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Dijets with large rapidity separation at the next-to-leading BFKL for search of new physics at colliders

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Outline

- BFKL evolution vs GLAPD evolution of QCD
- Mueller-Navelet dijets
- First direct evidence of BFKL evolution in pp collisions at 2.76 TeV by CMS at LHC
- Gravity with large extra dimensions as an example of possible new physics
- Predictions
- Summary

GLAPD vs BFKL

Gribov—Lipatov—Altarelli—Parisi—Dokshitzer

Bjorken limit

$$\sqrt{s} \to \infty; p_T \to \infty; x \sim \frac{p_T}{\sqrt{s}} \sim 1$$

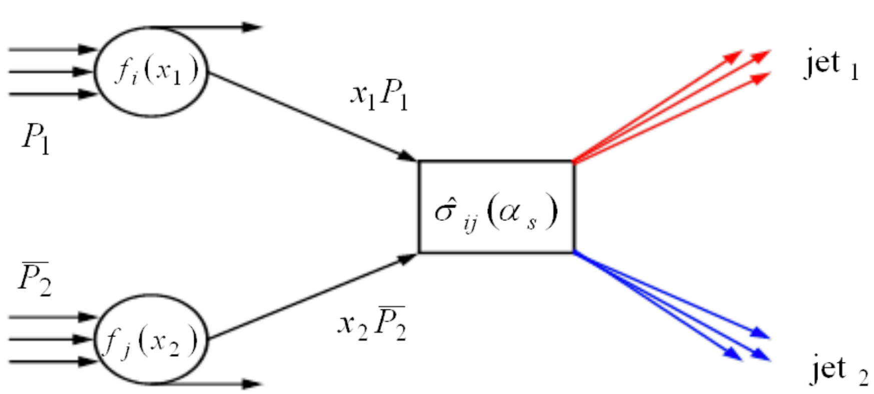
$$k_{Tn} \gg k_{Tn-1} \gg \ldots \gg k_{T2} \gg k_{T1}$$

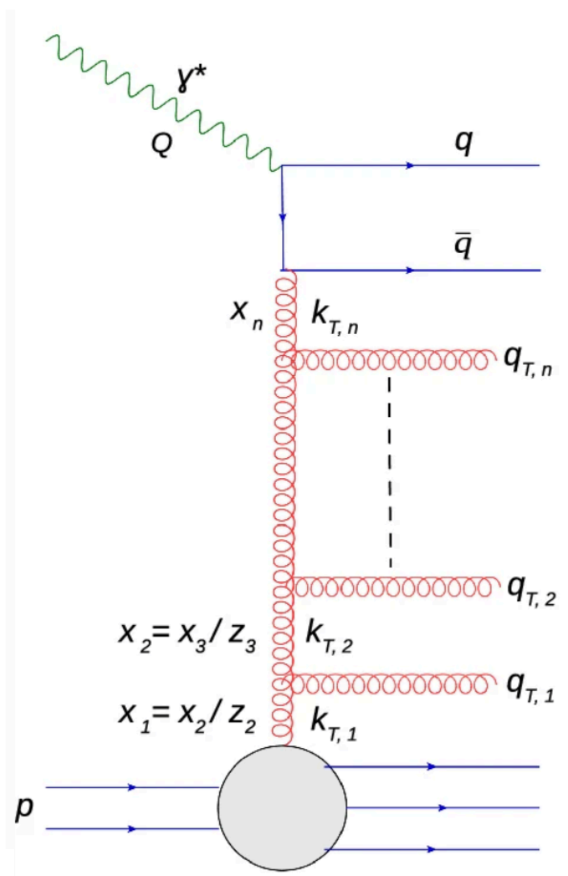
$$p_T \sim Q^2 \qquad \Sigma_n[\alpha_s \log Q^2]^n$$

GLAPD evolution

$$\frac{df_i}{d\log\mu^2} = \frac{\alpha_s}{2\pi}[P_{qq}\otimes f_i + P_{qg}\otimes f_g]$$

$$\frac{df_g}{d\log\mu^2} = \frac{\alpha_s}{2\pi}[P_{gq}\otimes \sum_i f_i + P_{gg}\otimes f_g]$$





Balitsky—Fadin—Kuraev—Lipatov

BFKL kinematics (LLA): Gribov-Regge limit

$$\sqrt{s} \to \infty; p_T - finite; x \sim \frac{p_T}{\sqrt{s}} \to 0;$$

with $p_T \gg \Lambda_{QCD}$

$$x_n \gg x_{n-1} \gg \ldots \gg x_2 \gg x_1$$

$$k_{Tn} \sim k_{Tn-1} \sim \ldots \sim k_{T2} \sim k_{T1}$$

$$\Sigma_n[\alpha_s \log(1/x)]^n$$

BFKL evolution

$$\frac{\partial f_g}{\partial \log 1/x} = K \otimes f_g = \omega f_g \quad \Rightarrow$$

$$f_g \sim \left(\frac{1}{x}\right)^\omega \sim \left(\frac{s}{s_0}\right)^\omega \sim e^{\omega \Delta y}$$

