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Nuclear Collective Vibrations Studied by Beyond Mean-Field Approach

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Outline

- **Nuclear collective vibrations**
- **Collective vibrations provide insight to**

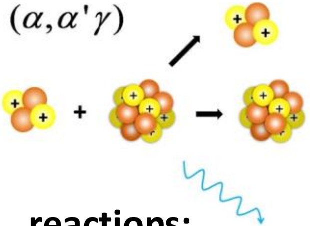
✓ **How were the heavy elements from iron to uranium made?**

✓ **What is the equation of state (EOS) of nuclear matter?**

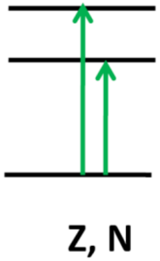
- **New possibilities to explore collective vibrations**
- **Summary and Perspective**

Nuclear Collective Vibrations

➢ **The vibration excitation of nucleus involving many nucleons**



- **Non-charge-exchange excitations**

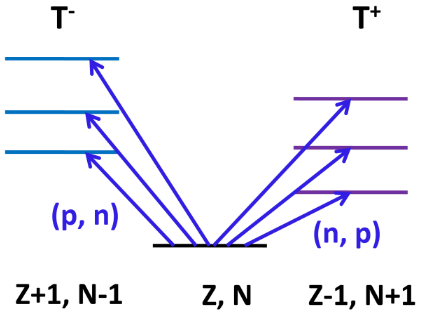


✓ Giant monople resonances (GMR)
✓ Giant dipole resonances (GDR)
✓ Giant quadrupole resonances (GQR)
...

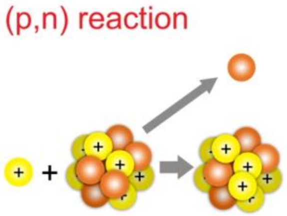
Strong
EM

reactions:
(α, α' γ),
(γ, γ'), (γ, n) ,(e, e')

- **Charge-exchange excitations**



✓ Isobaric Analogue States (IAS)
✓ Gamow-Teller Resonances (GTR)
✓ Spin-Dipole (SD) excitations
...

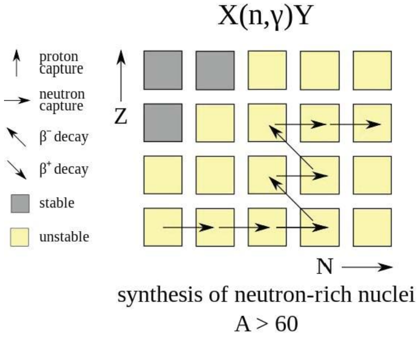


reactions:
(p,n) T^-, (n,p) T^+

Strong

Collective Vibrations Provide Insight to

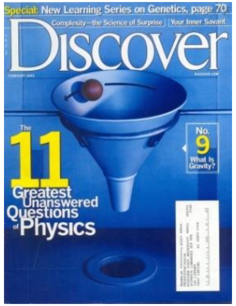
- ***How were the heavy elements from iron to uranium made?***
- ***What is the equation of state (EOS) of nuclear matter?***
- ***How do stars explode?***
- ***What are the masses of neutrinos?***
- ***What is the interaction between nucleons in nuclear medium that governs the properties of nuclei?***



How Were the Heavy Elements Made?

How were the heavy elements from iron to uranium made?

The 11 greatest unanswered questions of physics



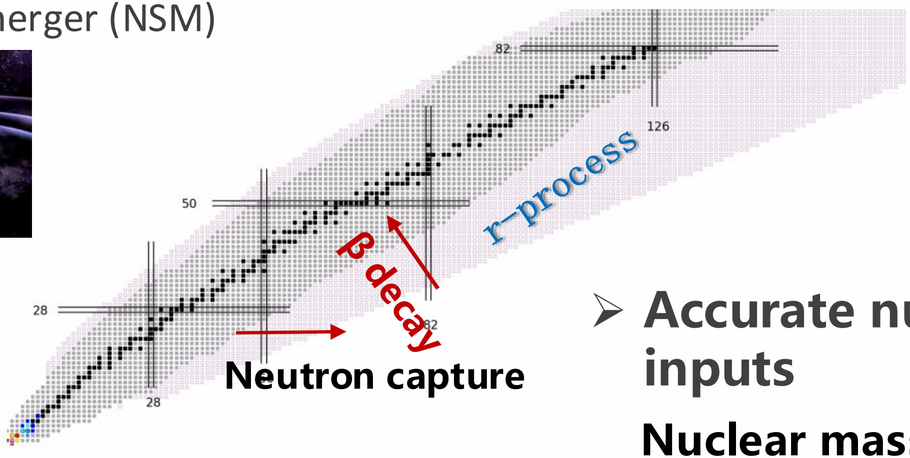
- **r-process**

➢ **Where does r-process happen?**

Supernova Neutron star merger (NSM)

GW170817 NSM:
One of the main r-process sites

Nature **551**,64; 67; 75; 80 (2017)
Science **358**, 1559 (2017)
ApJL **848**, L17; L19 (2017)



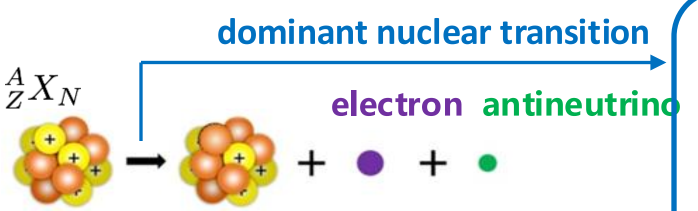
R-process path: far from stability, relies on theory!

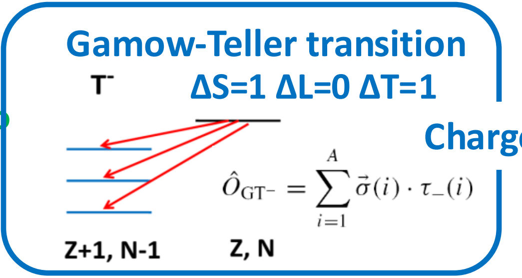
➢ **Accurate nuclear physics inputs**

Nuclear mass,
β decay half-lives,
Neutron-capture rates,
…

β-decay Half-life Is a Hard Problem

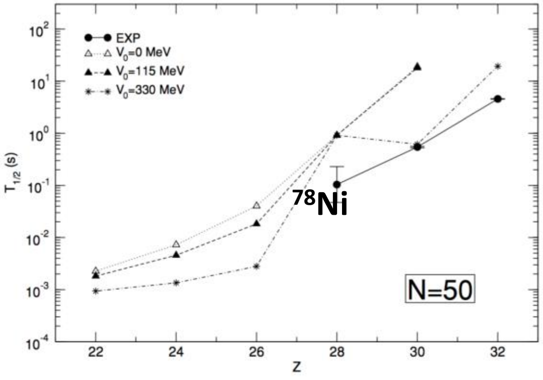
- **β-decay**





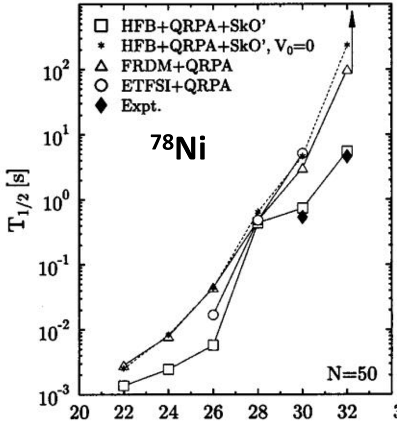
Charge-exchange mode

- **RHB+QRPA**
  (quasiparticle random phase approximation)



Niksic, et al., PRC 71, 014308 (2005)

- **Skyrme HFB+QRPA**



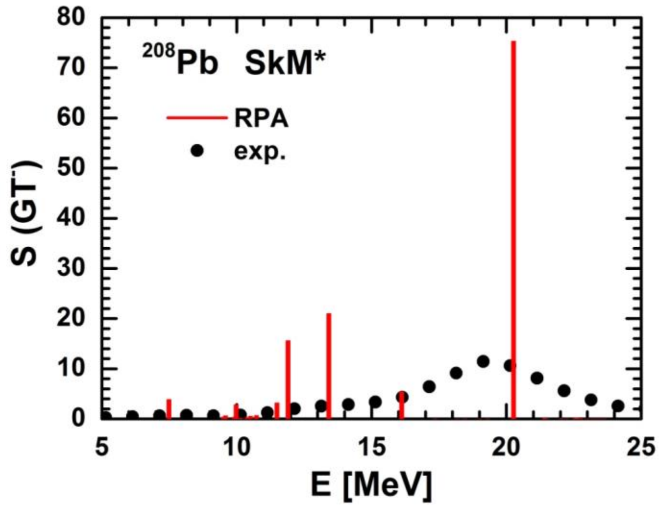
Engel, et al., PRC 60, 014302 (1999)

