Индивидуальные тороидальные состояния в ^{58}Ni

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Introduction

- Recently, the low-lying E1 individual toroidal states (ITS) were predicted within QRPA in strongly deformed ^{24}Mg.

V. O. Nesterenko, A. Repko, J. Kvasil, and P.-G. Reinhard, Phys. Rev. C 120, 182501 (2018).

In (e,e') DALINAC experiments states in spherical ^{58}Ni with enhanced transversal form-factors were observed. M1 states?

W. Mettner, A. Richter, W. Stock, B. C. Metsch, and A.G.M. Van Hees, Nucl. Phys. A473, 160 (1987)

(γ, γ') experiments:

F. Bauwens, et al., PRC 62, (2000)   T. Shizuma, et al., PRC 109, (2024)

(p,p') experiment:

I. Brandherm, P. von Neumann-Cosel, PRC 110, (2024)

We analyze low-energy states in spherical ^{58}Ni. The results of the work have been recently published in PRL 133, 232502 (2024).

PHYSICAL REVIEW LETTERS **133,** 232502 (2024)

Editors' Suggestion   Featured in Physics

Candidate Toroidal Electric Dipole Mode in the Spherical Nucleus ^{58}Ni

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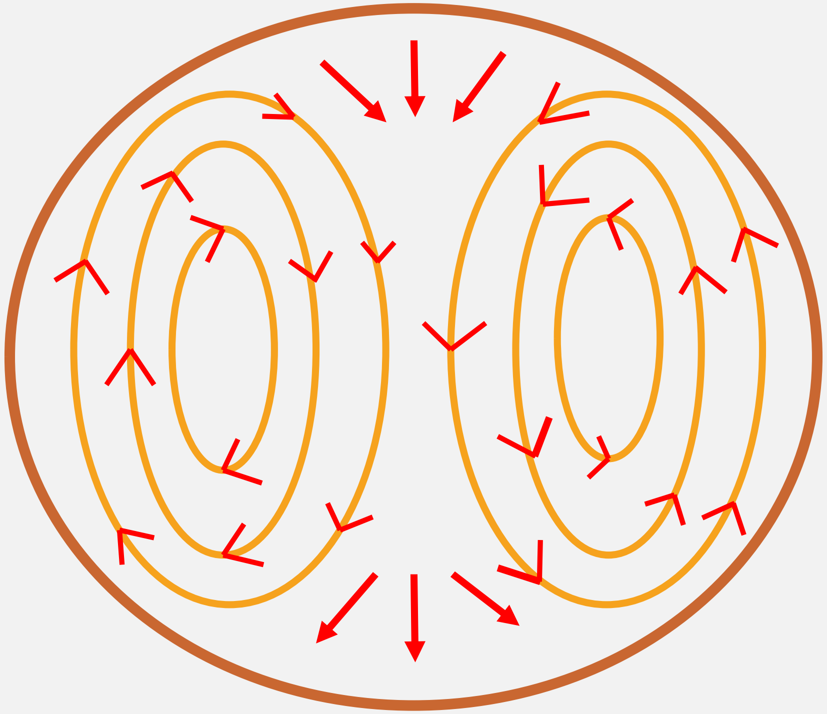
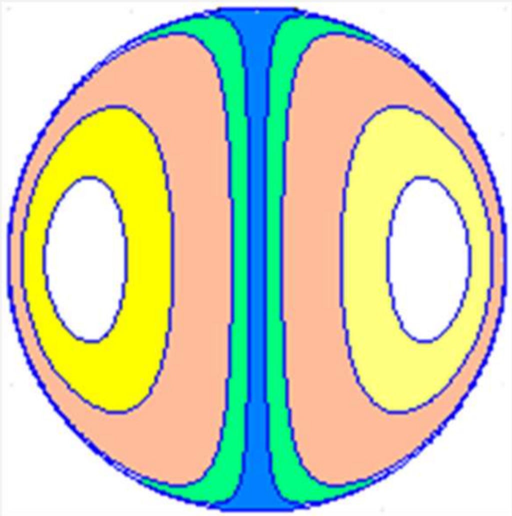
toroidal currents



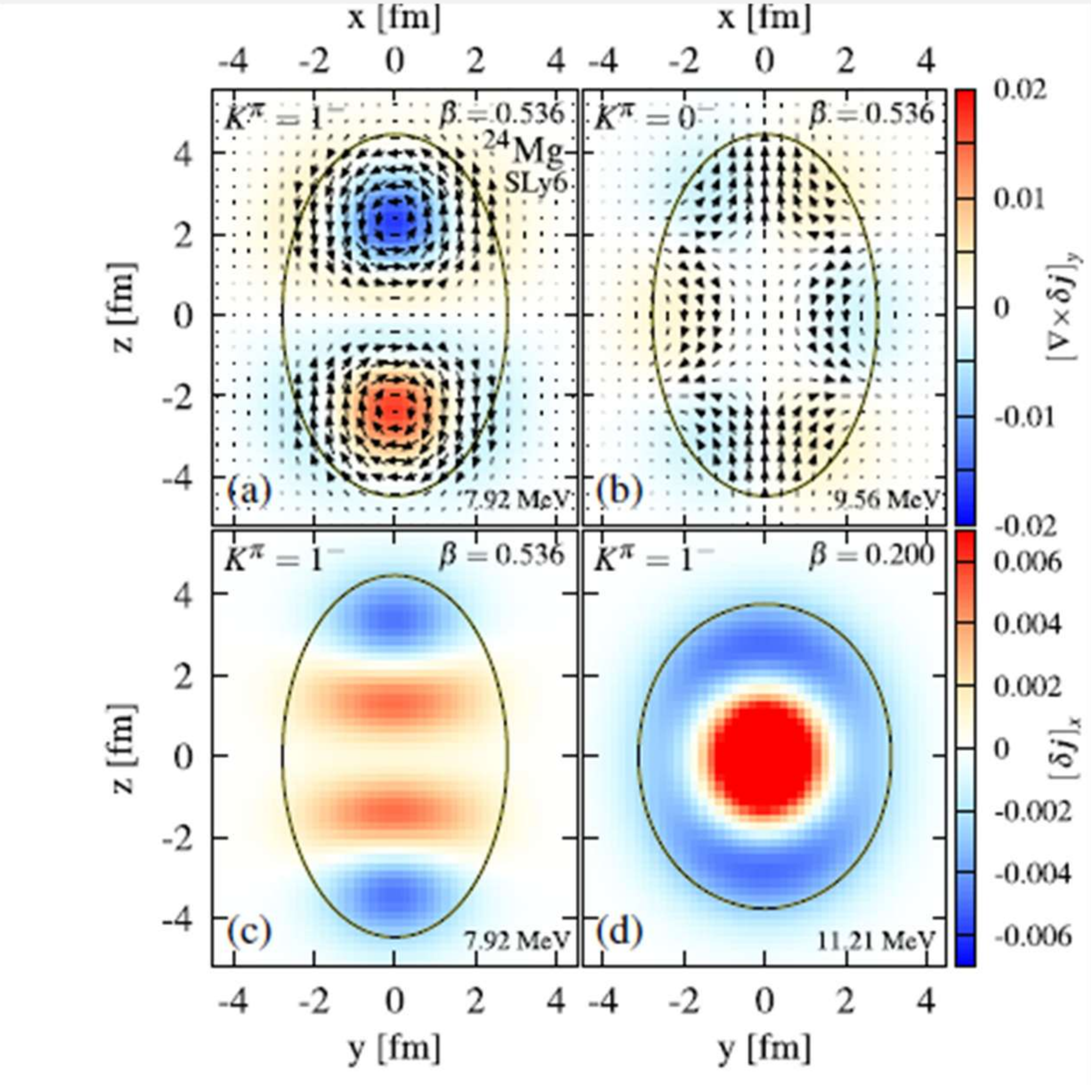
V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).
S.F. Semenko, SJNP 34 356 (1981).

Hill vortex ring



Hill showed that the incompressible Euler equations has a steady solution as a spherical vortex ring. The spherical vortex ring propagates without change of either velocity or shape.

M. J. M. Hill, PTRS 185, 213 (1894).
PhD thesis, M. M. Scase (2003)



Toroidal states in ^{24}Mg

- toroidal character CTD for 1^- state
- octupole caracter CTD for 0^- state
- two current domains for toroidal states

V. O. Nesterenko, A. Repko, J. Kvasil, and P.-G. Reinhard, PRL 120, 182501 (2018)

Model

- The Skyrme QRPA code (Repko) for spherical nuclei.

A. Repko, J. Kvasil, V. O. Nesterenko, and P.-G. Reinhard, arXiv:1510.01248[nucl-th].

- Fully self-consistent QRPA (mean field and residual interaction are derived from the initial Skyrme functional, p-p and p-h channels, residual interaction takes into account all terms from the initial functional).
- Single-particle basis includes 190 proton and 226 neutron levels.
- For example, 2qp basis includes 143 proton and 249 neutron pairs for SV-bas.
- Spurious admixture are removed.

