.

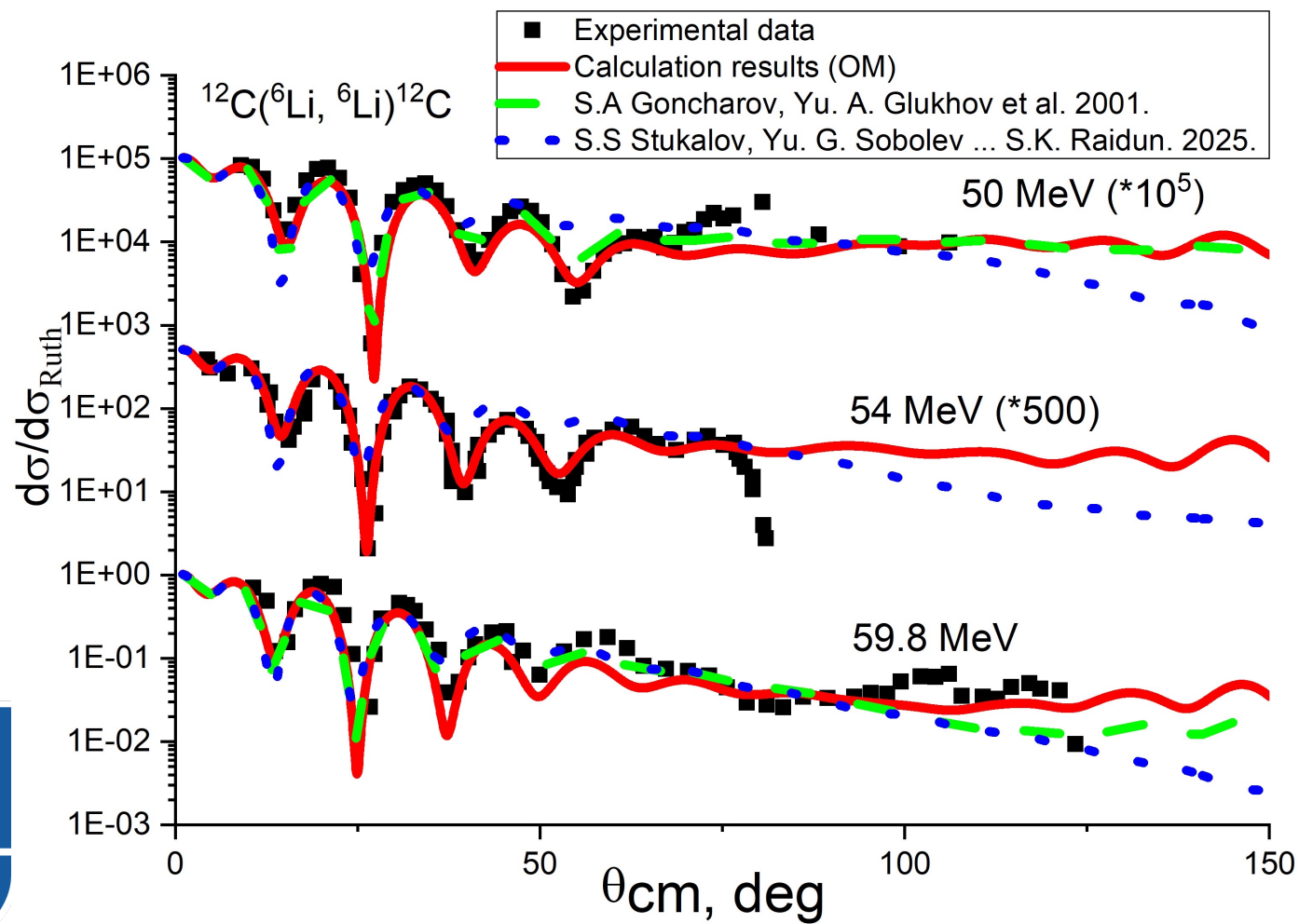
${}^7\text{Li} + {}^{10}\text{B}$



**Determination of  
optical model  
parameters for  
elastic scattering  
 ${}^7\text{Li} + {}^{10}\text{B}$**



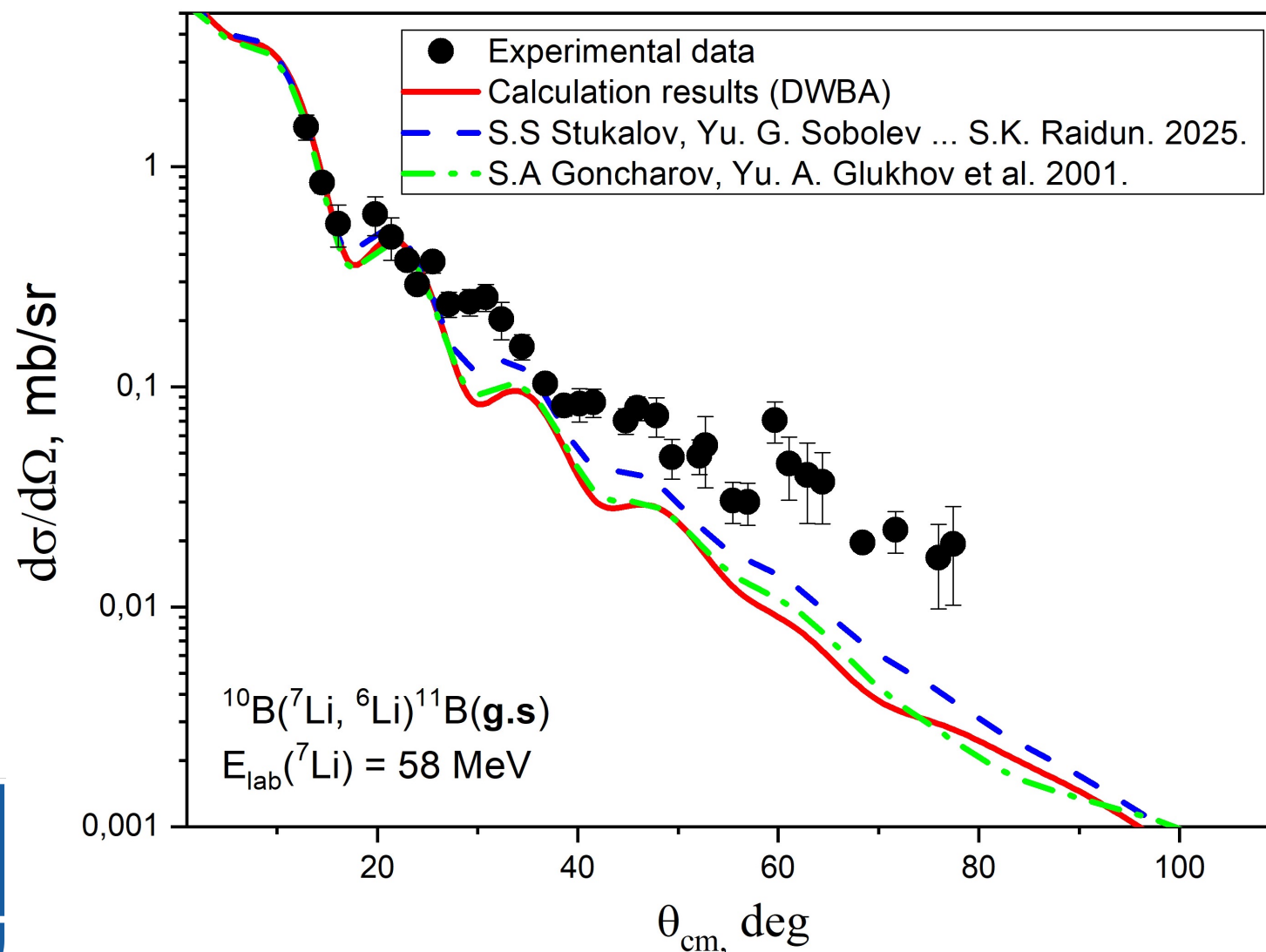
## ${}^6\text{Li} + {}^{11}\text{B}$ ( ${}^6\text{Li} + {}^{12}\text{C}$ )