- **Half-lives are overestimated**

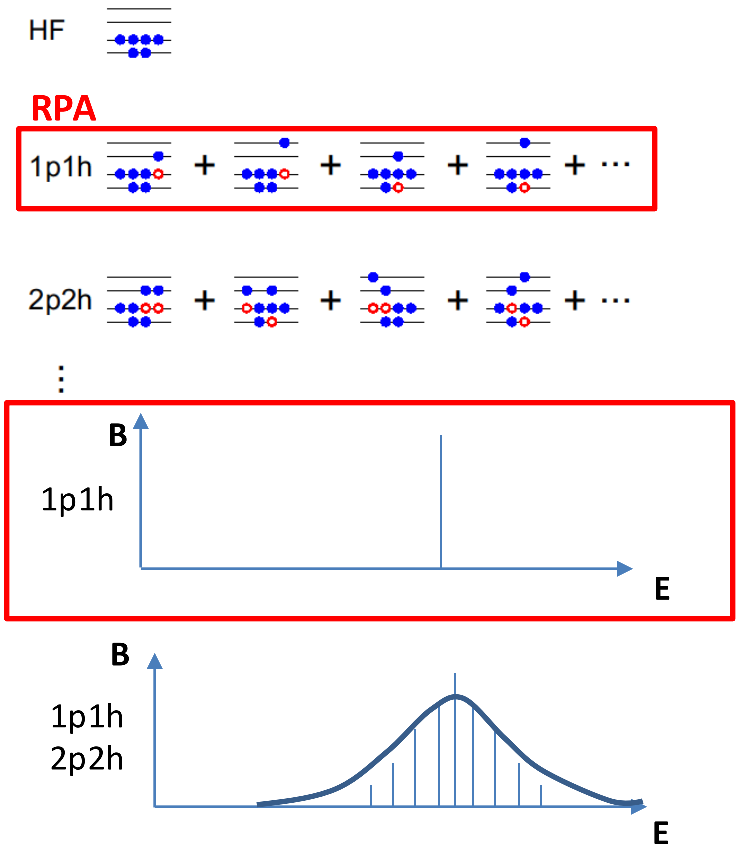
✓ **Due to the nuclear structure part – Gamow-Teller transition**

Limitations of (Q)RPA Description

- **(Q)RPA cannot describe the spreading width**



- **Correlations beyond RPA**

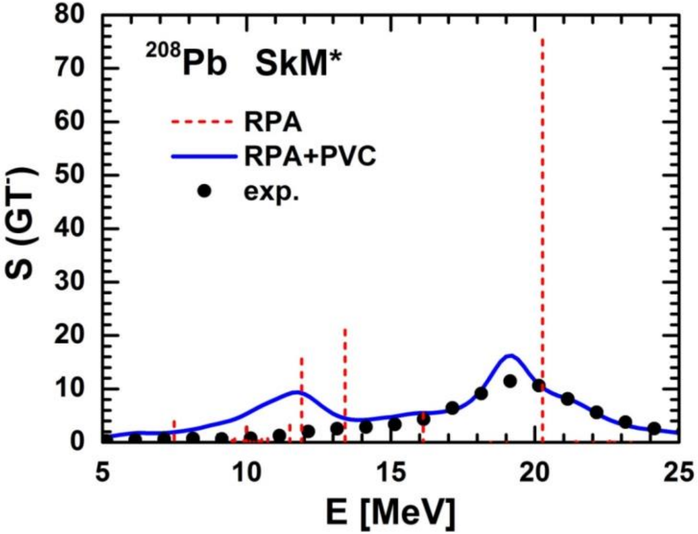


- **Spreading Width (Damping Width)**

  energy and angular momentum of coherent vibrations
  → more complicated states of 2p-2h, 3p-3h, … character

Limitations of (Q)RPA Description

- **Particle Vibration Coupling (PVC) effect**



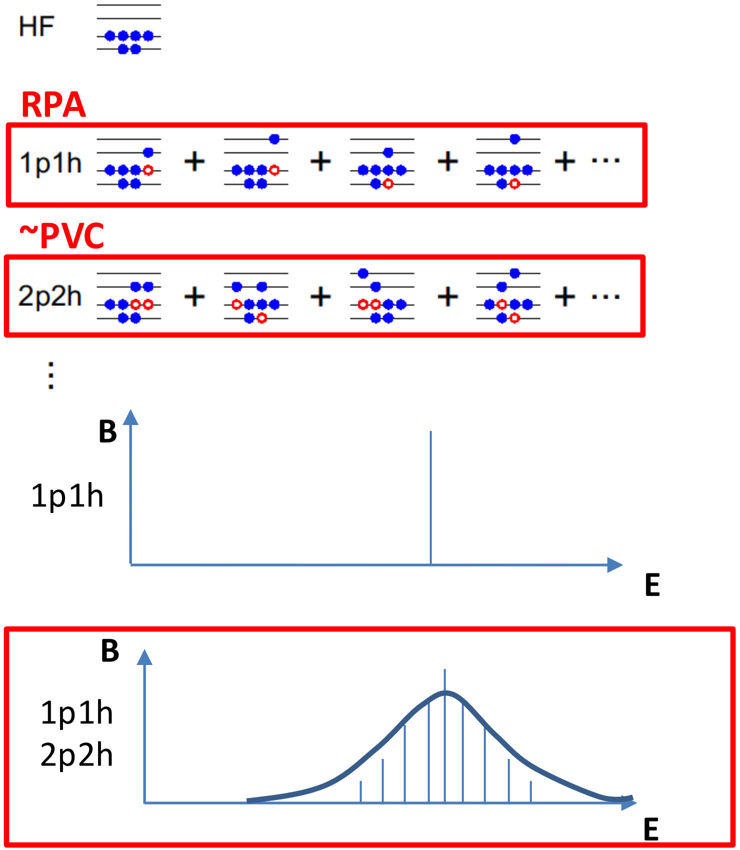
✓ **Develop a spreading width**

✓ **Reproduce resonance lineshape**

Y. F. Niu, G. Colo, and E. Vigezzi, **PRC** 90, 054328 (2014)

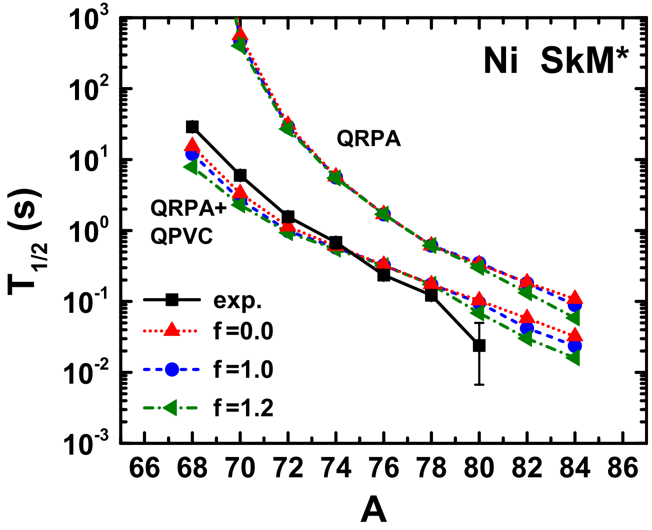
Y. F. Niu, G. Colo, and E. Vigezzi, **PRC** 94, 064328 (2016)