A. Repko, J. Kvasil, and V. O. Nesterenko, PRC 99, 044307 (2019).

- 5 Skyrme forces (SV-mas10, SV-bas, SV-mas08, SkM*, SLy6)
- Volume (SkM* and SLy6) and surface (SV-mas10, SV-bas, SV-mas08) pairings.

A. Repko, J. Kvasil, V. O. Nesterenko, and P.-G. Reinhard, EPJA 53, 221 (2017).

$$V^q_{\rm pair}(\mathbf{r},\mathbf{r}') = G_q\left[1-\eta\left(\frac{\rho(\mathbf{r})}{\rho_{\rm pair}}\right)\right]\delta(\mathbf{r}-\mathbf{r}')$$

	SV-mas10	SV-bas	SV-mas08	SkM*	SLy6
m/m*	1	0.9	0.8	0.79	0.69

Formalism

Given transition probabilities:

$$B_X(E1,\nu) = \sum_{\mu=0,-\pm1} |\langle\nu|\hat{M}_X(E1\mu)|0\rangle|^2$$

where: X=IV; tor; com

Transition operators:

$$\langle\nu|\hat{M}_{IV}(E1\mu)|0\rangle = e\sum_{q=p,n} e_{eff}^{IV,q}\int d^3r r Y_{1\mu}\delta\rho_q^\nu(\mathbf{r}) \quad (2)$$

$$\langle\nu|\hat{M}_{tor}(E1\mu)|0\rangle \quad (3)$$
$$= \frac{-e}{10\sqrt{2}c}\int d^3r r[r^2+d^s]\mathbf{Y}_{11\mu}\cdot(\nabla\times\delta\mathbf{j}_c^\nu(\mathbf{r}))\,,$$

$$\langle\nu|\hat{M}_{com}(E1\mu)|0\rangle \quad (4)$$
$$= \frac{-ie}{10c}\int d^3r r[r^2+d^s]Y_{1\mu}(\nabla\cdot\delta\mathbf{j}_c^\nu(\mathbf{r})),$$

where $Y_{1\mu}$ and $Y_{11\mu}$ are spherical and vector harmonics;

$d^s = -5/3\langle r^2\rangle_0$ is the center-of-mass correction

$$\langle r^2\rangle_0 = \int d^3r r^2\, \rho_0/A$$

where ρ_0 is the ground state density.

$\delta\rho_k^\nu(\vec{r}) = \langle\nu|\hat{\rho}_k|0\rangle(\vec{r})$ is the proton and neutron transition densities

Current transition density:

$$\delta\mathbf{j}^\nu(\mathbf{r}) = \langle\nu|\hat{\mathbf{j}}|0\rangle(\mathbf{r}) = \delta\mathbf{j}_c^\nu(\mathbf{r}) + \delta\mathbf{j}_m^\nu(\mathbf{r})$$

We included only convectional part of CTD:

$$\hat{\mathbf{j}}_c^q(\mathbf{r}) = -i\frac{e_{eff}^q}{2}\sum_k[\delta(\mathbf{r}-\mathbf{r}_k)\nabla_k + \nabla_k\delta(\mathbf{r}-\mathbf{r}_k)]).$$

	e_p	e_n
IV	0.517	-0.483
IS	1	1

Formalism

PWBA:

$$\frac{d\sigma}{d\Omega}(\theta, q_{\rm eff}, E_i) = 4\pi\sigma_{\rm Mott}(\theta, E_i) f_{\rm rec}(\theta, E_i)$$
$$\times \left[\left| F^C_{E\lambda}(q_{\rm eff}) \right|^2 + \left(\frac{1}{2} + \tan^2(\frac{\theta}{2}) \right) \left| F^T_{E\lambda}(q_{\rm eff}) \right|^2 \right]$$

where:

$$\sigma_{\rm Mott}(\theta, E_i) = \left[\frac{e^2 Z}{8\pi E_i} \frac{\cos(\frac{\theta}{2})}{\sin^2(\frac{\theta}{2})} \right]^2$$

In our calculations:

$f_{\rm rec}=1$

Form factors:

$$F^C_{E\lambda}(q) = \sqrt{2\lambda+1} \int_0^\infty drr^2 \delta\rho^\nu_\lambda(r) j_\lambda(qr)$$
$$F^T_{E\lambda}(q) = \frac{1}{c} \int_0^\infty drr^2 [\sqrt{\lambda+1}\, \delta J^\nu_{\lambda,\lambda-1}(r) j_{\lambda-1}(qr) - \sqrt{\lambda}\, \delta J^\nu_{\lambda,\lambda+1}(r) j_{\lambda+1}(qr)]$$

where: $\delta\rho^\nu_k(\vec{r}) = \langle \nu | \hat{\rho}_k | 0 \rangle (\vec{r})$ is the transition density,

$\delta J^\nu_{\lambda,\lambda\pm1}(r)$ are radial components of CTD

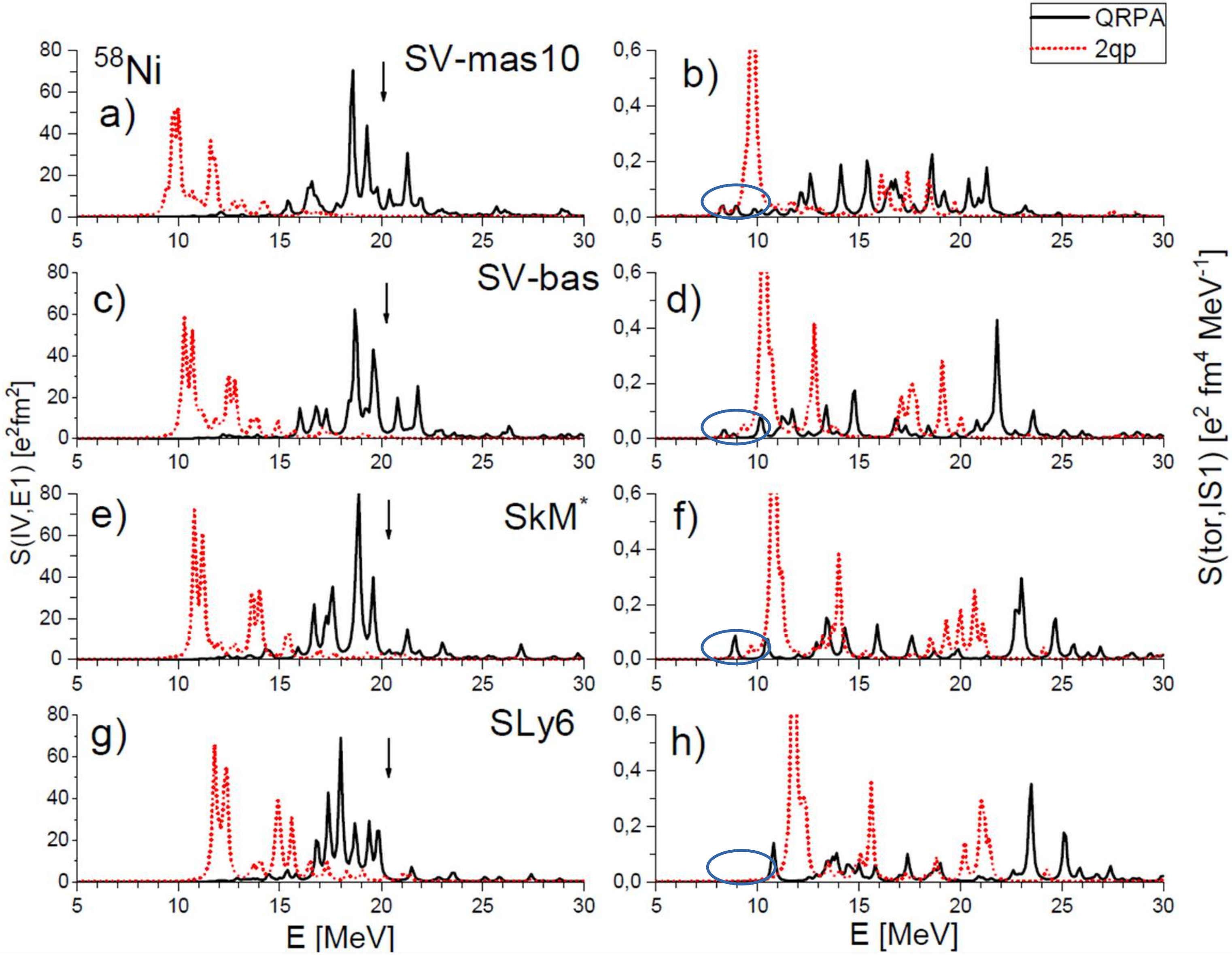
Transfer momentum:

$$q = \frac{2}{\hbar c} \sqrt{E_i E_f} \sin\left(\frac{\theta}{2}\right)$$

Here, $E_f = E_i - E_\nu$ is the final electron energy, E_ν is the nuclear excitation energy

The effective transfer momentum: $$q_{\rm eff} = q\left(1 + 1.5\frac{Z\alpha\hbar c}{E_i R}\right)$$