**Determination of optical model parameters from the integral characteristics of a nearby system ( ${}^6\text{Li} + {}^{12}\text{C}$ )**



# DWBA results for the ground state of the $^{11}\text{B}$ nucleus



The various parameters of the optical model do not affect the description of the main maximum of the angular distribution within the absolute error

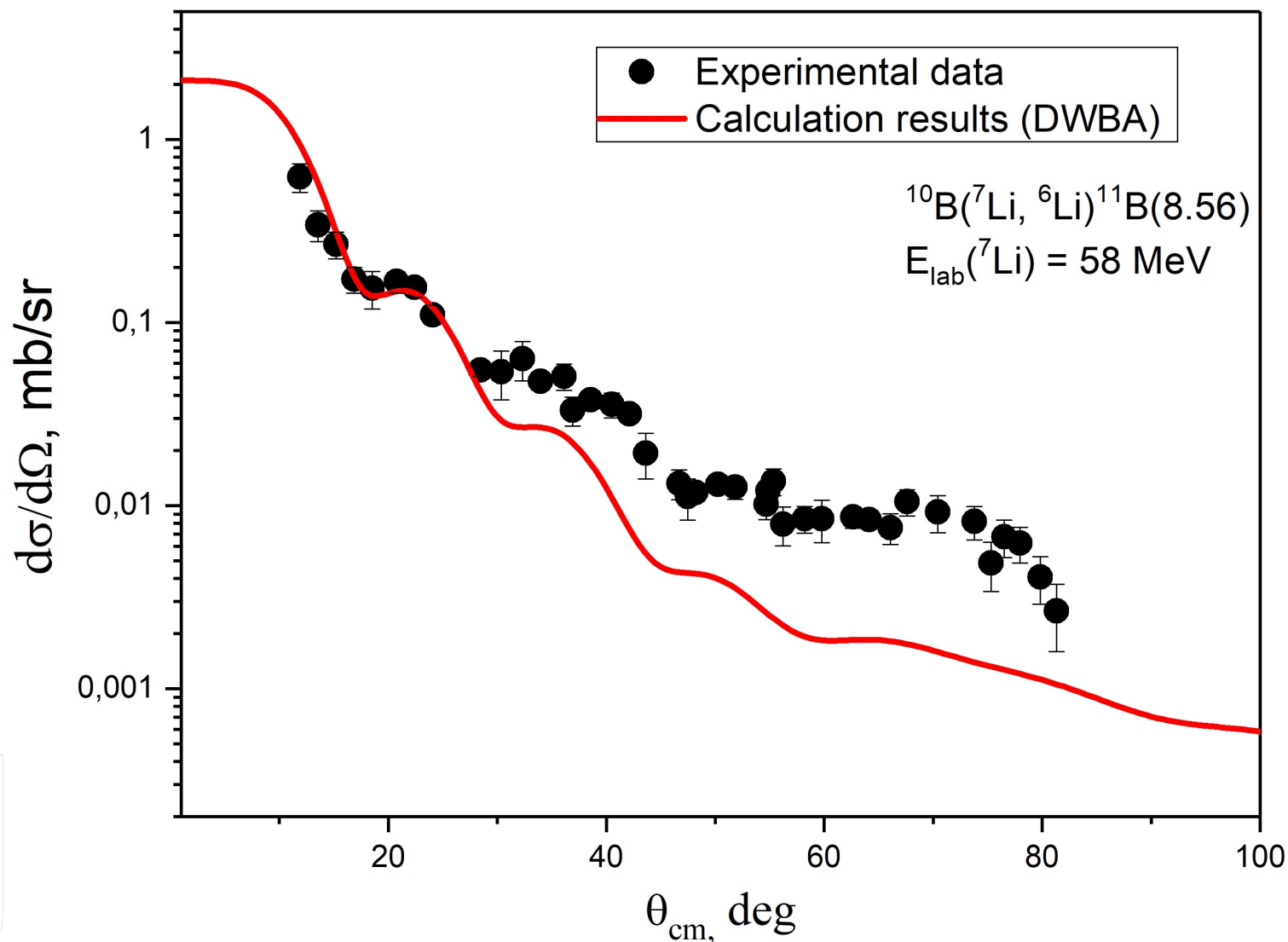
$$\text{ANC} = 13.6 \text{ fm}^{-1}$$

$$\text{ANC}^{\text{theor.}} = 13.1 \text{ fm}^{-1} [4]$$





# DWBA results for the 8.56 MeV state of the $^{11}\text{B}$ nucleus

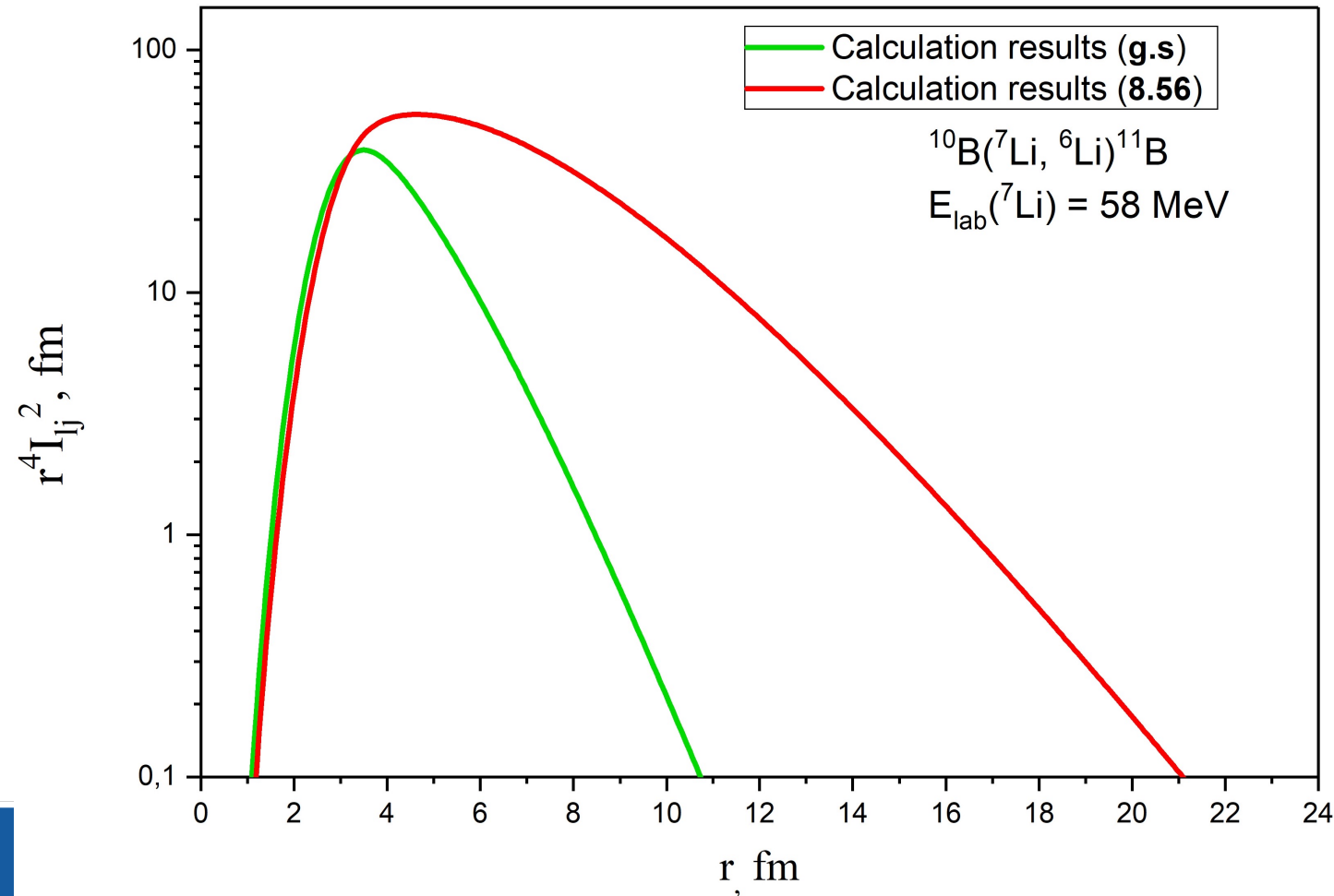


The found parameters of the optical potential and ANC make it possible to achieve a description of the main maximum of the angular distribution for the state 8.56 of the  $^{11}\text{B}$  nucleus

$$\text{ANC} = 0.26 \text{ fm}^{-1}$$



# The radial dependences of the form factors



$r^4 I_{lj}^2$  – the integrands in calculating the mean-square radii of these form factors:

$\{^{11}\text{B}(8.56) \rightarrow n + ^{10}\text{B}(\text{g.s.})\}$

и

$\{^{11}\text{B}(\text{g.s.}) \rightarrow n + ^{10}\text{B}(\text{g.s.})\}$