- **Correlations beyond RPA**



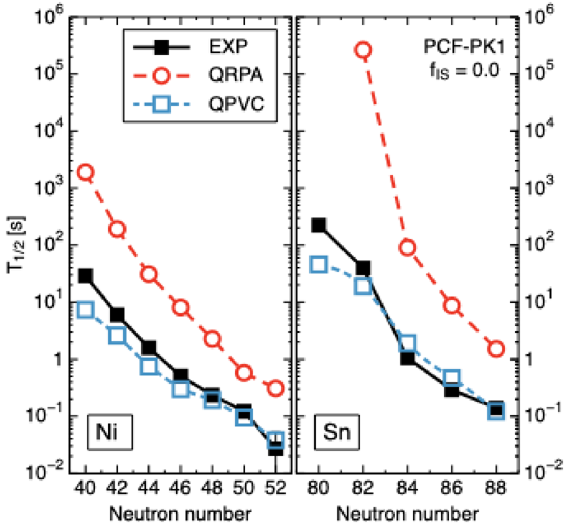
β-Decay Half-Lives in Ni and Sn isotopes

- Skyrme Quasiparticle PVC (QPVC)



- Isoscalar Pairing:
  - similar at QRPA and QRPA+QPVC level
  - not effective for Ni isotopes
  (nuclei before N=50 closed shell)

- Relativistic Quasiparticle PVC (QPVC)

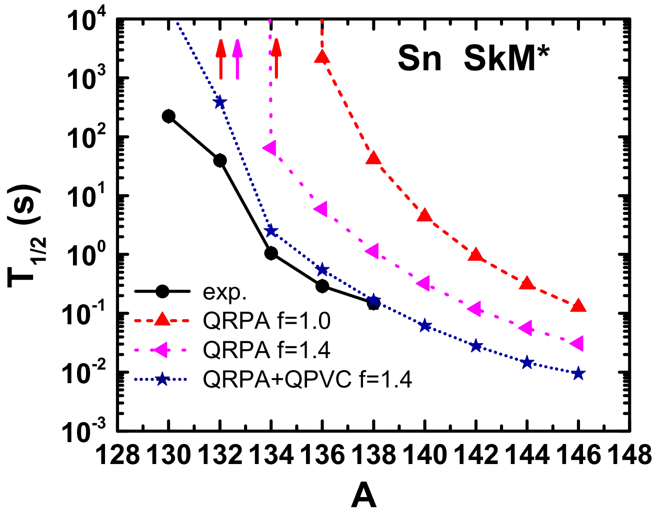


- QPVC:
  reduce the half-lives

Y. F. Niu, Z. M. Niu, G. Colo, and E. Vigezzi, **PLB** 780, 325 (2018)

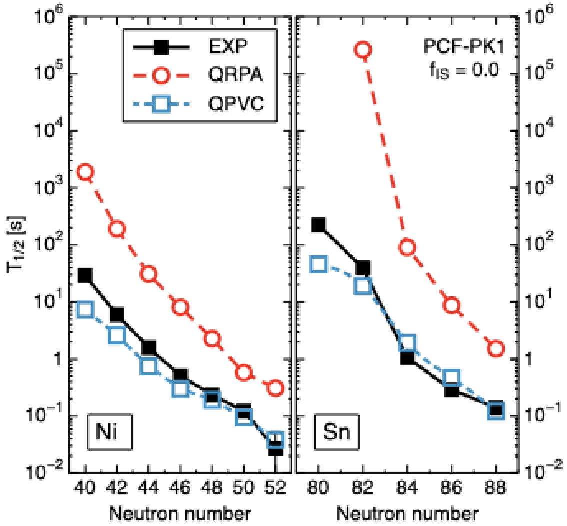
β-Decay Half-Lives in Ni and Sn isotopes

- Skyrme Quasiparticle PVC (QPVC)



- Isoscalar Pairing:
  - similar at QRPA and QRPA+QPVC level
  - effective for Sn isotopes
  (nuclei above N=82 closed shell)

- Relativistic Quasiparticle PVC (QPVC)



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Y. F. Niu, Z. M. Niu, G. Colo, and E. Vigezzi, **PLB** 780, 325 (2018)

Collective Vibrations Provide Insight to

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- *How do stars explode?*
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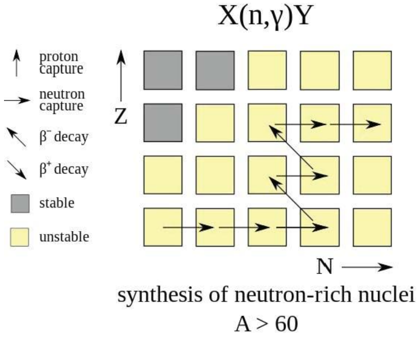
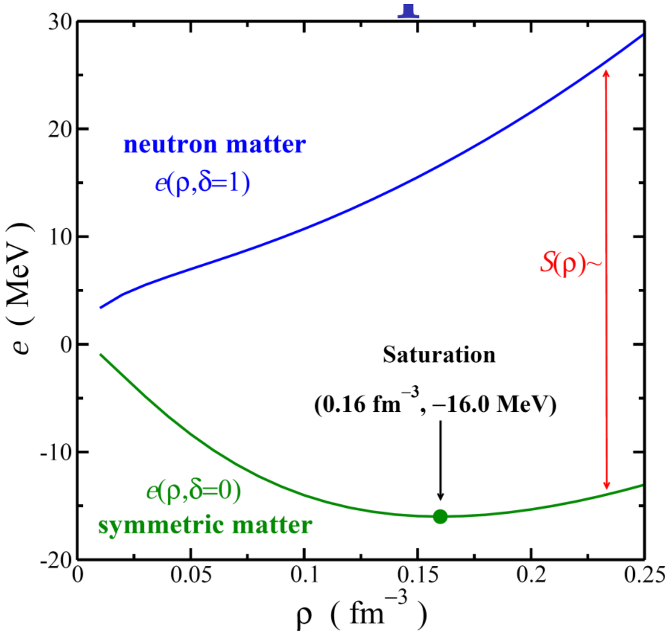


What Is the Equation of State of Nuclear Matter?

- **Nuclear equation of state (EoS)**



B. A. Li, L. W. Chen, and C. M. Ko, Phys. Rep. 464, 113 (2008)

Nuclear matter (NM) | **Symmetric NM** | **symmetry energy**

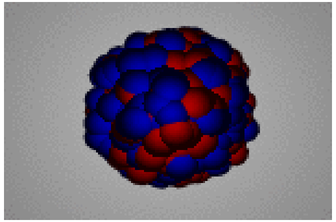
$$\frac{E}{A}(\rho,\delta) = \frac{E}{A}(\rho,\delta=0) + S(\rho)\delta^2$$

$$\delta \equiv (\rho_n - \rho_p)/\rho \qquad \xi = \frac{\rho-\rho_0}{3\rho_0}$$

$$\frac{E}{A}(\rho,\delta=0) = \epsilon_0 + \frac{1}{2}K\xi^2 + o(\xi^3)$$

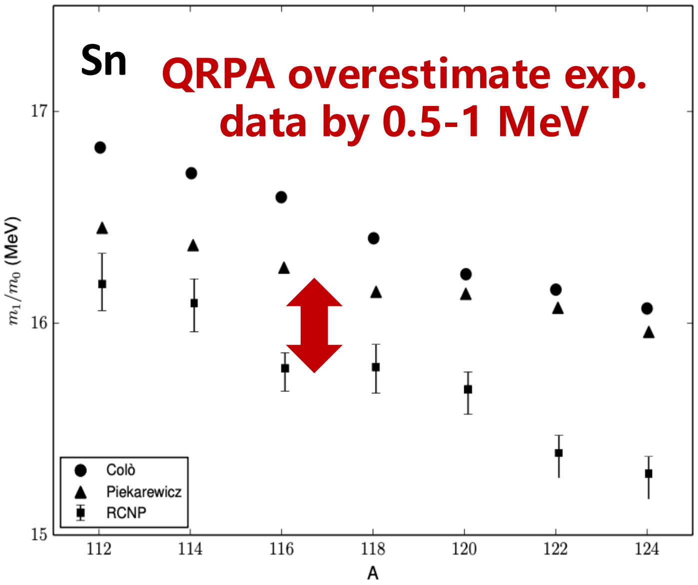
Nuclear incompressibility

Giant Monopole Resonance (GMR)



Non-charge-exchange mode

Puzzle: Why are tins so soft?