IV and tor IS1 strength functions



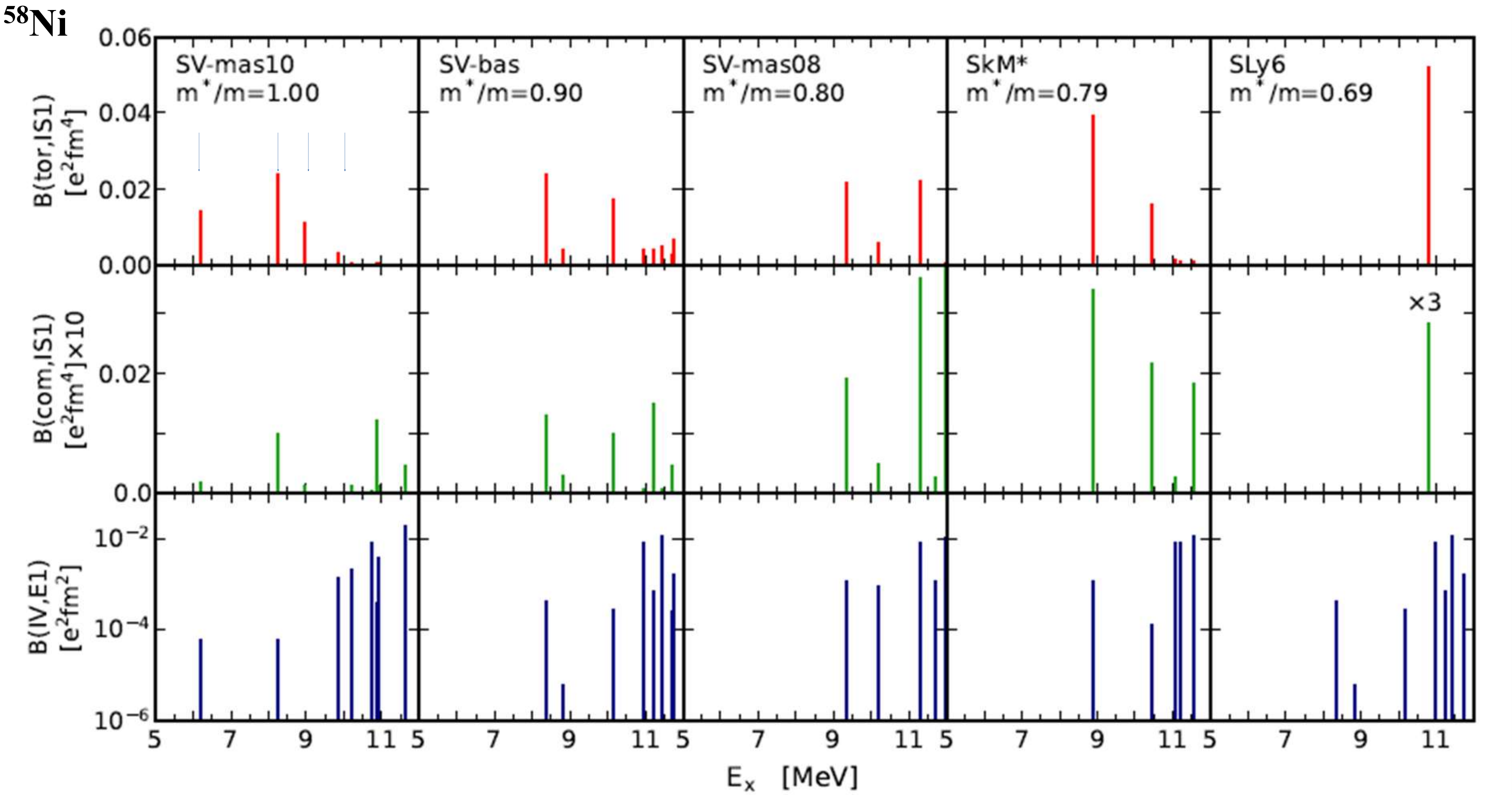
for IV:

- 2qp strength is located at 10-15 MeV;
- QRPA calculations correspond to IV GDR (16-22 MeV)

for tor IS1:

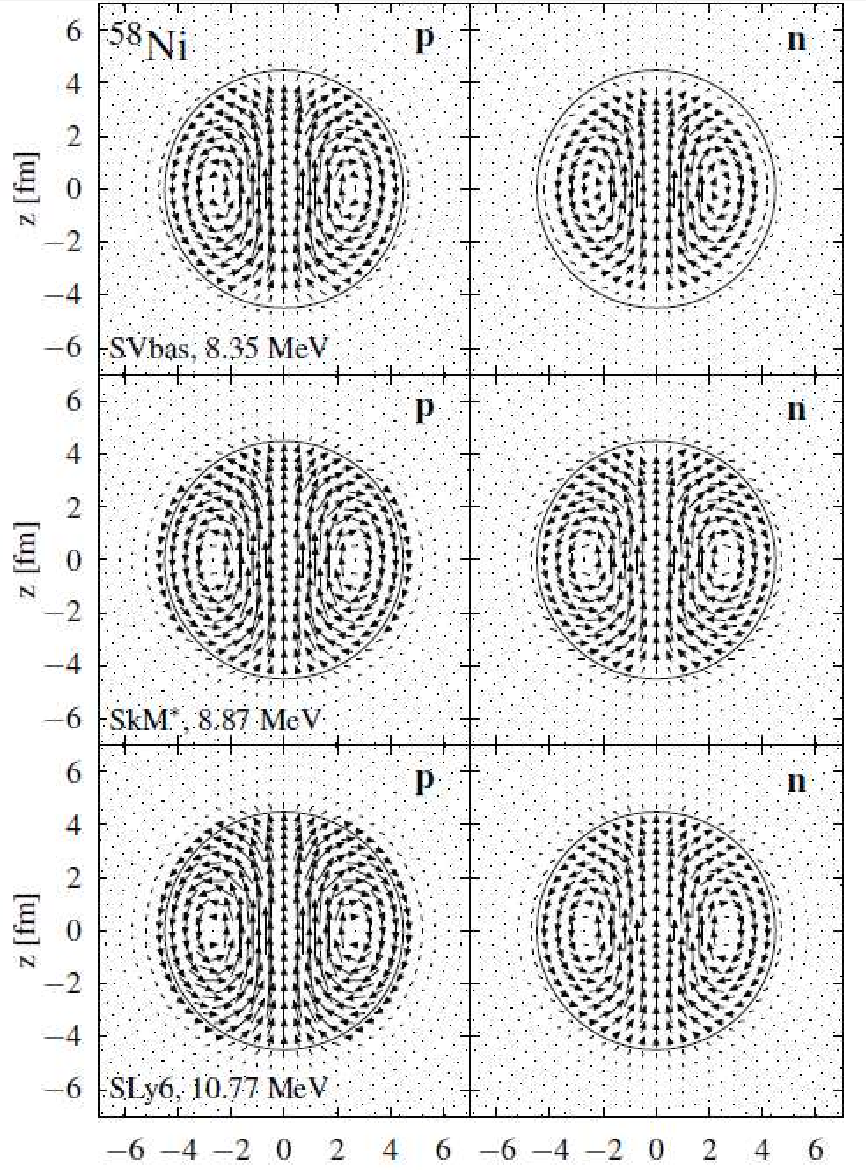
- two 2qp energy range;
- QRPA states at 6-11 MeV (our energy range!)

tor IS1, com IS1, IV E1 strength

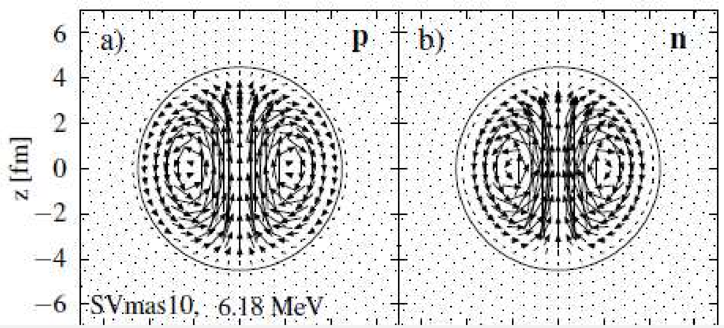


- The IS QRPA states have a toroidal character;
- Strength distribution depends on the effective mass.
- Good agreement for energies with SV-mas10