1. **A program has been developed** to automate the analysis of energy spectra.
2. **The angular distributions** of the differential cross section for the  $^{11}\text{B}$  nucleus in the ground state and 8.56 MeV state **were obtained** from reaction  $^{10}\text{B}(^7\text{Li}, ^6\text{Li})^{11}\text{B}$ .
3. **A theoretical analysis** of the obtained angular distributions **is performed**.
4. **It is shown** that the **wave function** of the 8.56 MeV state of the  $^{11}\text{B}$  nucleus has **increased spatial dimensions** compared to the ground state.

This may be an additional indication of the cluster structure of the 8.56 MeV state of the  $^{11}\text{B}$  nucleus.



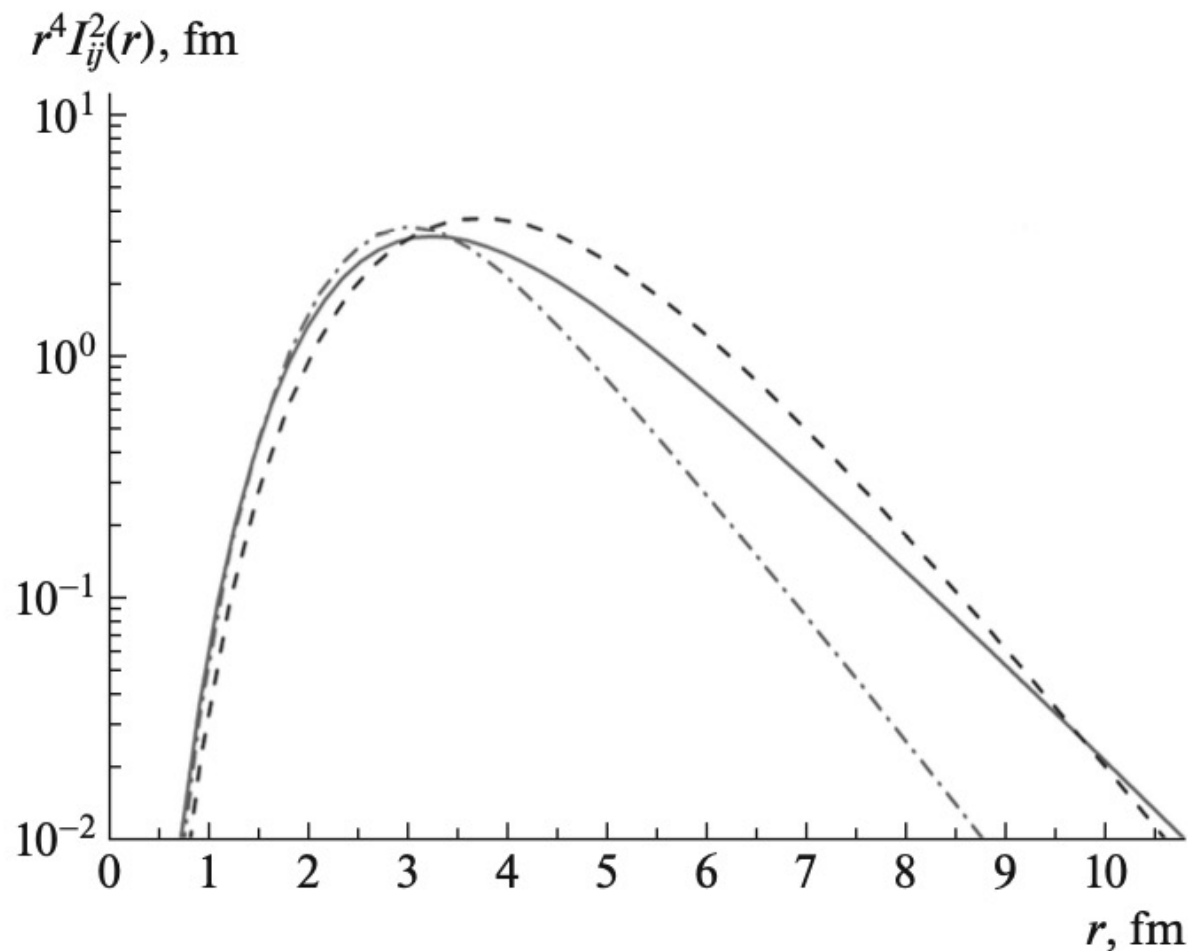
**Thank you for  
your attention!!!**



1. A.N. Danilov, A.S. Demyanova et al., *Physics of Atomic Nuclei* 78, 777 (2015).
2. A.S. Demyanova, *AIP Conf. Proc.* 3020, 020002 (2024).
3. A. Ozawa, T. Suzuki, I. Tanihata, *Nuclear Physics A* 693, Issues 1 - 2 (2001).
4. N. K. Timofeyuk, *Phys. Rev C* 81, 064306 (2010).
5. Stukalov, Sobolev et al. Study of the  ${}^6\text{Li}$   $0^+$  Excited State. *Physics of Atomic Nuclei* 87, (2025).