PRL 99, 162503 (2007) PHYSICAL REVIEW LETTERS week ending 19 OCTOBER 2007

Isotopic Dependence of the Giant Monopole Resonance in the Even-A $^{112-124}$Sn Isotopes and the Asymmetry Term in Nuclear Incompressibility

T. Li,[1] U. Garg,[1] Y. Liu,[1] R. Marks,[1] B. K. Nayak,[1] P. V. Madhusudhana Rao,[1] M. Fujiwara,[2] H. Hashimoto,[2] K. Kawase,[2] K. Nakanishi,[2] S. Okumura,[2] M. Yosoi,[2] M. Itoh,[3] M. Ichikawa,[3] R. Matsuo,[3] T. Terazono,[3] M. Uchida,[4] T. Kawabata,[5] H. Akimune,[6] Y. Iwao,[7] T. Murakami,[7] H. Sakaguchi,[7] S. Terashima,[7] Y. Yasuda,[7] J. Zenihiro,[7] and M. N. Harakeh[8]

The strength distributions of the giant monopole resonance (GMR) have been measured in the even-A Sn isotopes ($A = 112$–124) with inelastic scattering of 400-MeV α particles in the angular range 0°–8.5°. We find that the experimentally observed GMR energies of the Sn isotopes are lower than the values predicted by theoretical calculations that reproduce the GMR energies in ^{208}Pb and ^{90}Zr very well. From the GMR data, a value of $K_\tau = -550 \pm 100$ MeV is obtained for the asymmetry term in the nuclear incompressibility.

In Sn the ISGMR centroid energy is overestimated by about 1 MeV by the same models which reproduce the ISGMR energy well in ^{208}Pb.

Focus issue on open problems in nuclear structure theory

^{208}Pb GMR → K ~ 230 MeV
v.s.
Sn GMR → K ~ 205 MeV

J. Piekarewicz, **JPG** 37, 064038 (2010)

Garg and Colo, **PPNP** 101, 55 (2018)

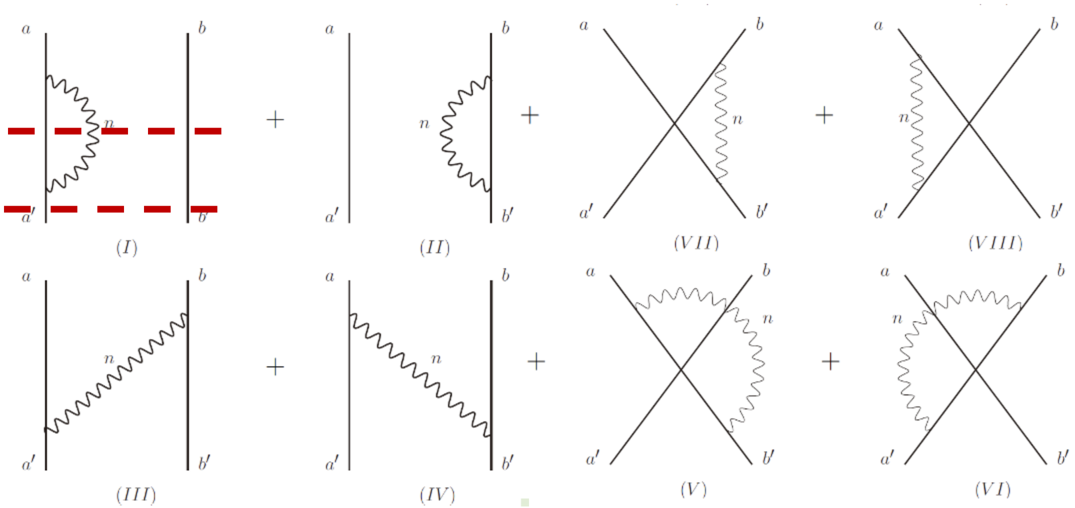
Our attempt to solve the puzzle

- Quasiparticle RPA + quasiparticle vibration coupling
  (**QRPA**) + (**QPVC**) for non-charge-exchange modes

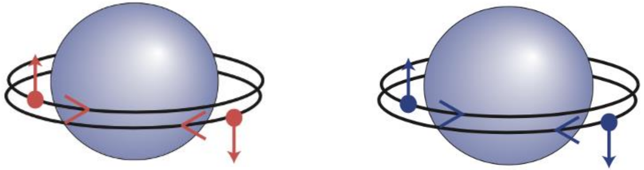
✓ QPVC effect

2 qp ⊗ phonons

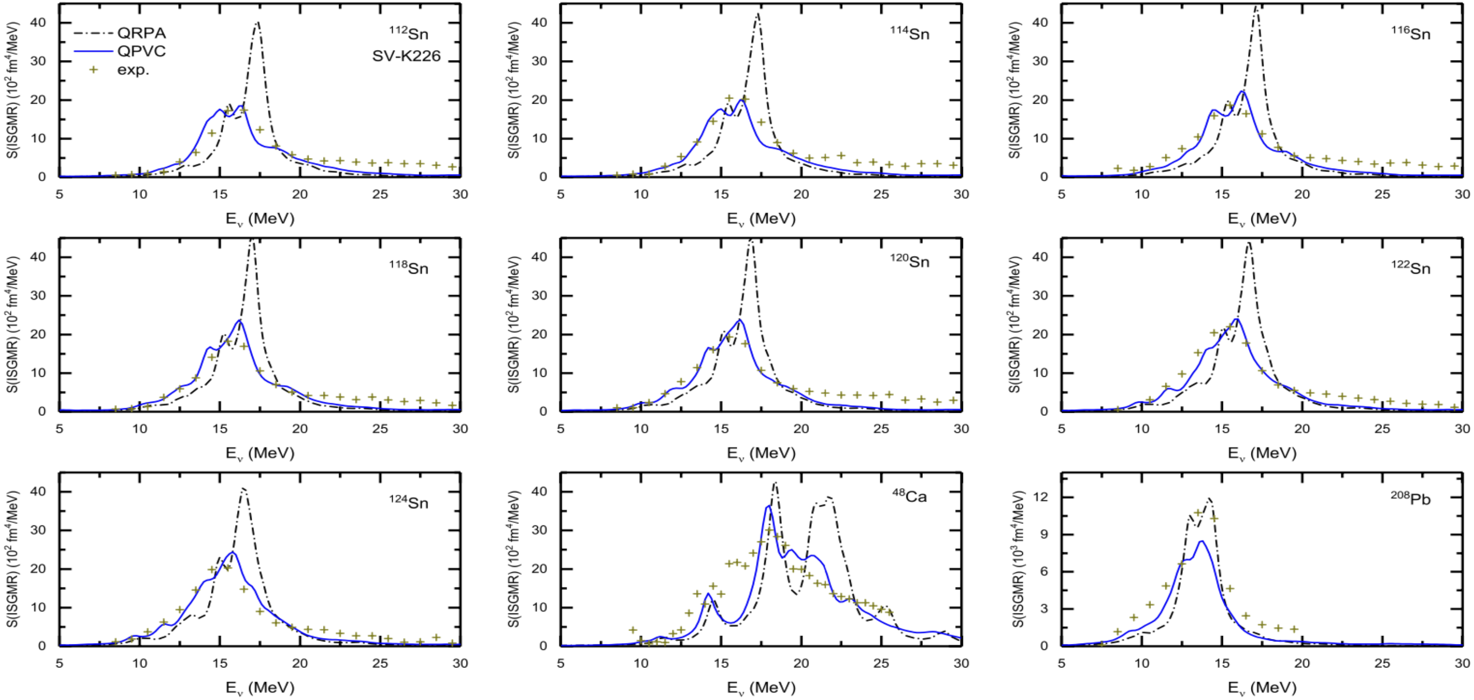
2 quasiparticles(qp)