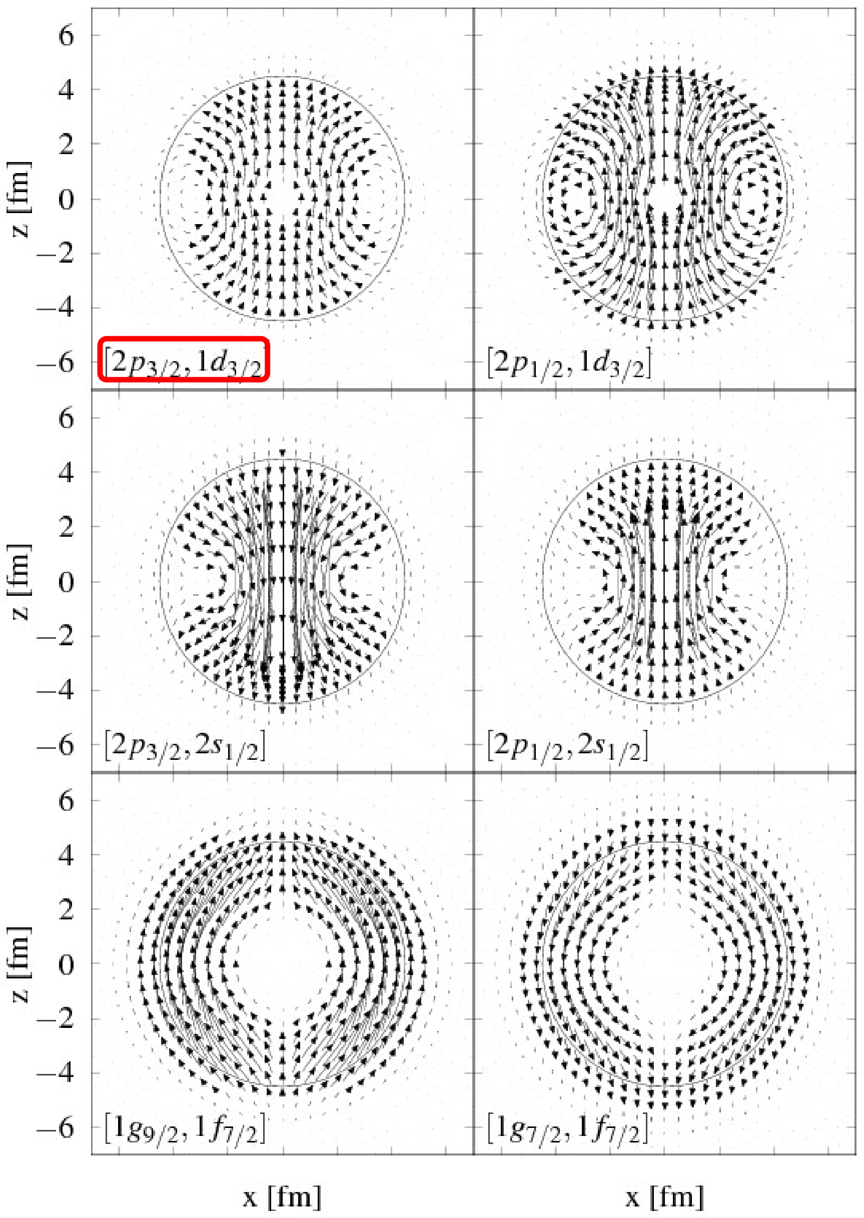
Currents: lowest dipole states



Force	E [MeV]	main 2qp components	%
SV-mas10	6.18	pp [$2p_{3/2}$,$1d_{3/2}$] pp [$2p_{1/2}$,$1s_{1/2}$]	42 11
SV-bas	8.35	pp [$2p_{3/2}$,$1d_{3/2}$] nn [$2p_{3/2}$,$1d_{3/2}$]	64 9
SkM*	8.87	pp [$2p_{3/2}$,$1d_{3/2}$] pp [$1g_{9/2}$,$1f_{7/2}$]	56 10
SLy6	10.78	pp [$2p_{3/2}$,$1d_{3/2}$] pp [$2p_{3/2}$,$2s1_{/2}$]	29 16



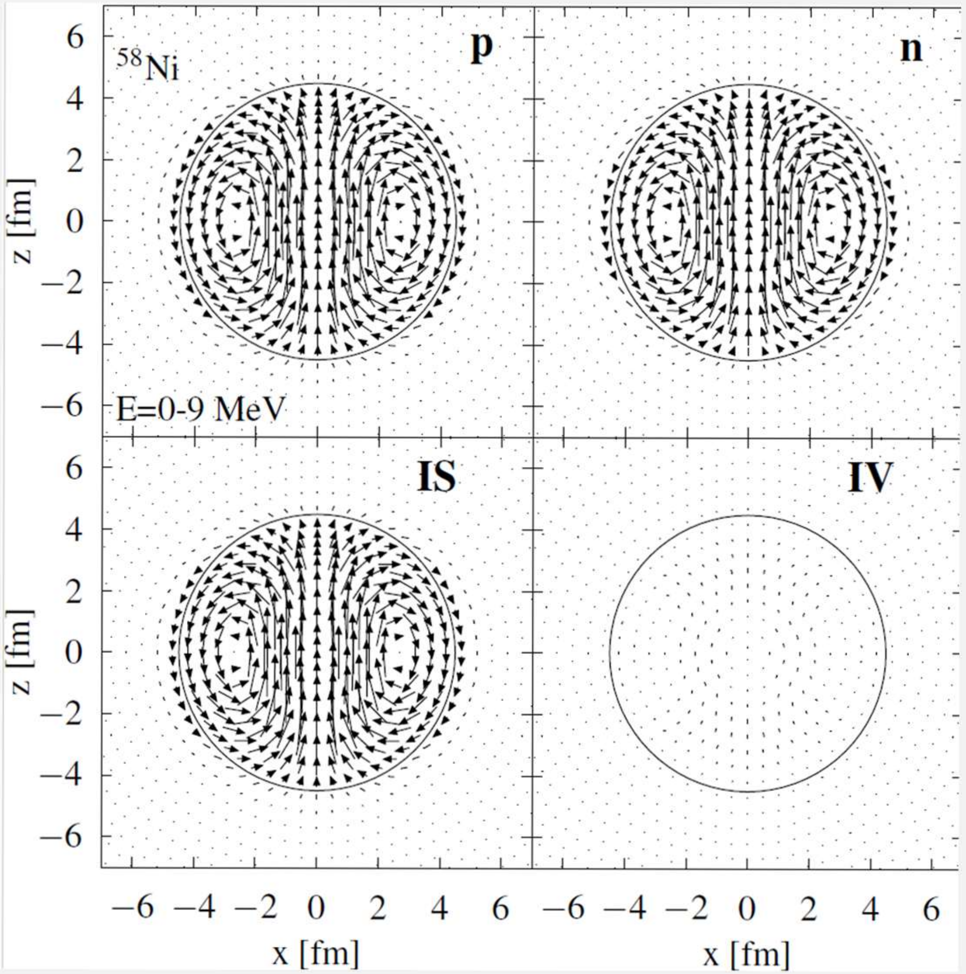
- the lowest E1 IS states have a toroidal character;
- the same basic 2qp states for all QRPA states