$$\omega_{\max} = \alpha_{I\!P}(0) - 1$$

LL BFKL provides too large intercept

$$\alpha_{I\!P}^{LL}(0) \approx 1.5$$

NLL BFKL: $\alpha_{I\!P}^{NLL} \approx 1.2$

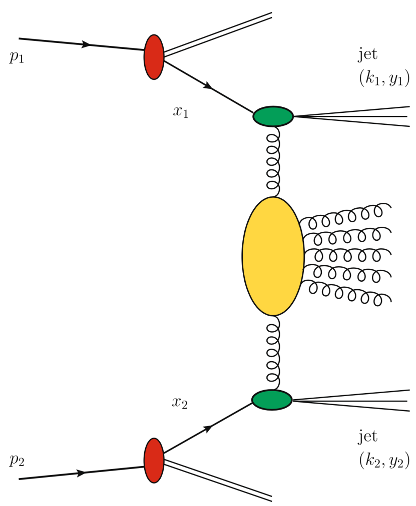
S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, G.B. Pivovarov

[JETP Lett. 70 (1999) 155-160]

Mueller-Navelet (most forward/backward) dijets

A. H. Mueller and H. Navelet [Nucl. Phys. B 282 (1987) 727]

σ^{MN}

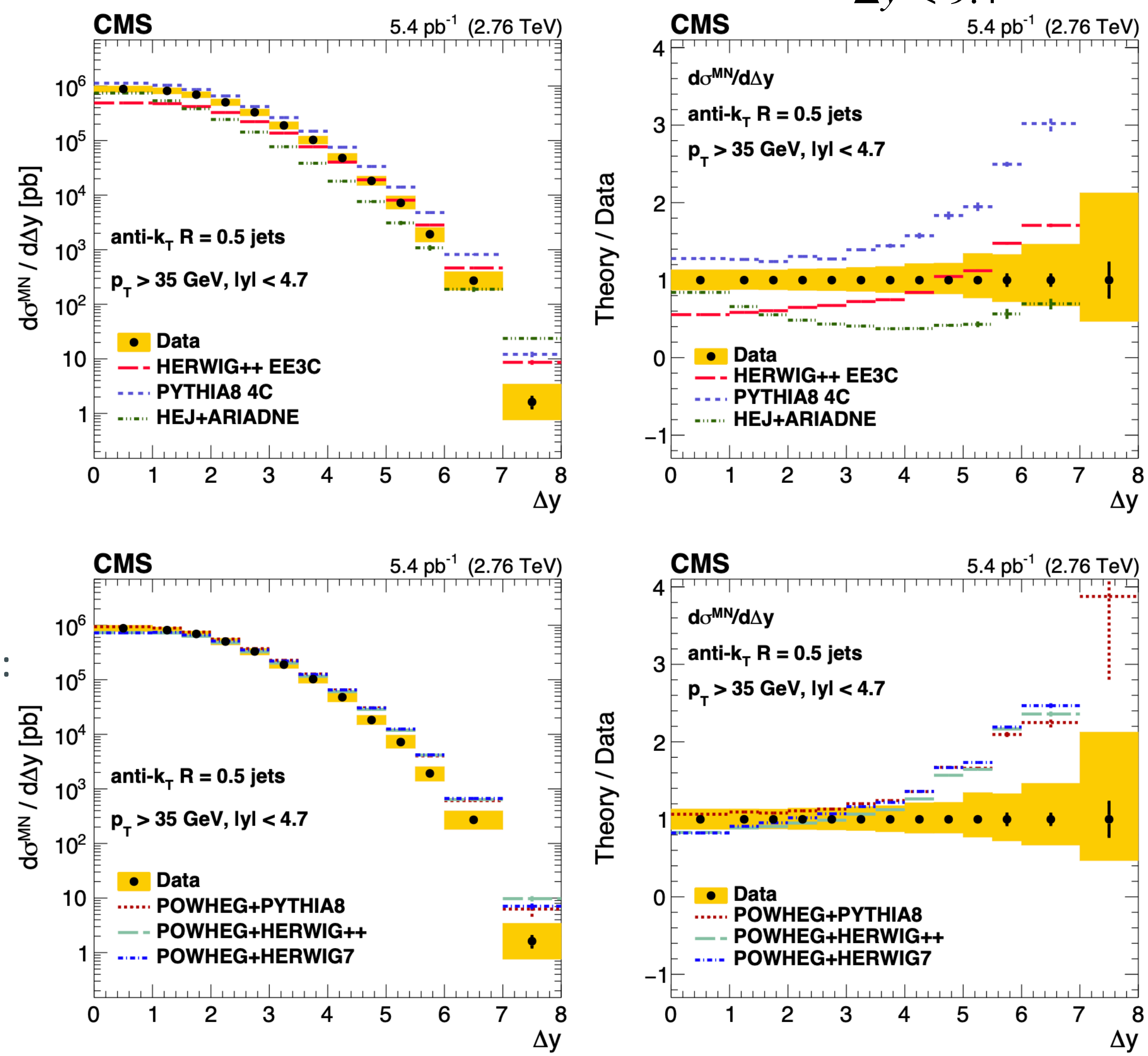


$\sqrt{s} = 2.76$ TeV

$p_{T\min} = 35$ GeV

$\Delta y < 9.4$

CMS [JHEP03(2022)189]