✓ Pairing effect



Puzzle Solved: Unified description of GMR in Ca, Sn and Pb



SV-K226: $K_\infty = 226$ MeV

Z. Z. Li, Y. F. Niu, and G. Colo, Phys. Rev. Lett. **131**, 082501, 2023

Outline

- **Nuclear collective vibrations**

- **Collective vibrations provide insight to**

✓ **How were the heavy elements from iron to uranium made?**

✓ **What is the equation of state (EOS) of nuclear matter?**

- **New possibilities to explore collective vibrations**

- **Summary and Perspective**

Photo-nuclear reaction

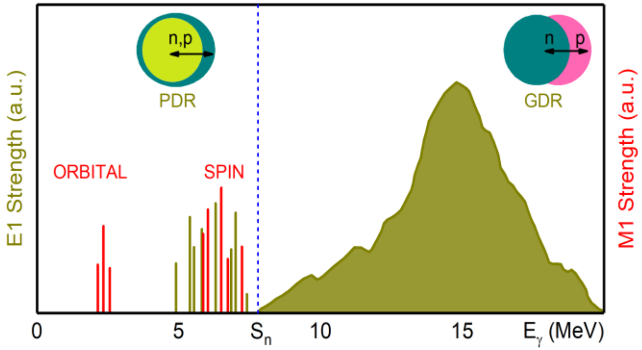
- **Advantages**

The transition strengths can be extracted in a model-independent way due to the well-known electromagnetic force

$$\sigma^{(pl)}_{\mu J}(E_\gamma) = \frac{8\pi^3 e^2}{3}\frac{(2J+1)(J+1)}{J((2J+1)!!)^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2J-1} \boxed{S_{\mu J}(E_\gamma)}$$

cross section　　transition strengths

- **Main modes: Dipole**



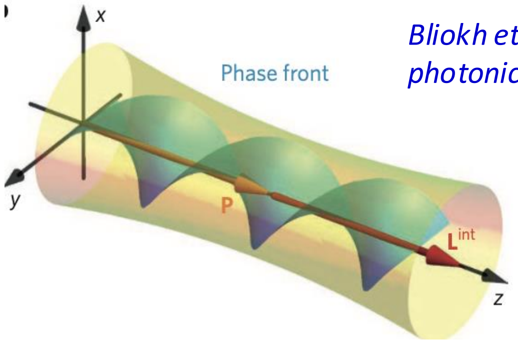
$$T_E(L+1)/T_E(L) \sim 10^{-3}$$
$$T_M(L+1)/T_M(L) \sim 10^{-3}$$
$$T_M(L)/T_E(L) \sim 10^{-3}$$

T: transition probability
E: electric
M: magnetic

Is it possible to study giant resonances with higher multipolarity by photons?

Vortex gamma photon (with Orbital Angular Momentum)

- **Coordinate space**



Bliokh et al., Nature photonics 9, 796(2015)

✓ **Vortex wavefunction**:

$$\psi\,(\boldsymbol{r},t) = u(\rho,z)e^{il\phi}e^{ik_z z}e^{-i\omega t}$$

Eigenfunctions for **OAM** operator

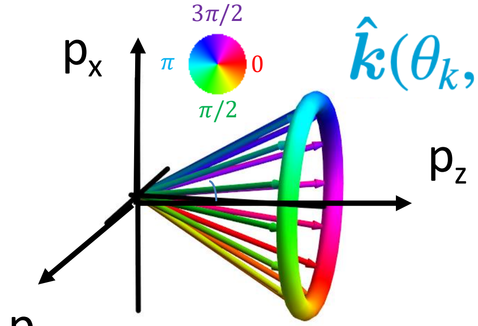
$L_z = -\frac{i\partial}{\partial\phi}$ OAM $m_l = l\hbar,\ m_\gamma = m_l + m_s$

Mode function: $u(\rho,z)$

(Bessel or LG beam modes)

Helical phase: $e^{il\phi}e^{ik_z z}$

- **Momentum space**



✓ **Superposition of plane waves**

$$\psi_{\varkappa m k_z}(\mathbf{r}) = \exp(\mathrm{i}k_z z)\int a_{\varkappa m}(\mathbf{k}_\perp)\exp(i\boldsymbol{k}_\perp\rho)\frac{d^2k_\perp}{(2\pi)^2}$$

$$= \int a_{\varkappa m}(\mathbf{k}_\perp)\exp(ikr)\frac{d^2k_\perp}{(2\pi)^2},$$

where the Fourier amplitude

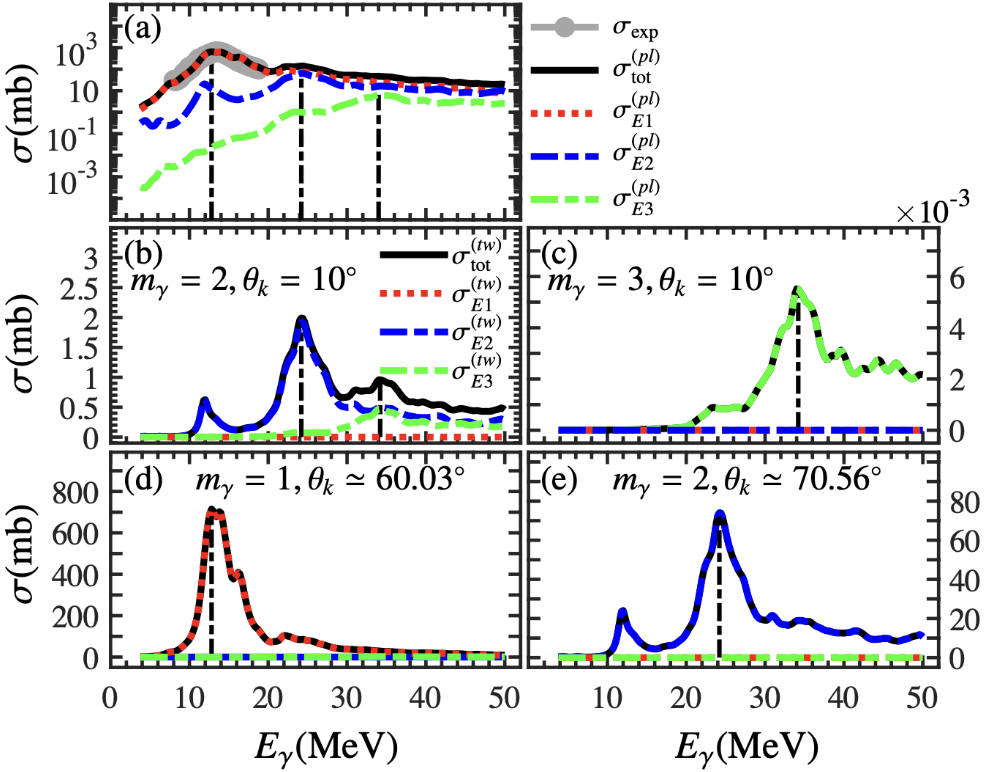
$$a_{\kappa m}(\mathbf{k}_\perp) = \mathrm{i}^{-m}\exp(im\varphi_k)\frac{2\pi}{k_\perp}\delta(k_\perp - \kappa)$$

Knyazev and Serbo, Physics uspekhi 61, 449 (2018)

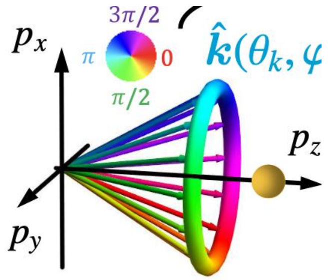
Photo-absorption cross section: plane wave vs. vortex