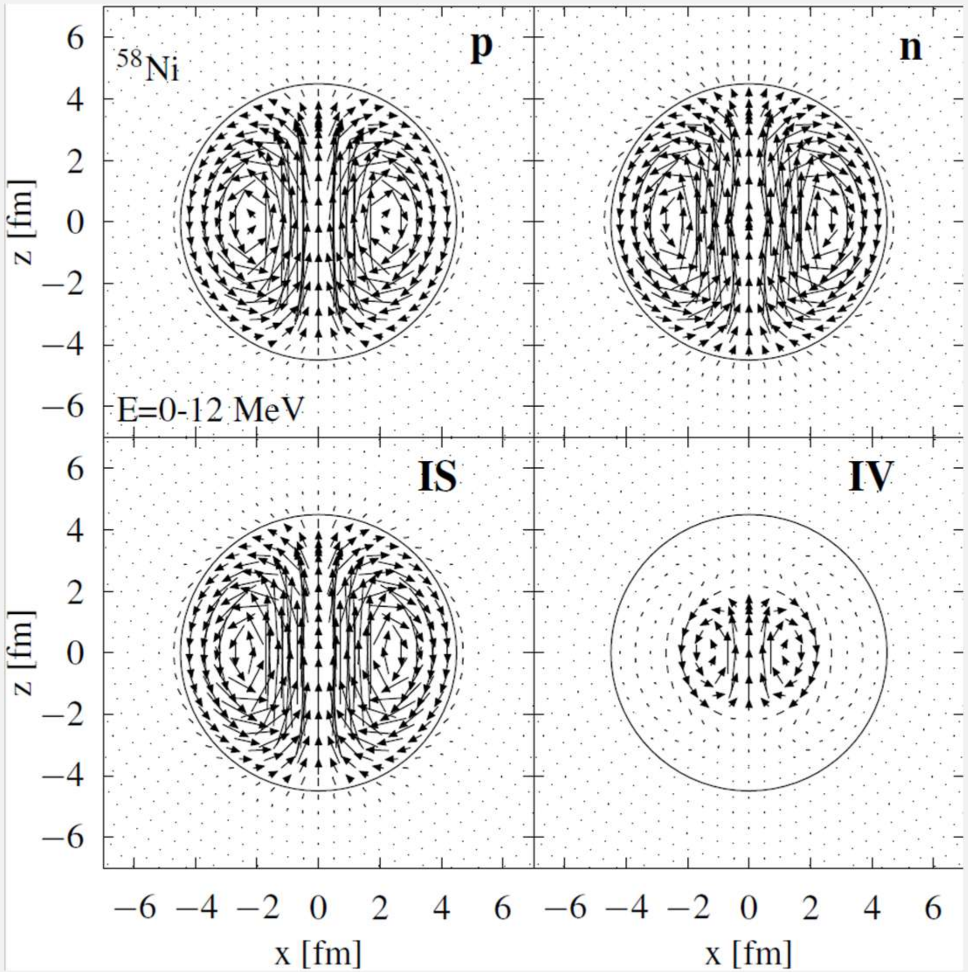


Currents: 2qp proton configurations

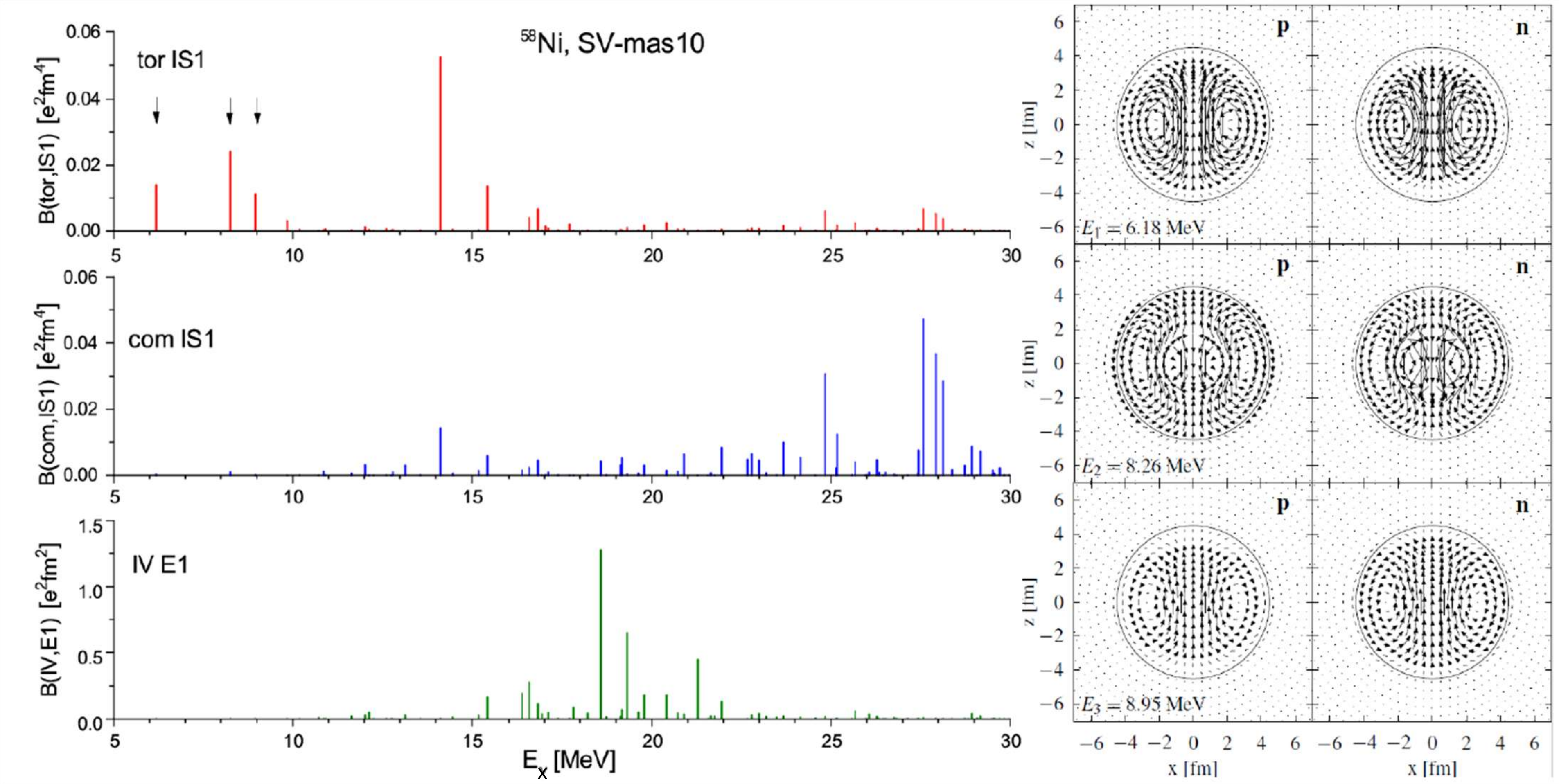
- The currents of these 2qp configurations very remind the typical toroidal flow
- the vorticity of the 2qp states leads to the vorticity of the QRPA states.
- vortical flow has a mean-field origin

SV-mas10: averaged currents



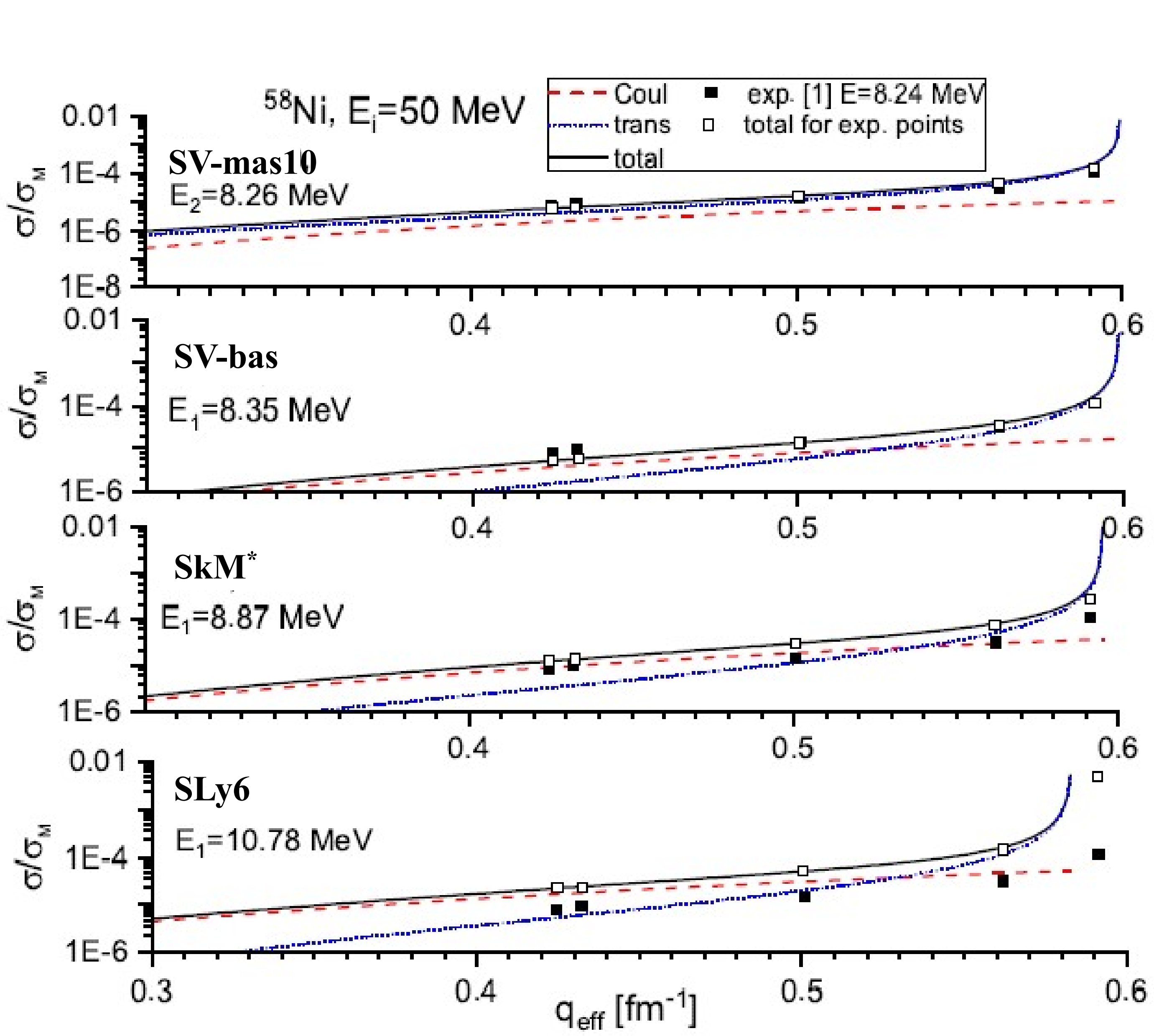


SV-mas10: B(E1) and lowest states



All three lowest states are toroidal nature.