The radial dependences of the form factors for the  ${}^6\text{Li}$  и  ${}^{11}\text{B}$  nuclei [5]:



Comparison of radial dependencies for: solid line – transition  $\{{}^7\text{Li}(\text{g.s.}) \rightarrow n + {}^6\text{Li}(\text{g.s.})\}$ , dotted line –  $\{{}^7\text{Li}(\text{g.s.}) \rightarrow n + {}^6\text{Li}(\mathbf{3.56.})\}$ , dash-dotted line –  $\{{}^{11}\text{B}(\text{g.s.}) \rightarrow n + {}^{10}\text{B}(\text{g.s.})\}$





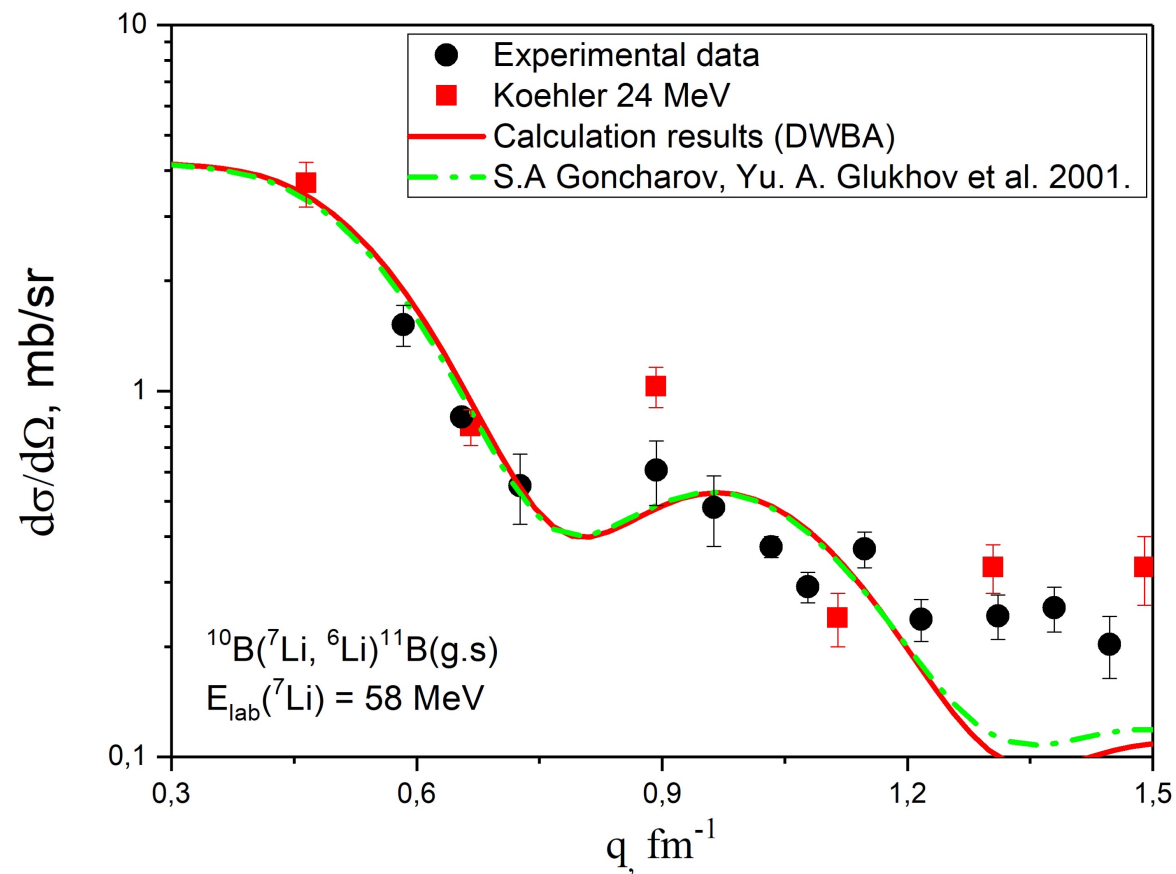
## The equation for the form factor [5]:

$$(T_r - V_{lsj}(r) - e_{lsj})I_{lsj}(r) = 0,$$

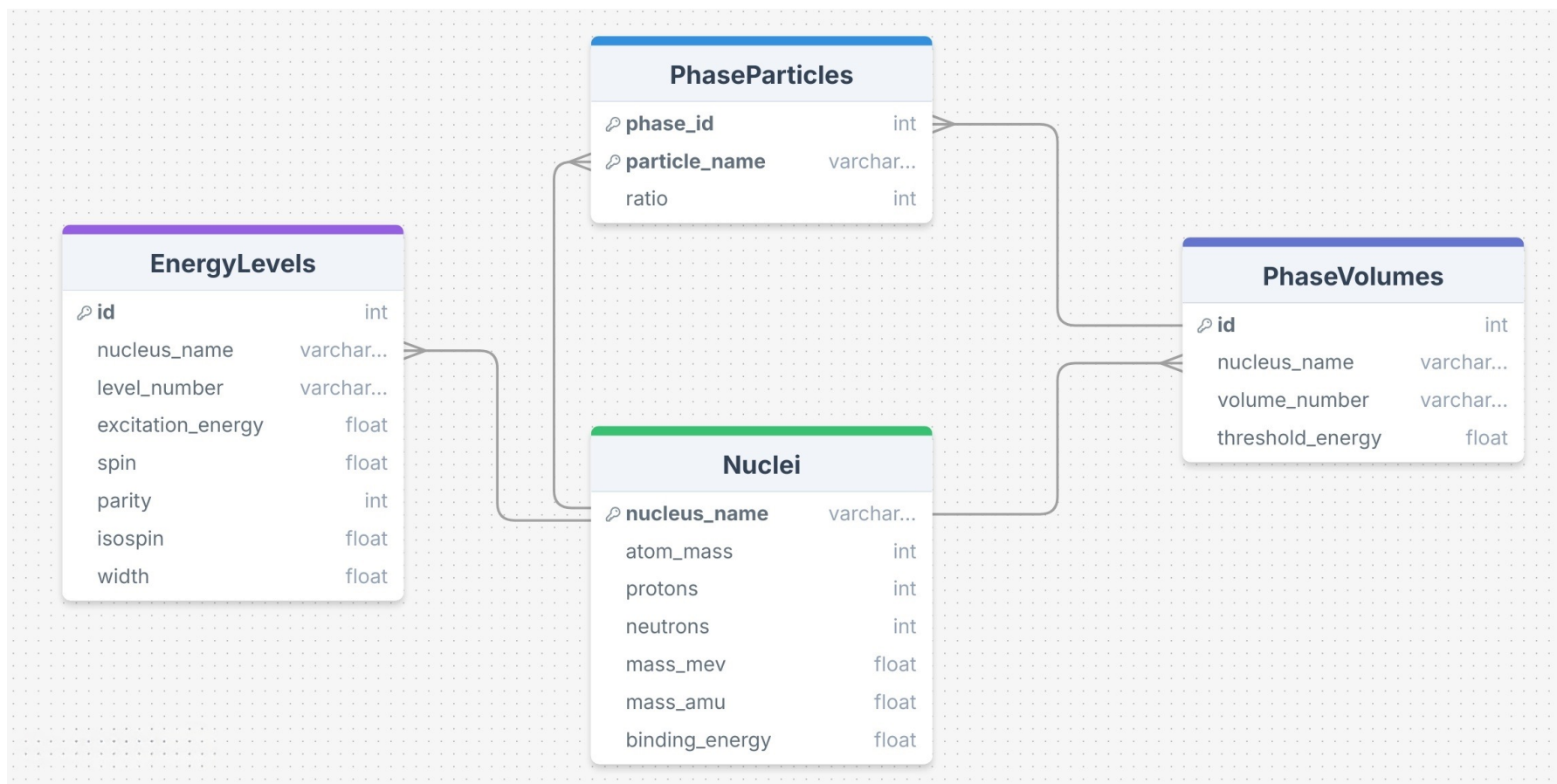
где  $T_r$  – the operator of the kinetic energy of the relative motion of the transferred particle and the core,  $V_{lsj}(r)$  – effective potential in Woods-Saxon parameterization,  $e_{lsj}$  – is the binding energy of the transferred particle in a given state of the nucleus  $B(a)$ .