$\Lambda = 1, \quad b = 0$ RPA+PVC calculation SAMi-T

◆ Plane-wave photon case $M_f - M_i = \Lambda$



◆ Vortex photon case

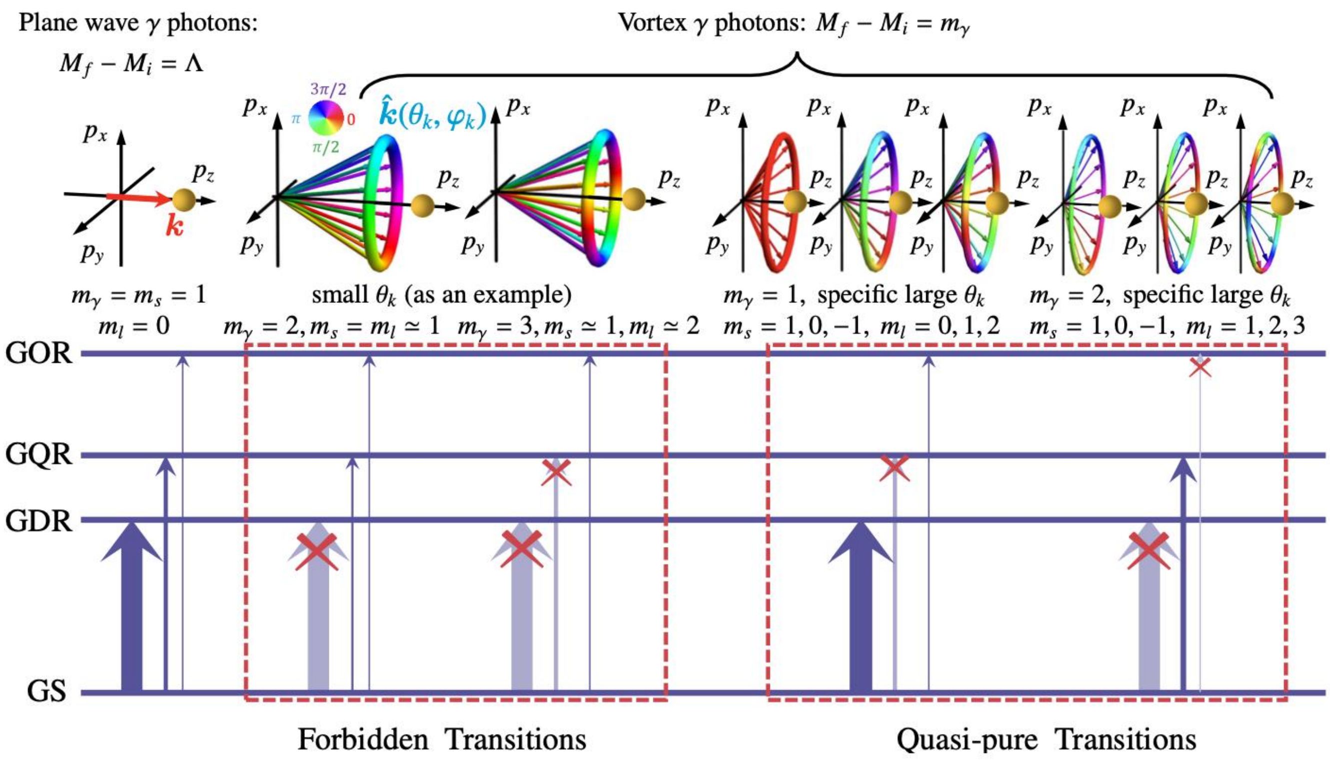


- ☐ Forbidden of $J < |m_\gamma|$ due to selection rule:

$$M_f - M_i = m_\gamma$$

- ☐ Forbidden of $J = |m_\gamma| + 1$ due to properties of Wigner d-function $d^{J_f}_{m_\gamma \Lambda}(\theta_k)$ which vanishes at specific angle.

Manipulation of giant resonances via vortex photon



Z. W. Lu, L. Guo, Z. Z. Li, M. Ababekri, F. Q. Chen, C. B. Fu, C. Lv, R. R. Xu, X. J. Kong, Y. F. Niu, and J. X. Li, PRL 131, 202502 (2023)

Summary and Perspectives

Summary

- Fully self-consistent QRPA+QPVC based on Skyrme density functional for charge-exchange channel and non-charge-exchange channel are both developed

✓ Improve the beta-decay half-lives, which provides accurate inputs for r-process simulation

✓ Achieve unified description of GMR in Sn and Pb, which solves long-standing puzzle “Why is Sn so soft?”

- New possibilites to study nuclear collective vibrations with vortex photons are explored.

Perspectives

- Nuclear double beta decay
- Electron-capture in stellar environment
- Neutrino-nucleus scattering

Acknowledgement

Collaborators:

LZU: Z. Z. Li L. Guo X. L. Zhi F. Q. Chen PKU: J. Meng Anhui Uni.: Z. M. Niu

Aizu Univ. and RIKEN: H. Sagawa Milan Univ. : G. Colo, E. Vigezzi

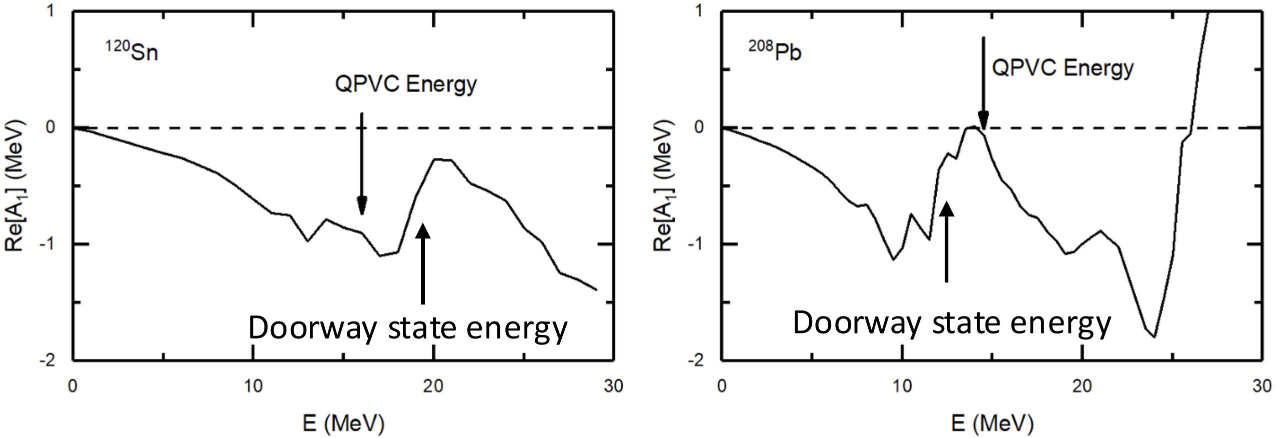
Xian Jiaotong Uni.: Z. W. Lu, M. Ababekri, J. X. Li

CIAE: R. R. Xu, C. Lv Fudan Uni.: C. B. Fu, X. J. Kong

Thank you!

Mechanism: Self-energy

- Real part of self-energy for ^{120}Sn and ^{208}Pb: determines the energy shift

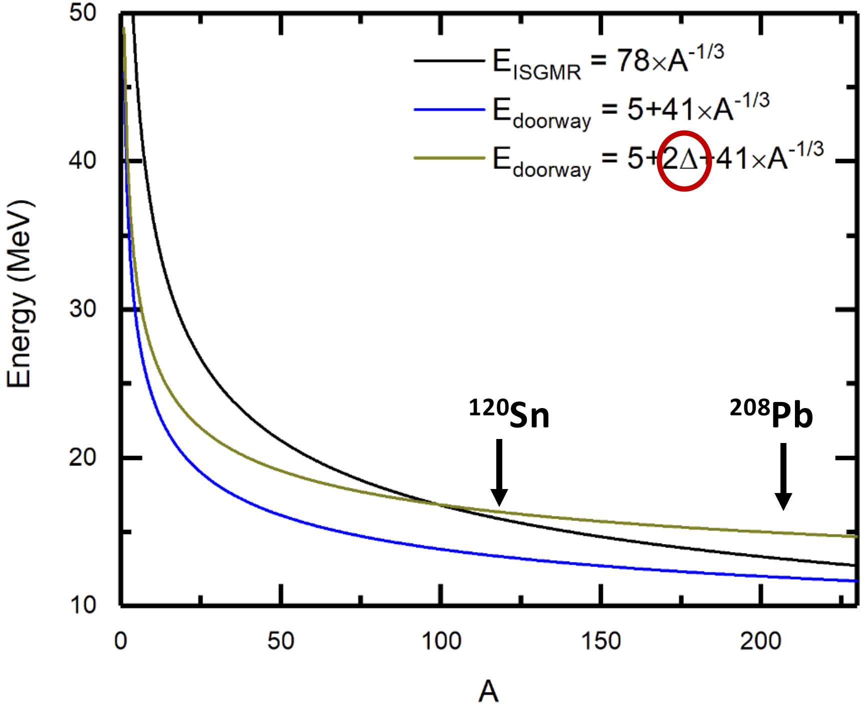


Self-energy: $$\Sigma(E) \approx \int dE' \, \frac{V^2}{E - E' + i\epsilon}$$

- E: QPVC energy of GMR peak
- E': Doorway-state energy (the energy of important doorway state 2qp⊗phonon)

^{120}Sn: QPVC energy < doorway state energy → larger self-energy
^{208}Pb: QPVC energy > doorway state energy → smaller self-energy

Mechanism: Role of pairing gap



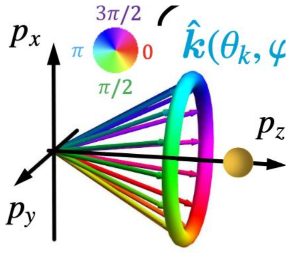
The pairing gap makes the relative energy position of GMR and doorway state different!