not only the lowest state a toroidal

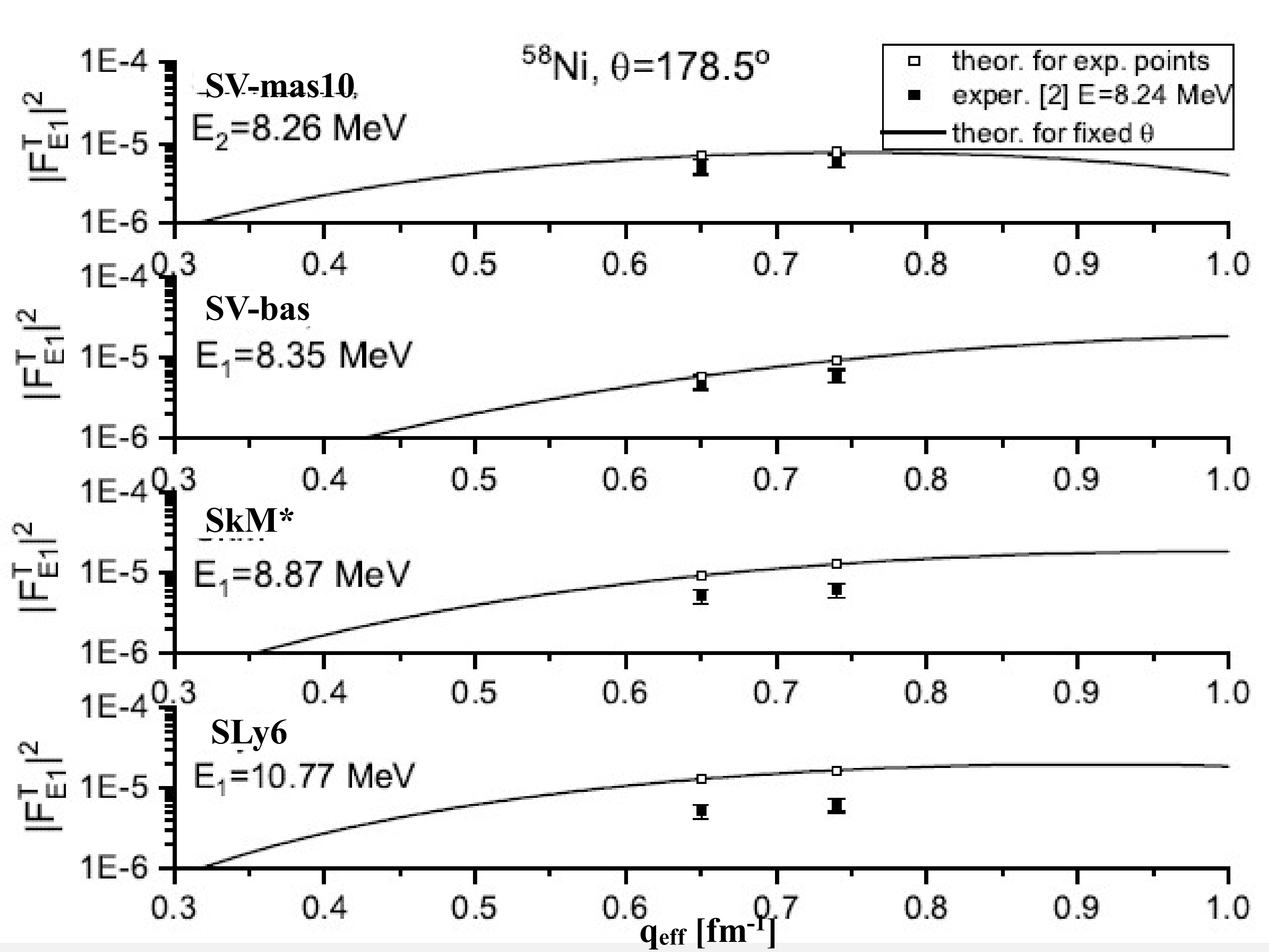


Cross-sections

E [MeV]	θ [deg]	q_eff [fm^{-1}]	σ/σ_M
49.4	92.9	0.4358	7.943E-06
50.4	92.9	0.4359	9.525E-06
49.9	116.9	0.5111	1.434E-05
50.4	140.9	0.5647	3.128E-05
50.4	164.9	0.5937	1.124E-04

W. Mettner, A. Richter et al, NPA 473, 160 (1987).

- QRPA well describes experimental data.
- The best agreement with experiment for SV-bas and SV-mas10 forces.



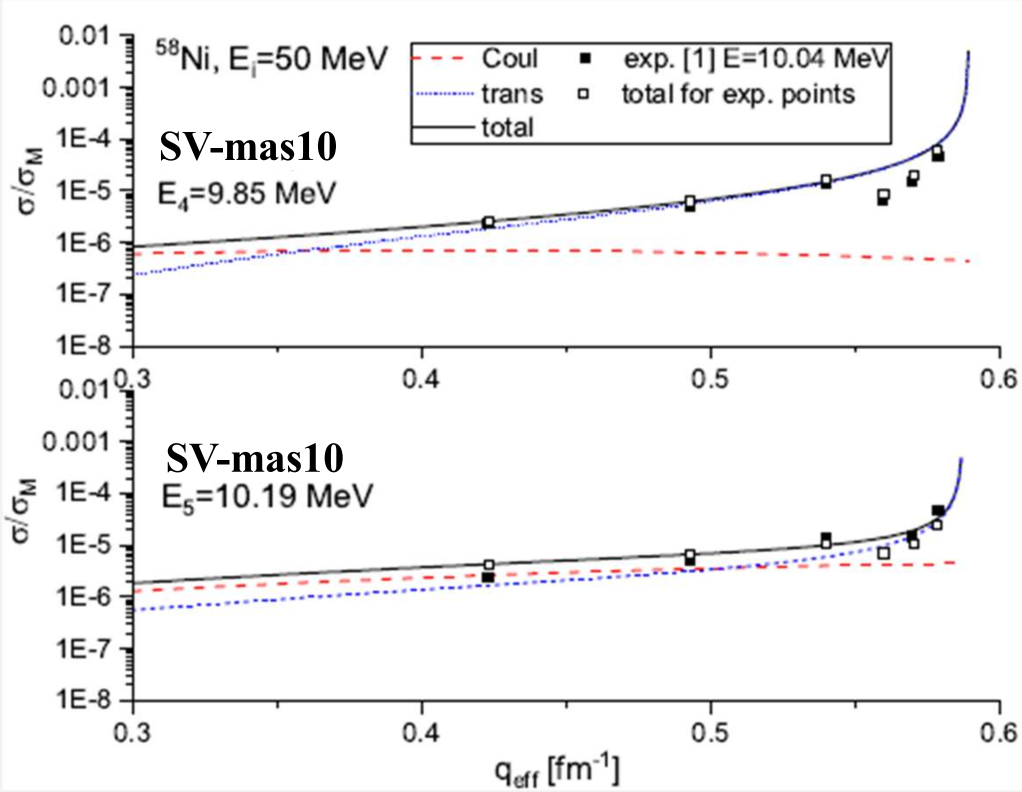
B. Reitz, (1986) private communication

θ=178.5

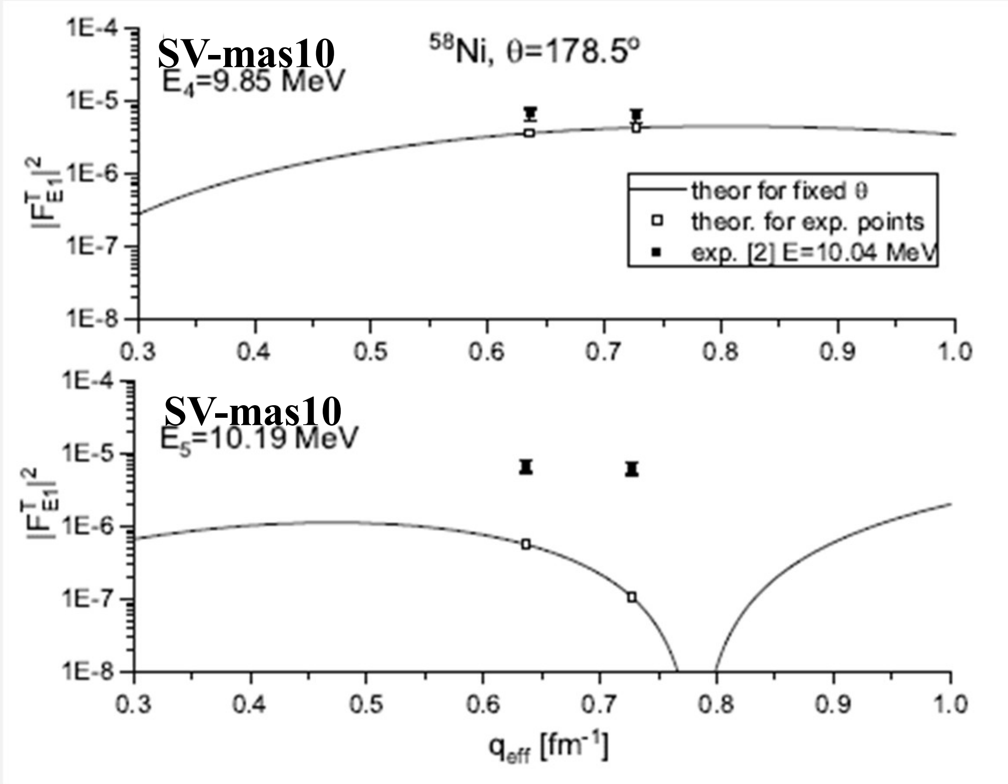
E	θ	q_eff	$\|F_T\|^2$
[MeV]	[deg]	[fm^{-1}]	
56.6	178.5	0.65	5.13E-06
65.4	178.5	0.74	6.1E-06

- QRPA well describes the experimental data of squared form factors.
- The best agreement for the SV-mas10 force.