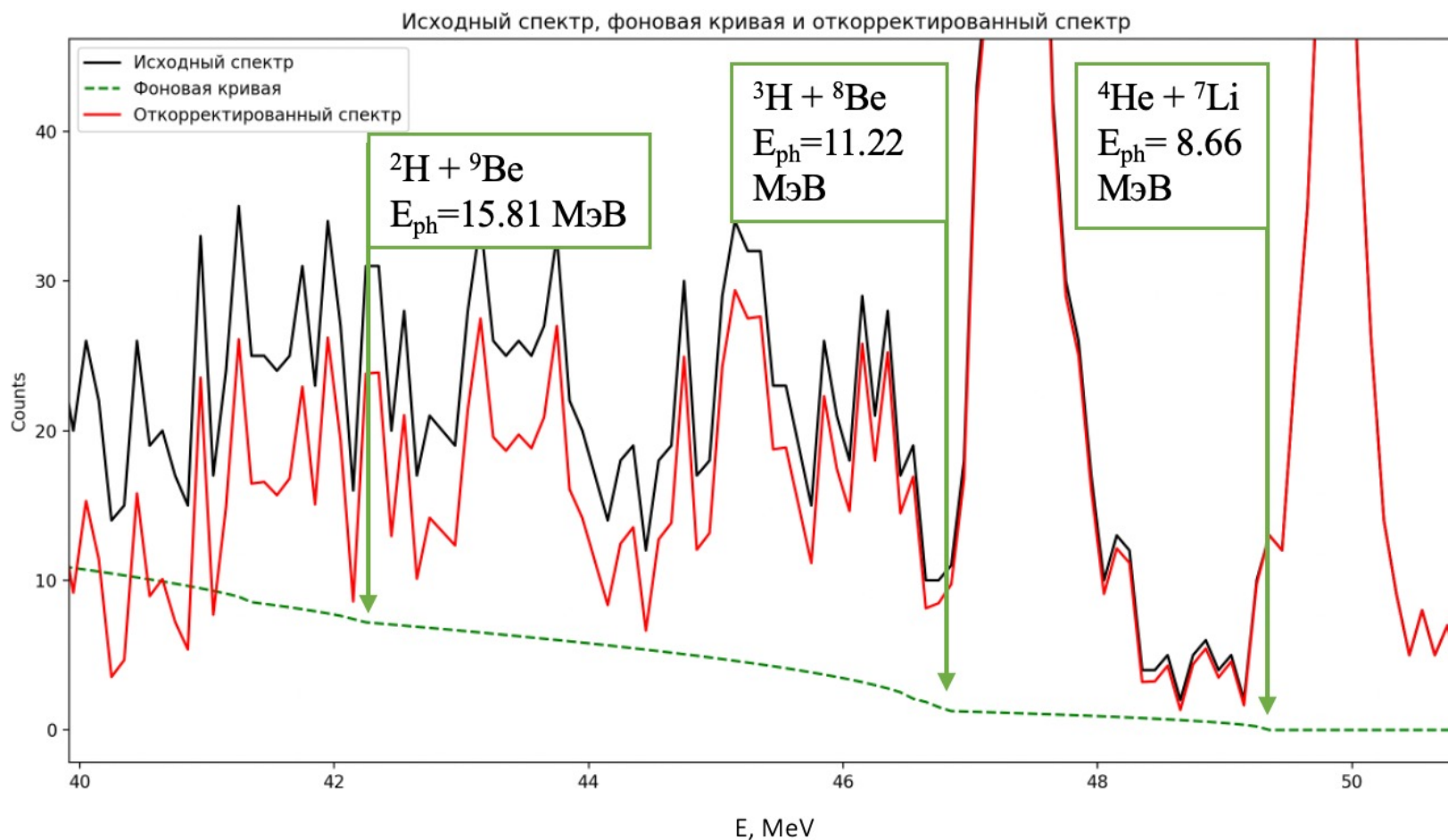
## Description of the main peak for the ground state:



## Database of light nuclei with mass numbers $A = 1 - 20$ :



## Background events:



### The stage of the program.

The energy spectrum, taking into account background events. The X-axis is the energy of the ejected particles, the Y-axis is the number of events. The original spectrum is shown in black. The red color is responsible for the spectrum with the background taken into account. The green curve is the phase volume.



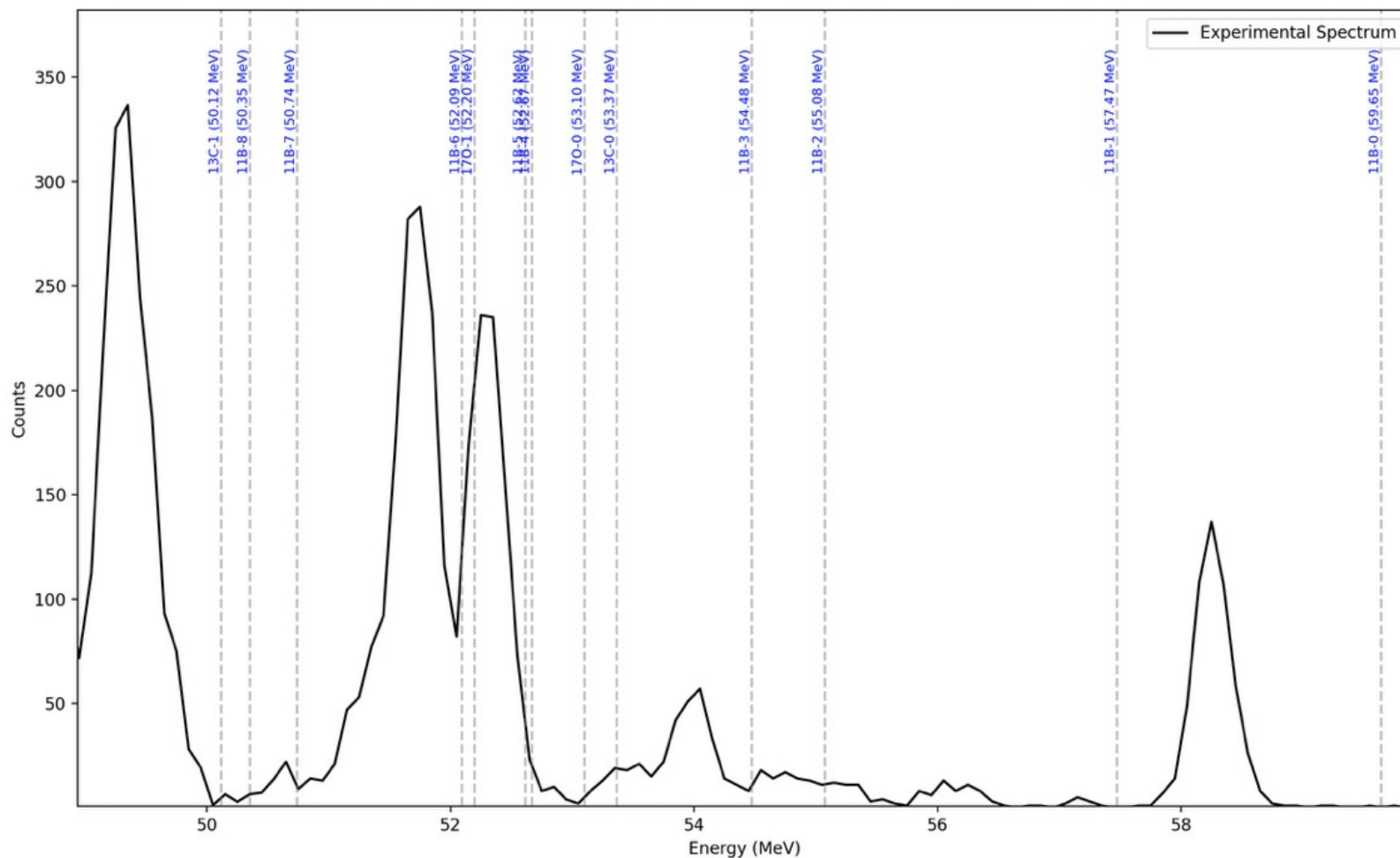
## Background events:

$$C * E_1^{1/2} * \left[ \frac{(M - m_1) * E_{cm}}{(M - E_1)} + 2 * \left( \frac{m_1 * m_p * E_p}{(m_p + m_t)^2} \right)^{1/2} * E_1^{1/2} \cos t - \frac{m_1 * m_p * E_p}{(m_p + m_t)^2} \right]^{\frac{3N-8}{2}}$$

where  $E_p$  is the energy of the projectile,  $m_p$  is the mass of the projectile,  $m_t$  is the mass of the target,  $E_1$  is the energy of the scattered particle,  $m_1$  is the mass of the scattered particle,  $M$  is the total mass in the output channel,  $E_{cm}$  is the total kinetic energy in the output channel in c.m.,  $N$  is the number of particles in the output channel,  $t$  is the scattering angle,  $C$  is the selected constant.



## Recalibration of the energy spectrum:



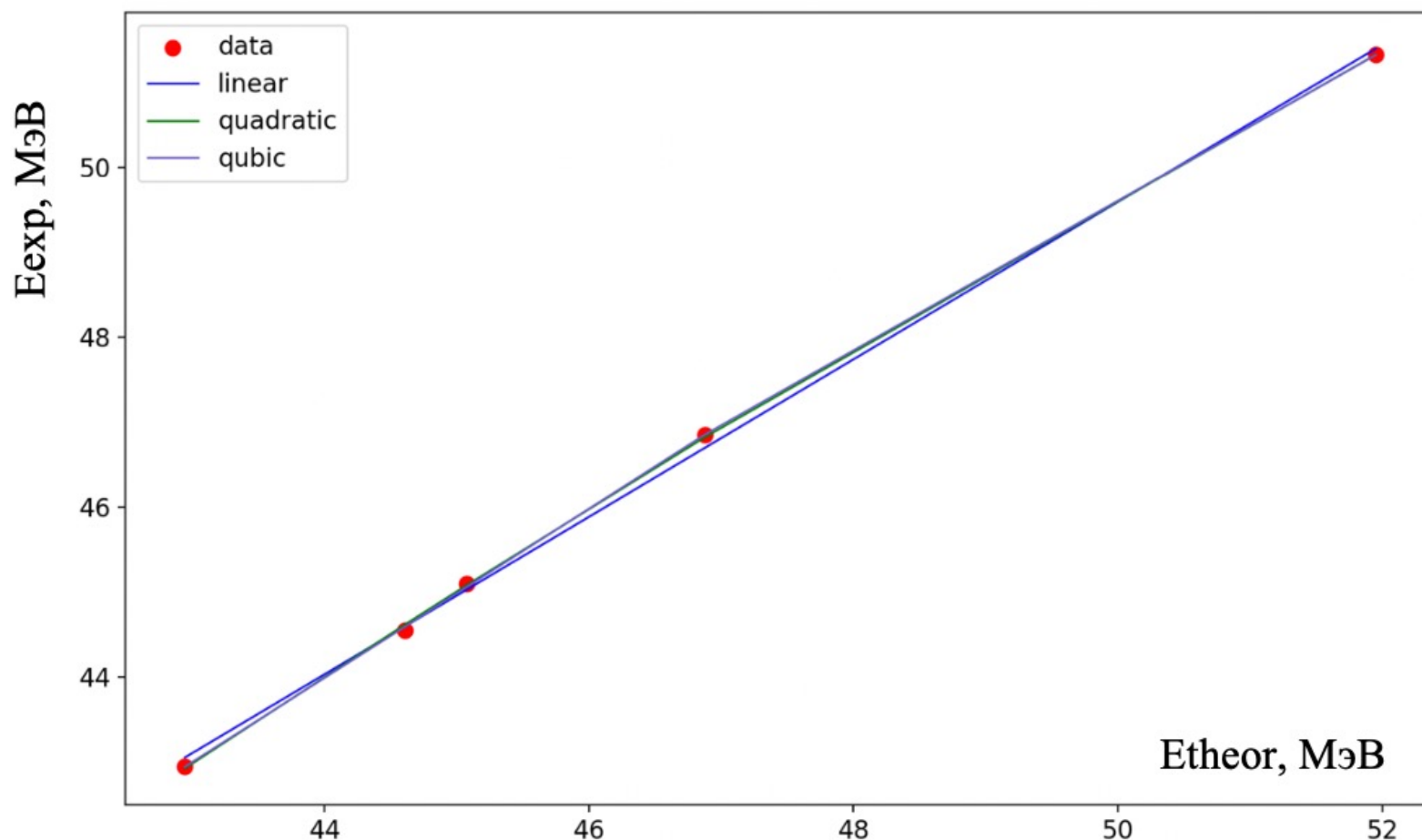
**The stage of the program.**

Determination of the position of energy levels. The X-axis is the energy of the ejected particle in MeV, the Y-axis is the number of events.