Cross section for vortex photon

- Nuclear photo-absorption cross section for vortex gamma photon

$$\sigma^{(tw)} = \frac{2\pi\delta(E_\gamma - E_f + E_i)}{(2J_i+1)\bar{J}_z^{(tw)}} \sum_{M_i M_f} M^{(tw)}_{M_i M_f}(\boldsymbol{b}) M^{(tw)*}_{M_i M_f}(\boldsymbol{b}).$$

✓ Transition amplitude



$$M^{(tw)}_{M_i M_f} = -\frac{1}{c}\langle J_f M_f \mid \int \hat{\mathbf{j}}(\boldsymbol{r}) \cdot \boldsymbol{A}^{(tw)}_{\varkappa m_\gamma k_z \lambda}(\boldsymbol{r}, t) d\boldsymbol{r} \mid J_i M_i \rangle$$
$$= \int \frac{d^2 k_\perp}{(2\pi)^2} \alpha_{\varkappa m_\gamma}(\boldsymbol{k}_\perp) e^{-i\boldsymbol{k}_\perp b} M^{(pl)}_{M_i M_f}(\theta_k, \varphi_k),$$

By rotating the nucleus from the propagation axis to the ***k*** direction (plane-wave component)

$$M^{(pl)}_{M_i M_f}(\theta_k, \varphi_k) = e^{-i(M_f - M_i)\varphi_k} \sum_{M_i' M_f'} d^{J_i}_{M_i M_i'}(\theta_k) d^{J_f}_{M_f M_f'}(\theta_k) M^{(pl)}_{M_i' M_f'}(0).$$

Finally

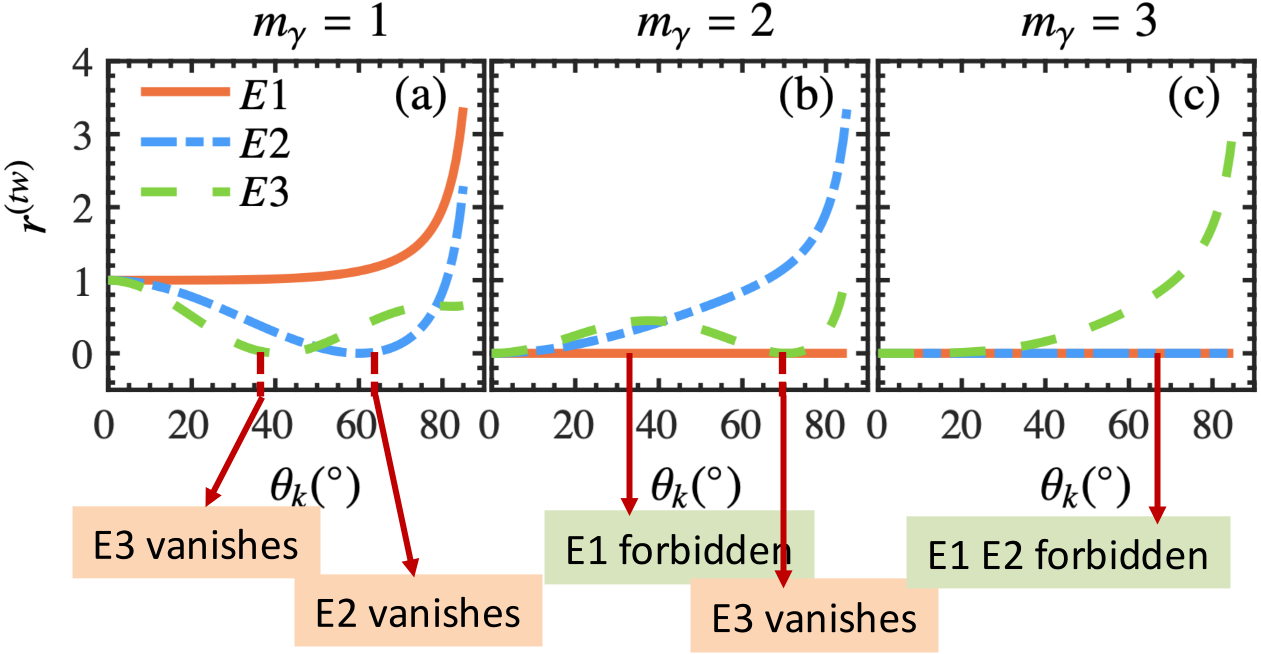
$$M^{(tw)}_{M_i M_f}(\boldsymbol{b}) = -i^{M_f - M_i - 2m_\gamma} e^{i(m_\gamma + M_i - M_f)\varphi_b} J_{m_\gamma + M_i - M_f}(\varkappa b) \sum_{M_i' M_f'} d^{J_i}_{M_i M_i'}(\theta_k) d^{J_f}_{M_f M_f'}(\theta_k) M^{(pl)}_{M_i' M_f'}(0).$$

✓ Average flux density of vortex gamma beam: $\bar{J}_z^{(tw)} = k\cos\theta_k/(2\pi)$

Ratio of the vortex and plane-wave cross section

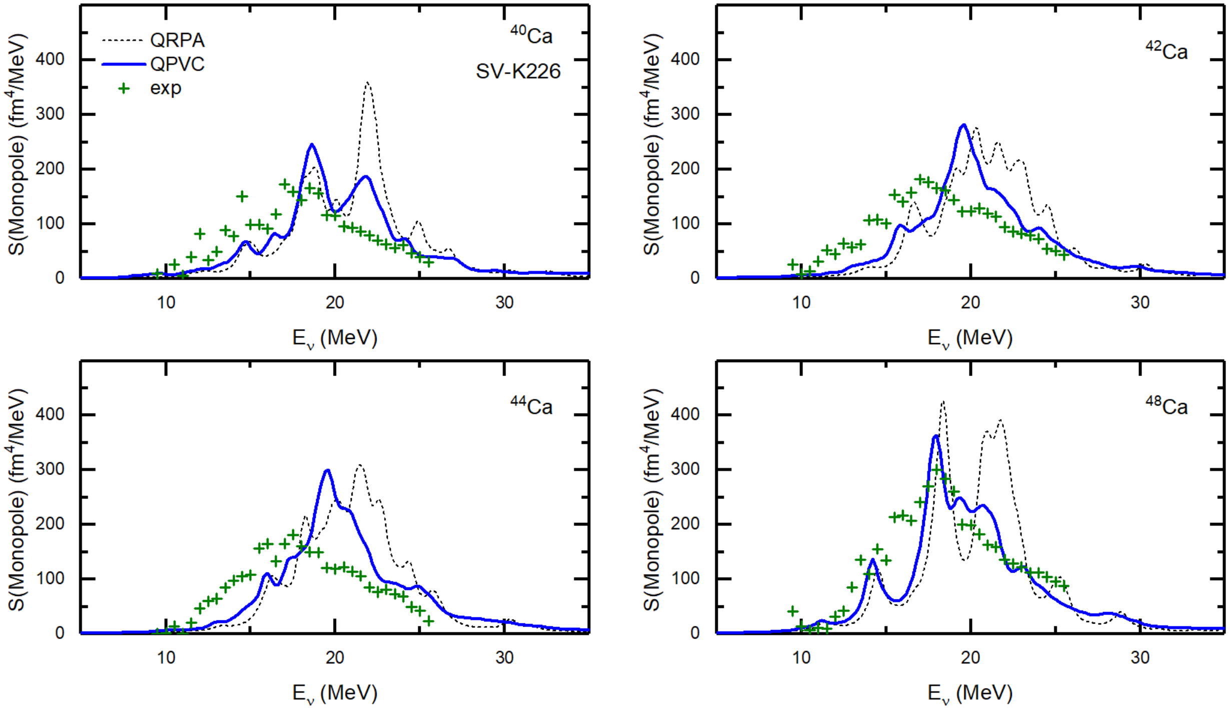
$\Lambda = 1, \quad b = 0$

polar angle $\theta_k = \arctan(\varkappa/k_z)$



- $m_\gamma = 1, \theta_k = 0$: back to the plane-wave case $r^{(tw)}=1$
- dependence on θ_k comes from Wigner d-function $d^{J_f}_{m_\gamma \Lambda}(\theta_k)$