10.04 MeV



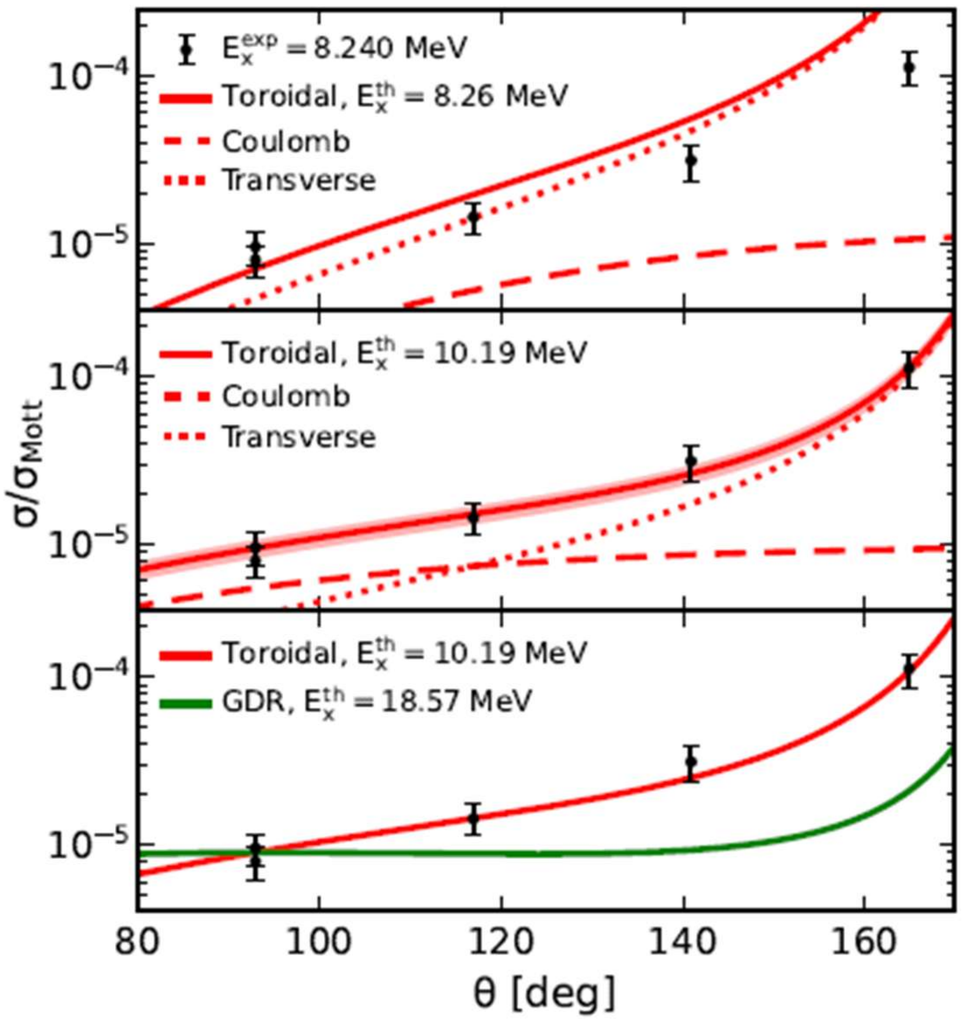
W. Mettner, A. Richter et al, NPA 473, 160 (1987).



B. Reitz, (1986) private communication

- It is reasonable to associate 10.04 MeV state with 9.85 and 10.19 MeV states obtained in calculations with SV-mas10.

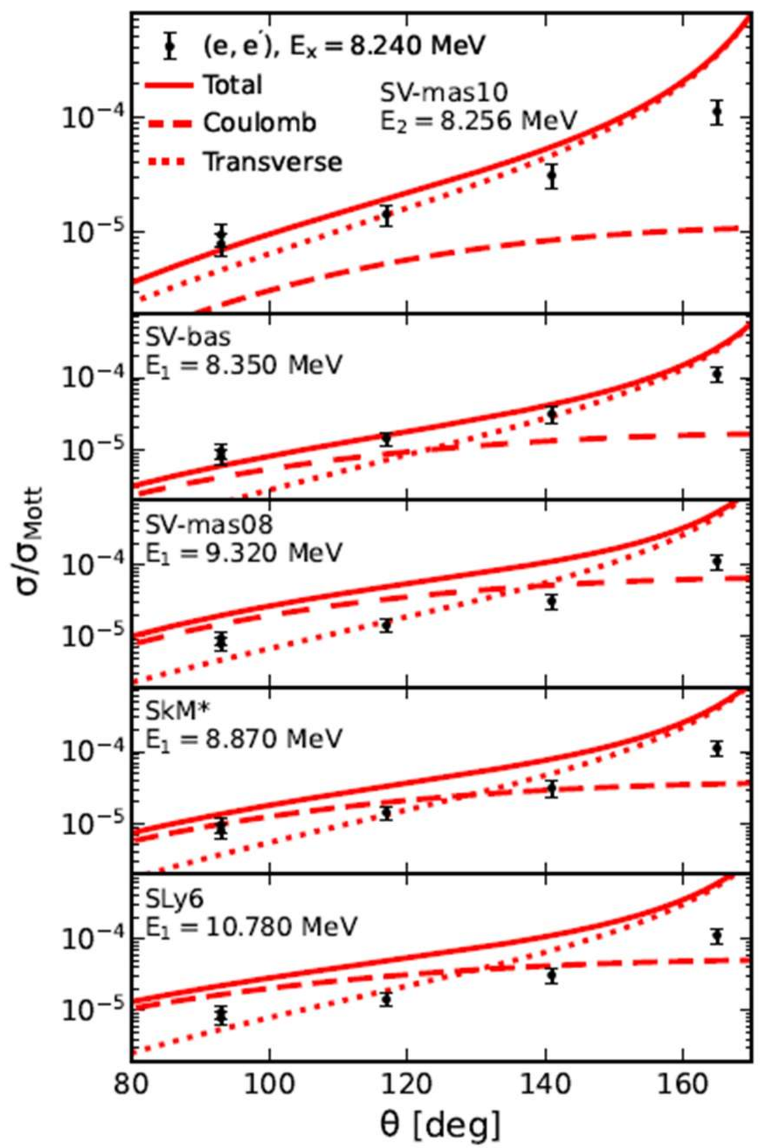
Cross section for SV-mas10



Electron scattering cross sections of the toroidal candidate at 8.240 MeV compared to QRPA predictions using the SV-mas10 interaction.

The strong slope of the cross section for toroidal state in comparison with the GDR.

The dependence of the cross sections depends at θ

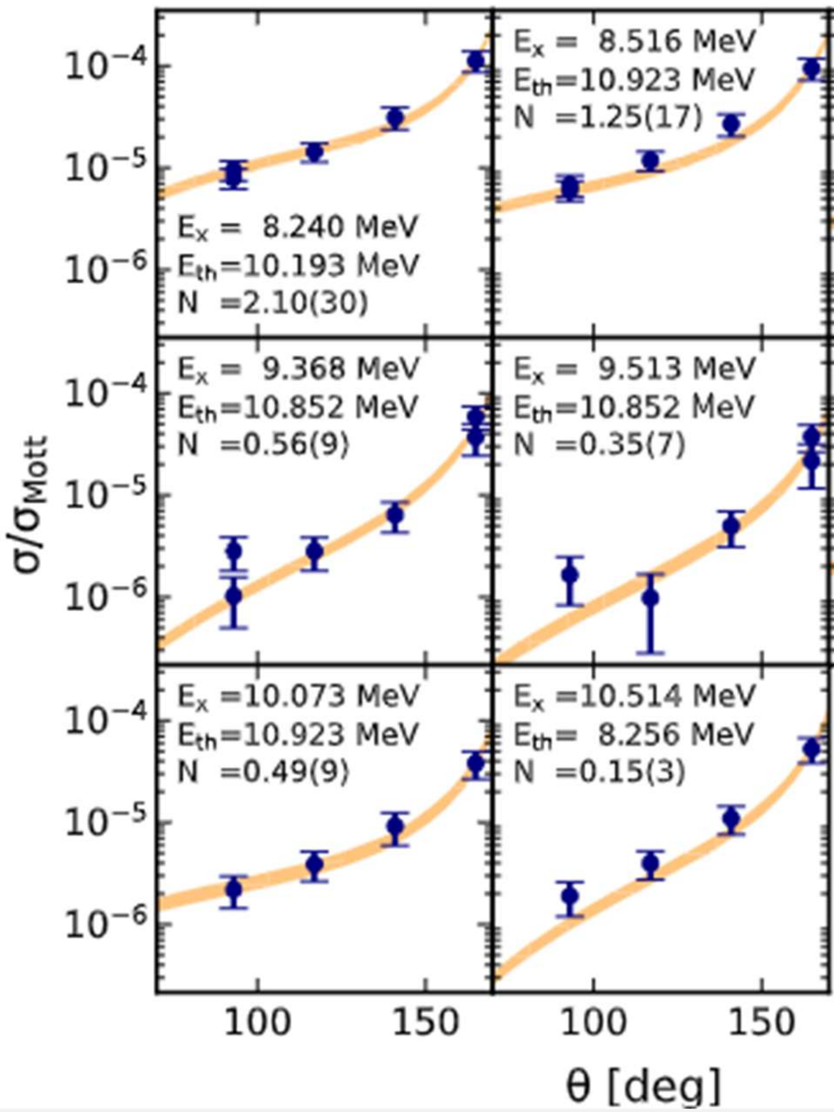


A superior agreement with experiment is obtained for SV-mas10 and SV-bas.