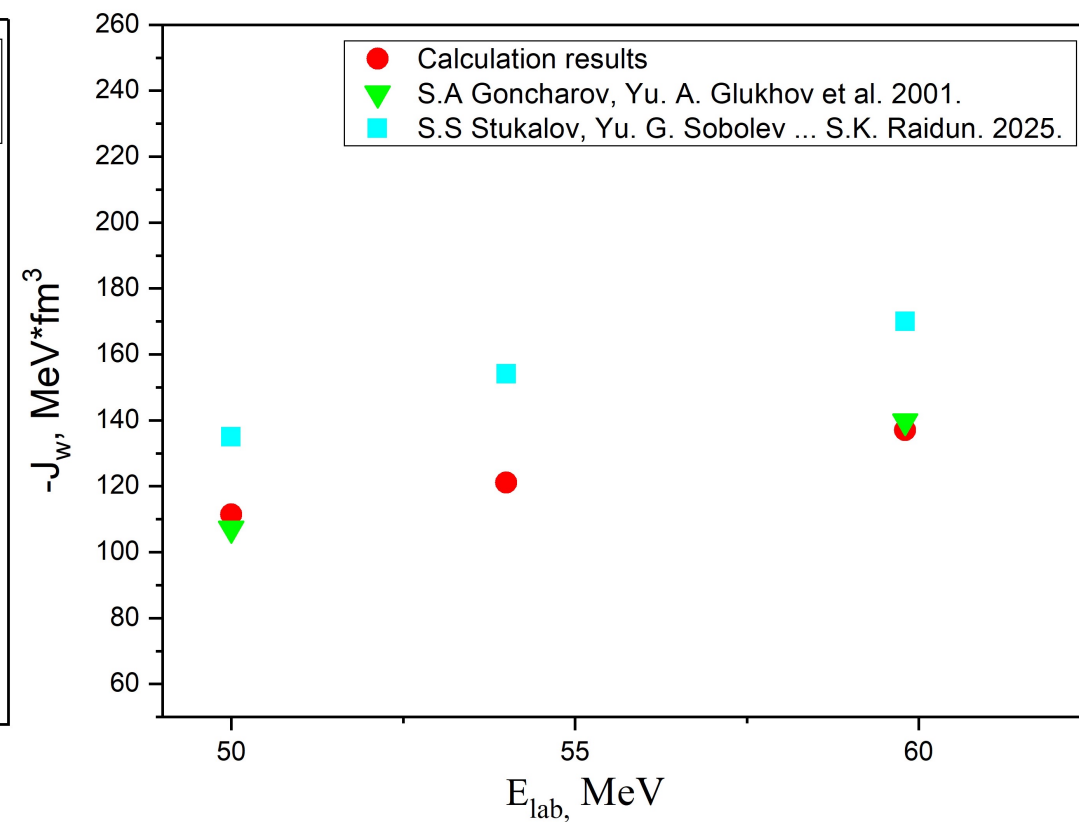
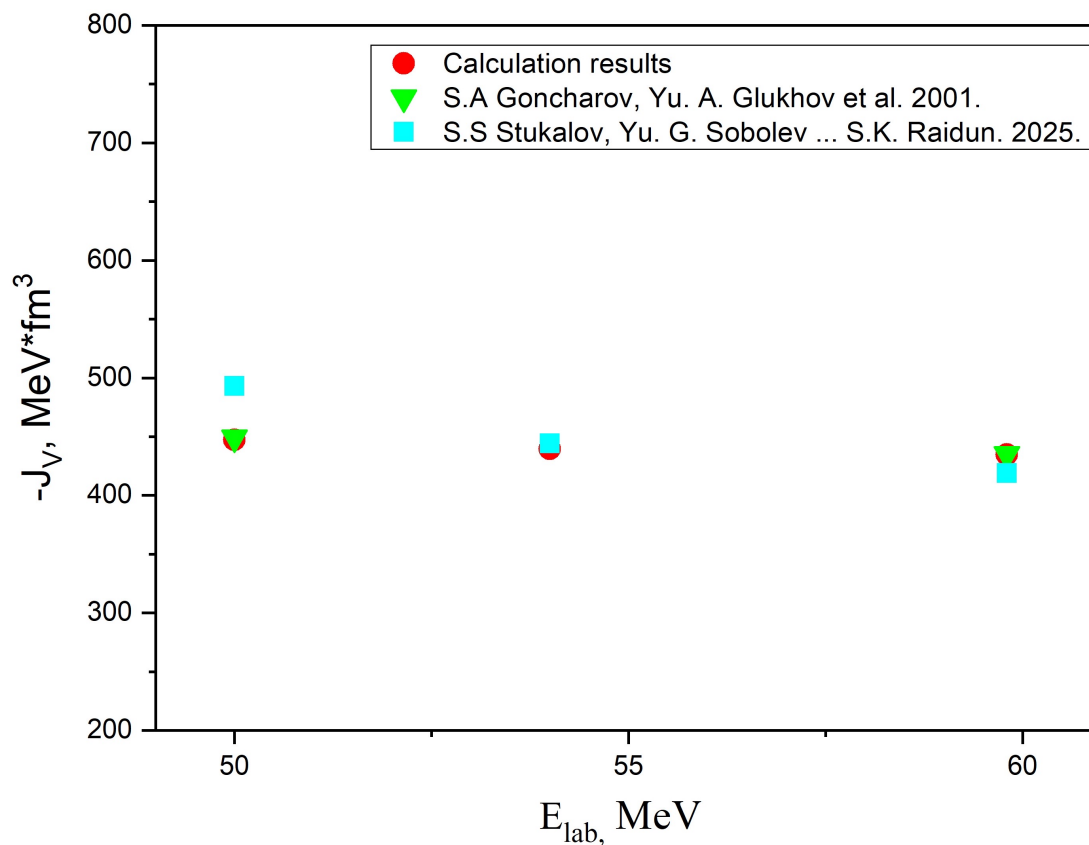
## Calibration curves:



**The stage of the program.** An example of choosing the type of approximation. The X-axis is the total energy of the ejected particles, calculated theoretically, the Y-axis is the user-selected values.



## Integral characteristics of elastic scattering ${}^6\text{Li} + {}^{12}\text{C}$

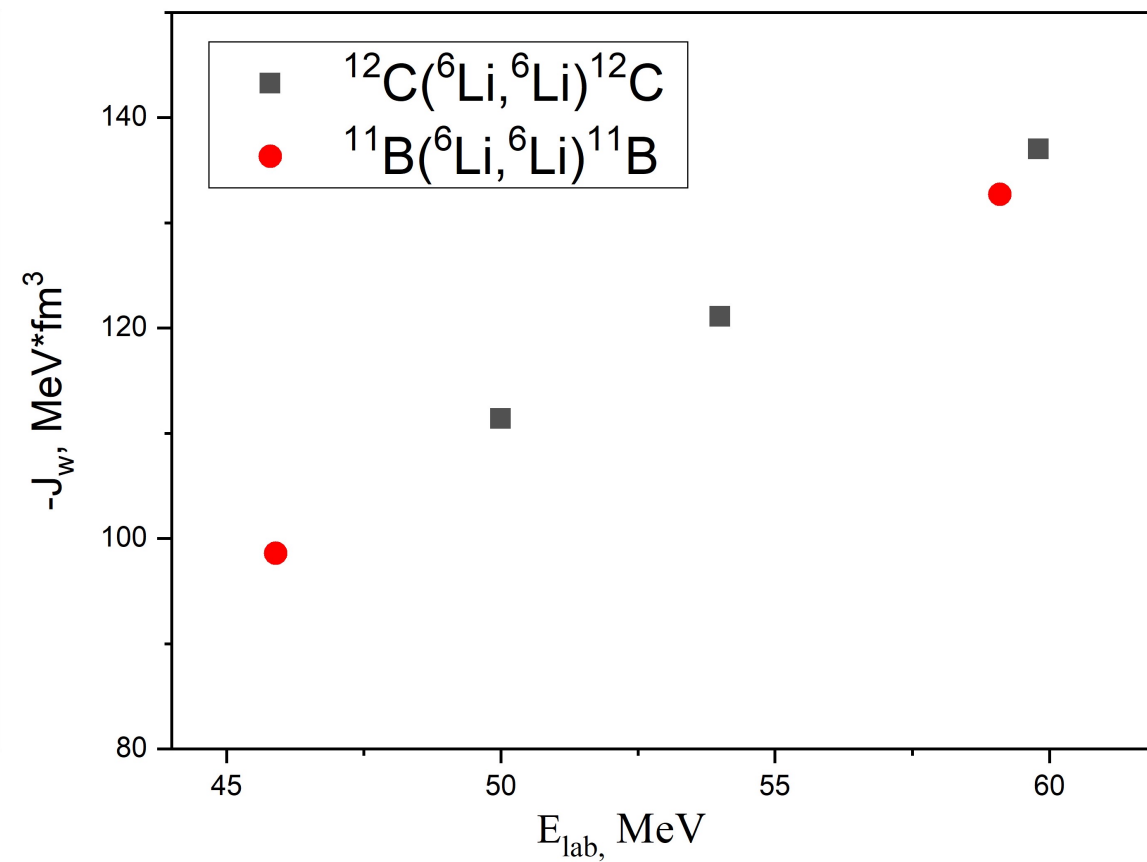
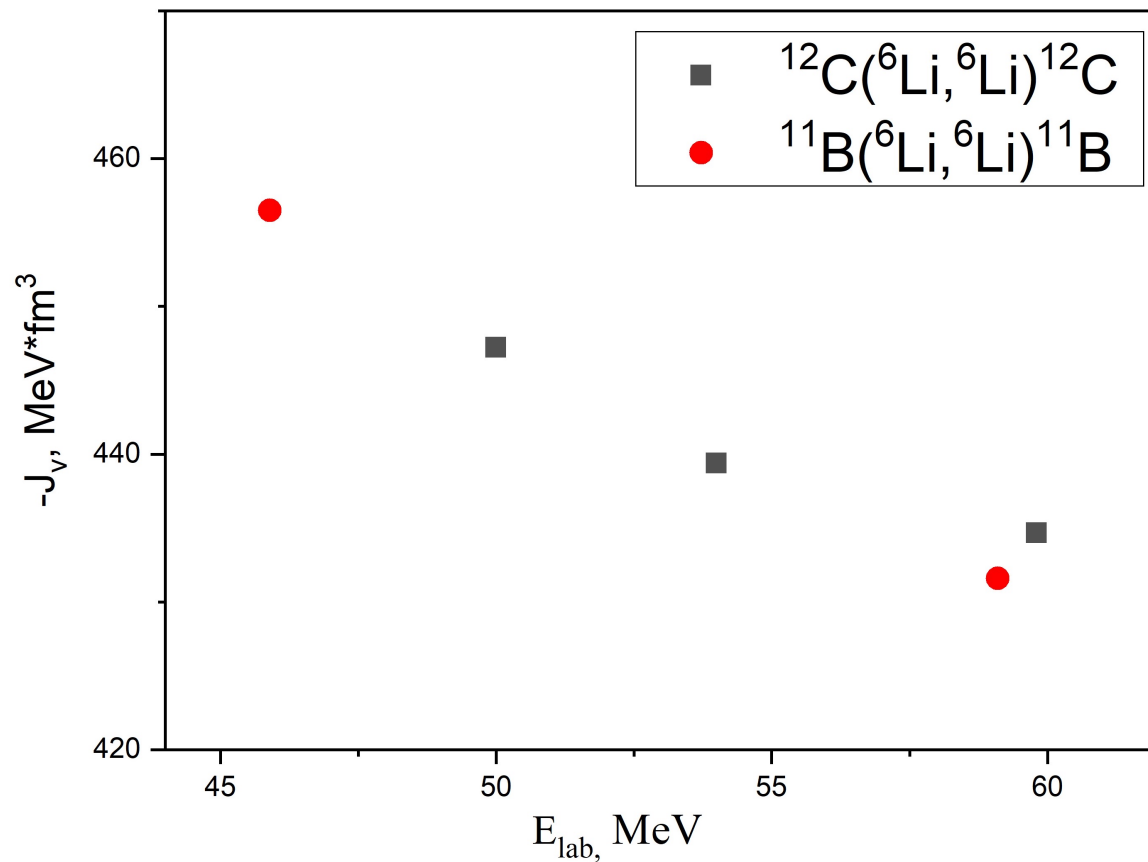