GMR of Ca isotopes studied by QRPA+QPVC

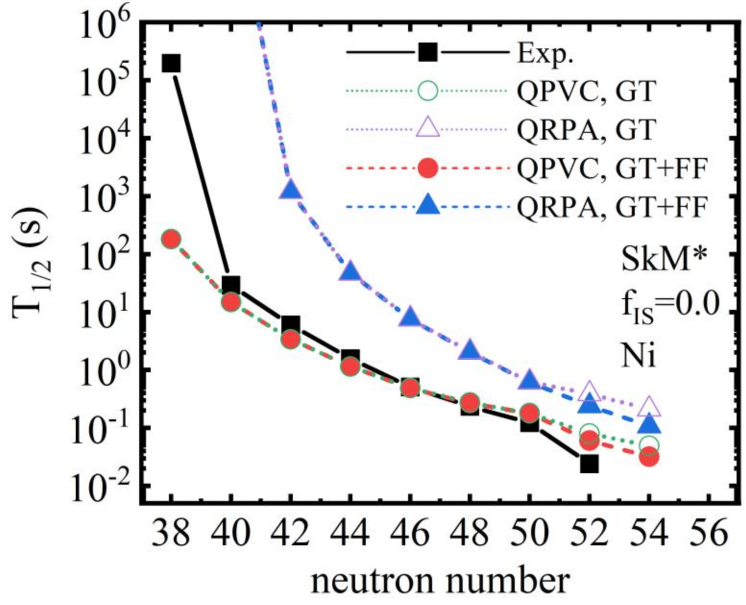


- QRPA:
  - ✓ Much overestimate GMR energies
- QPVC:
  - ✓ Much improve the strength function

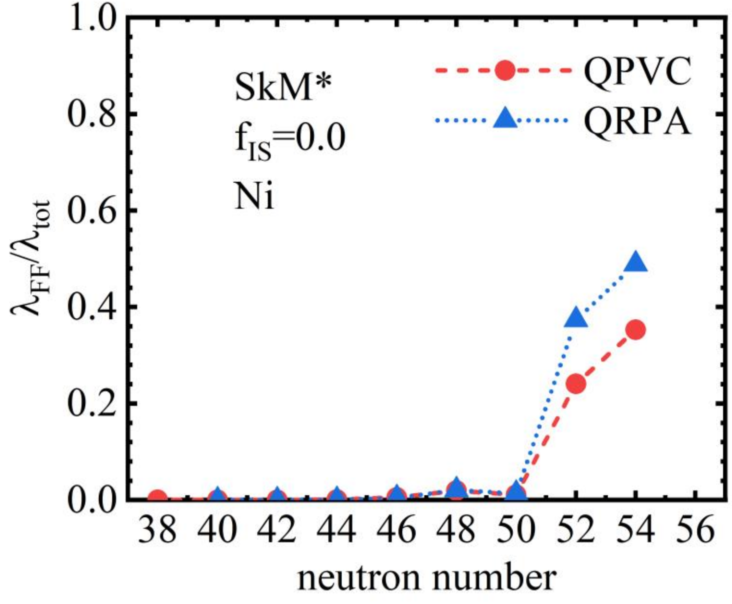
Exp.: *Olorunfunmi et al., PRC 105, 054319 (2022)*

The role of Forbidden Transitions

- Allowed transitions vs. Forbidden transitions

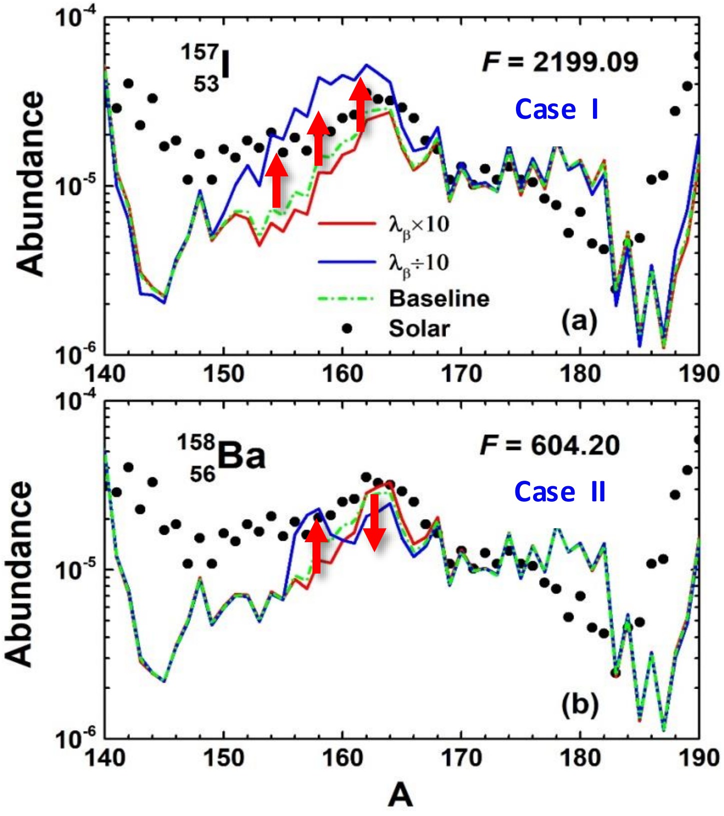


- Contribution of forbidden transitions



- Forbidden transition starts to contribute after N=50 for both QRPA and QPVC
- In QPVC, the contributions from forbidden transitions become smaller

Impact of β-decay half-life on rare-earth peak



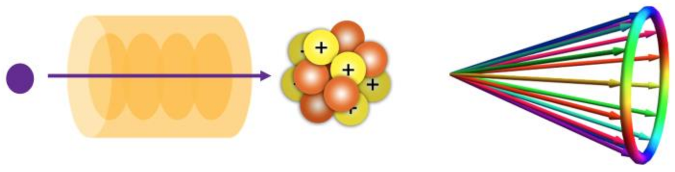
When the β-decay rate decreases:

- **Case I:** Abundance increases in the range of more than 10 mass numbers
- **Case II:** The abundance increases at mass number A of this nucleus and decreases at higher mass numbers in the rare-earth mass region

Y. W. Hao, Y. F. Niu, and Z. M. Niu, PRC 108, L062802 (2023)

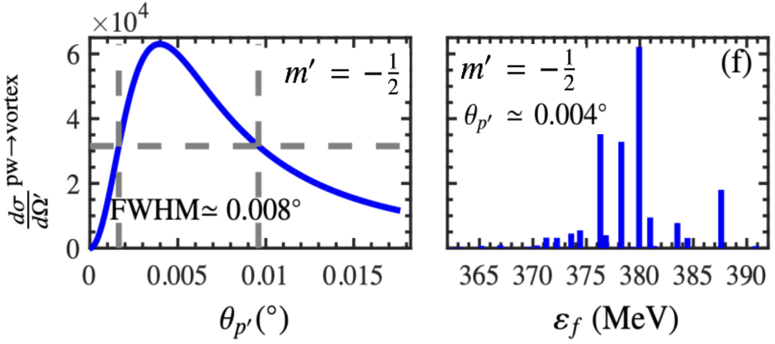
Giant resonances studied by inelastic electron scattering

- Incoming electrons: plane wave (polarized)
- Outgoing electrons: ? Vortex!



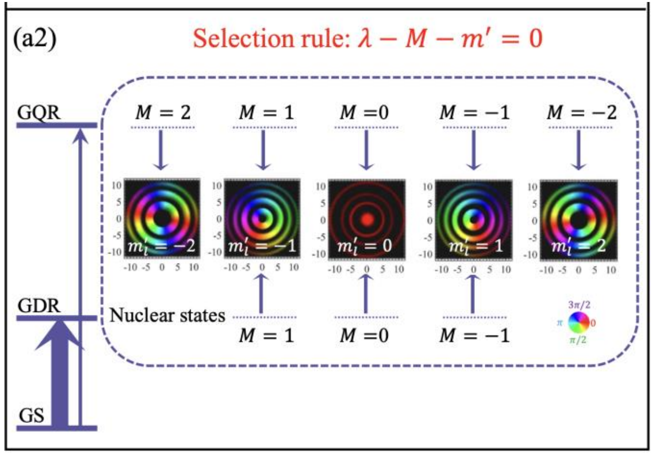
ε_i=400 MeV $\lambda = \lambda' = \frac{1}{2}$

✓ Properties of outgoing electrons



✓ Selection rule: $\lambda - M - m' = 0$

m': Angular momentum projection of outgoing electron
M: Angular momentum projection of giant resonances



By detecting m'=λ +/-2, the transition strength of GQR can be extracted model independently.

Z. W. Lu, et al., Phys. Rev. Lett. 134, 052501 (2025)