The description gets worse with further decreasing effective mass.

The angular dependence of the transverse part is similar for all Skyrme parametrizations.

Cross sections of all torodal candidates



For each experimental state, the theoretical one with the lowest xi^2 deviation

Conclusions

- 1^- states from 6 to 11 MeV in ^{58}Ni with strong slope of transversal form factors were analyzed as possible candidate for toroidal dipole excitations
- We have shown the toroidal nature of these states.
- The vorticity is produced by the dominant 2qp components
- This study is the **first prediction of the individual toroidal states in spherical nuclei** supported by detailed calculations and experimental data from different reaction

Thanks for your attention!

Theoretical studies:

Many publications on toroidal and compressional (ISGDR) modes:

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M.N. Harakeh et al, PRL 38, 676 (1977).
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M. Urban, PRC, 85, 034322 (2012).
…….

(e,e') data

Exp. data for 8.24 MeV

E [MeV]	θ [deg]	q_eff [fm^{-1}]	σ/σ $_M$
49.4	92.9	0.4358	7.943E-06
50.4	92.9	0.4359	9.525E-06
49.9	116.9	0.5111	1.434E-05
50.4	140.9	0.5647	3.128E-05
50.4	164.9	0.5937	1.124E-04

W. Mettner, A. Richter et al, NPA 473, 160 (1987).

E [MeV]	θ [deg]	q_eff [fm^{-1}]	$\|F_T\|^2$
56.6	178.5	0.65	5.13E-06
65.4	178.5	0.74	6.1E-06

B. Reitz, (1986) private communication

Exp. data for 10.04 MeV

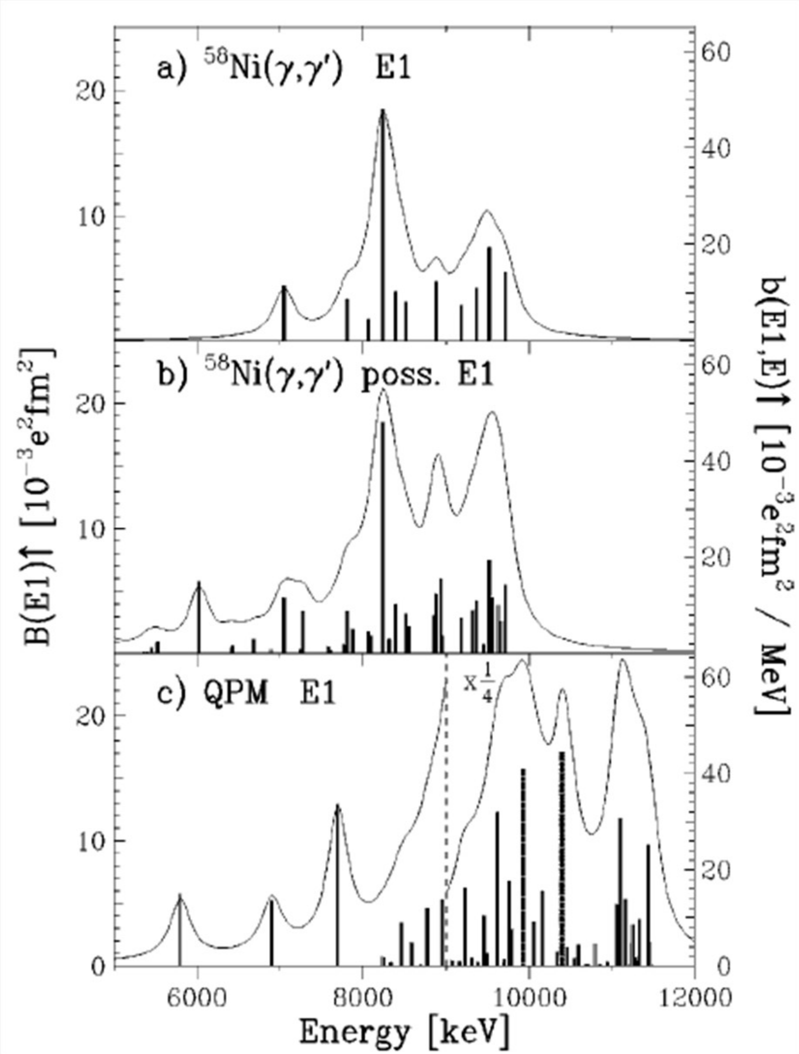
E [MeV]	θ [degrees]	q_eff [fm^{-1}]	σ/σ $_M$
50.4	92.9	0.423	2.373E-06
49.9	116.9	0.493	5.054E-06
49.4	140.9	0.540	1.374E-05
50.4	164.9	0.579	4.613E-05
52.4	140.9	0.570	1.493E-05
57.4	116.9	0.560	6.561E-06

W. Mettner, A. Richter et al, NPA 473, 160 (1987).

E [MeV]	θ [deg]	q_eff [fm^{-1}]	$\|F_T\|^2$
56.6	178.5	0.65	1.33E-06
65.4	178.5	0.74	1.17E-06

B. Reitz, (1986) private communication

B(E1) in other models



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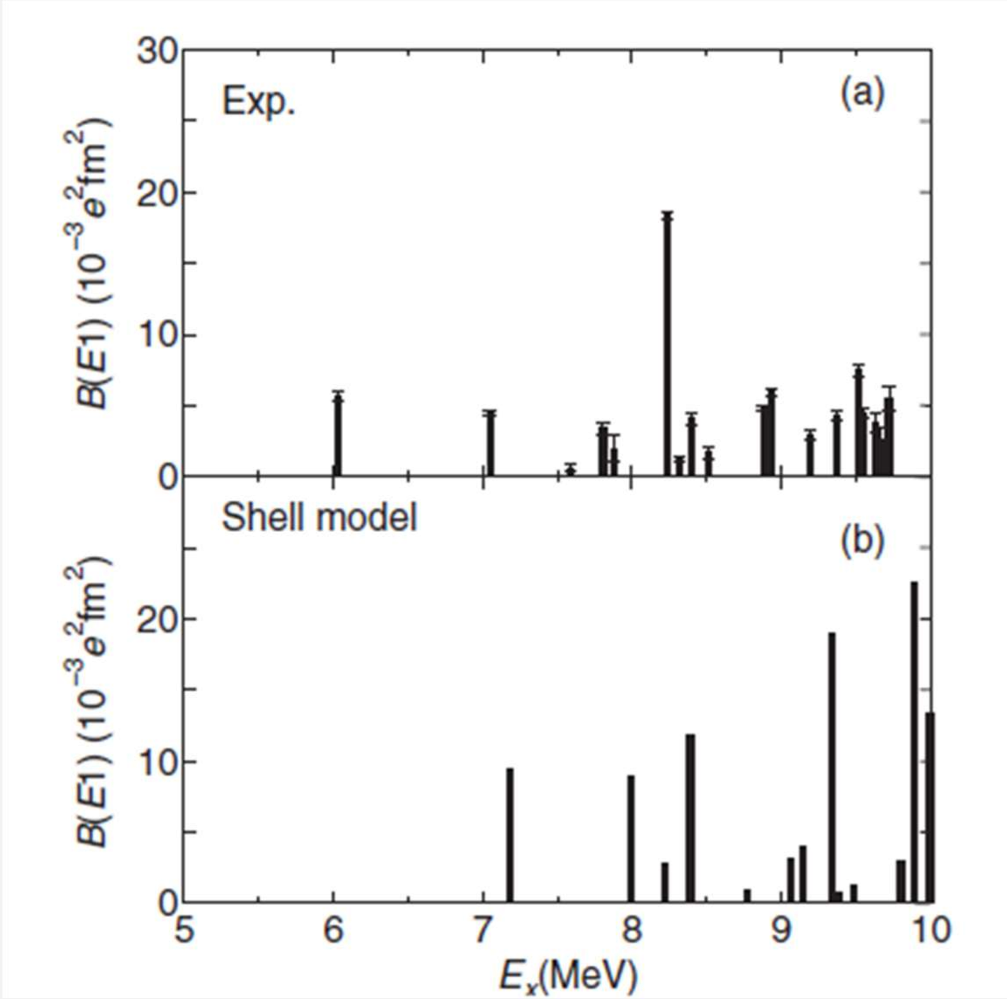

Shizuma, et al., PRC 109, (2024)

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General treatment of TDR

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TDR vs pygmy resonance

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TDR as a measure of vorticity

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,
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Deformation effects in TDR

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"Toroidal resonance: relation to pygmy mode, vortical properties and anomalous deformation splitting",
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154Sm

**V.O. Nesterenko, A. Repko, J. Kvasil, and P.-G. Reinhard,
"Individual low-energy toroidal dipole state in 24Mg",
PRL 120, 182501 (2018)**

24Mg