## Exit channel optical model parameters ( ${}^6\text{Li} + {}^{12}\text{C}$ )

$E_{\text{lab.}}$ MeV	$V_R$ MeV	$W_S$ MeV	$W_D$ MeV	$J_V$ MeV fm <sup>3</sup>	$J_W$ MeV fm <sup>3</sup>	$\sigma_r$ mb
<b>59.8</b>	277	14	17	435	137	1361
<b>54.0</b>	280	13	15	439	121	1333
<b>50.0</b>	285	12	13.8	447	111	1317

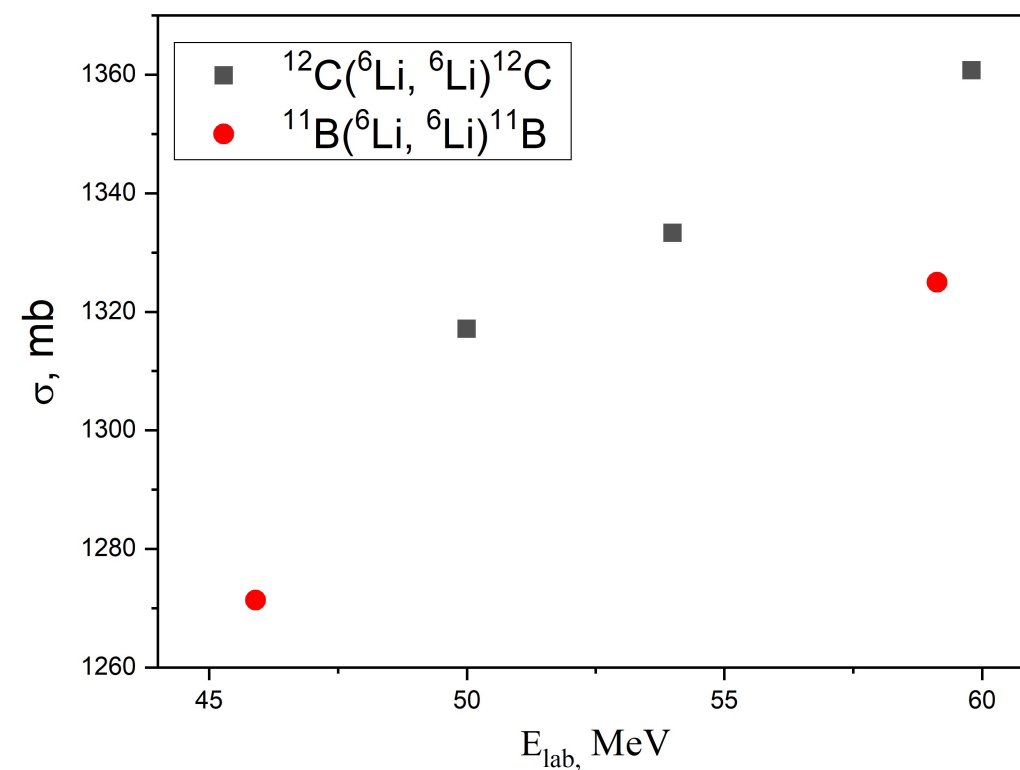


## Integral characteristics of the potentials:



## Exit channel optical model parameters ( ${}^6\text{Li} + {}^{11}\text{B}$ )

	$E_{\text{lab.}}$ MeV	$V_R$ , MeV	$W_S$ , MeV	$W_D$ , MeV	$-J_V$ , MeV*fm <sup>3</sup>	$-J_W$ , MeV* fm <sup>3</sup>	$\sigma_r$ , mb
g.s.	59.1	260	13	15.5	432	133	1325
8.56	45.9	275	10	11.5	456	99	1271



## The results of the theoretical analysis



$A+1(I^{\pi}T, E')$ , $nA(I^{\pi}T, E')$	$(I, j)$	$-V$ , MeV	$R$ , fm	$a$ , fm	$\langle r^2 \rangle^{1/2}$ , fm	$NC^2$ , fm <sup>-1</sup>	$NC^2_{\text{theor}}$ , fm <sup>-1</sup>
$^{11}\text{B}(3/2^-$ $1/2, \text{g.s.}),$ $n^{10}\text{B}(3^+0, \text{g.s.})$	(1, 3/2)	39.40	3.49	0.37	2.91	13.60	13.10
$^{11}\text{B}(3/2^-$ $1/2, 8.56.),$ $n^{10}\text{B}(3^+0, \text{g.s.})$	(1, 3/2)	25.40	3.40	0.30	3.62	0.26	-