MN dijet is a pair of jets with $p_T > p_{T\min}$

most separated in rapidity $\Delta y = |y_1 - y_2|$

GLAPD-based generators (justified at small Δy):

- PYTHIA8 - LO + LL GLAPD+ Color coherence
- HERWIG - LO + LL GLAPD+ Color coherence
- POWHEG - NLO

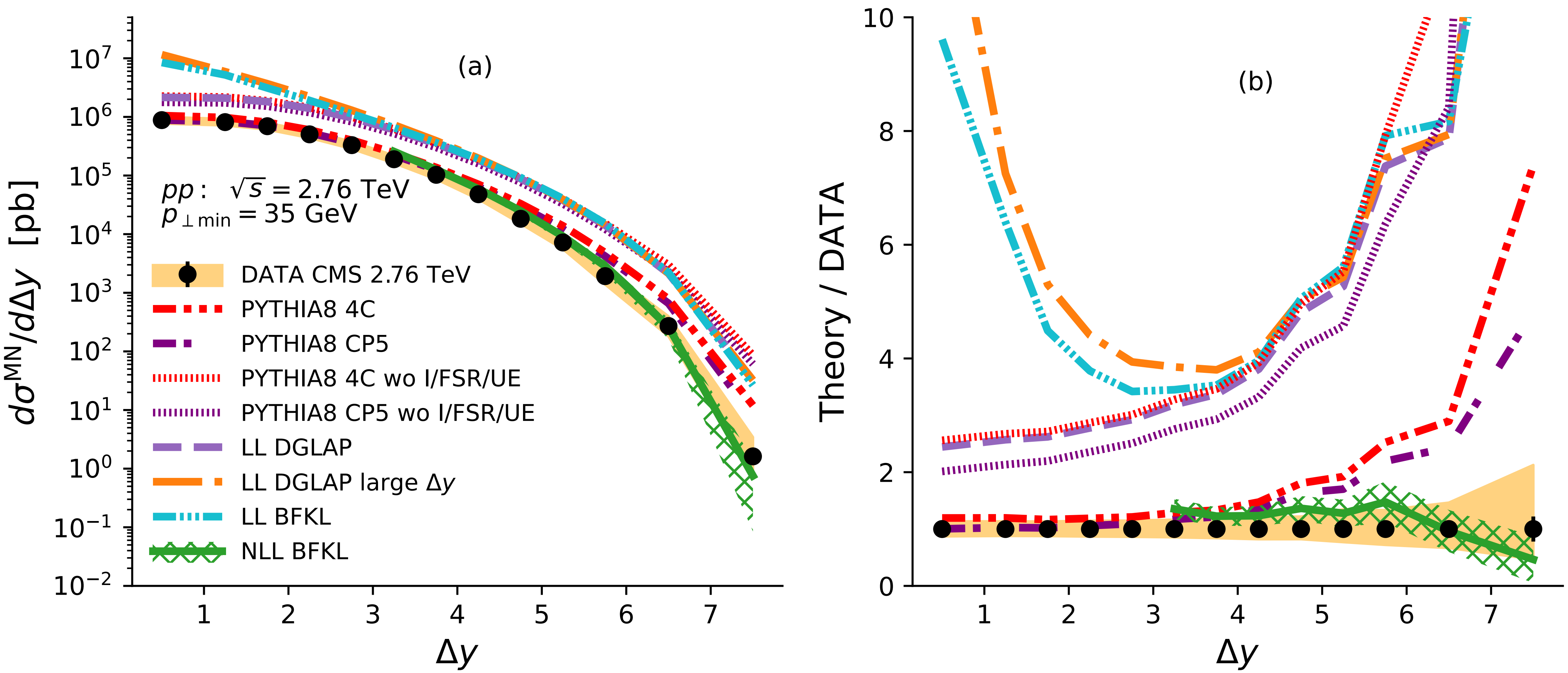
BFKL-based generator (valid at $\Delta y \gg 1$):

- HEJ+ARIADNE - LL BFKL

First comparison: NLL BFKL for MN dijets @ 2.76 TeV

NLL BFKL dijets: A. Iu. Egorov and V. T. Kim [Phys. Rev. D 108 (2023) 014010]

CMS data 2.76 TeV [JHEP03(2022)189]



- MN dijet cross section: NLL BFKL agrees with the CMS data at large Δy
- The all other calculations based on LO/NLO+LL DGLAP overestimates the CMS data at large Δy (analytical LL DGLAP, PYTHIA8, HERWIG, POWHEG)

Example of possible new physics: Gravity with large extra dimensions

N. Arkani-Hamed, S. Dimopoulos, G. Dvali (ADD) [Phys. Lett. B 429 (1998) 263-272]

$$V(r) \sim \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2}} \frac{1}{r^{n+1}} \qquad (r \ll R)$$

$$V(r) \sim \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2} R^n} \frac{1}{r} \qquad (r \gg R)$$

$$M_{Pl}^2 \sim M_{Pl(4+n)}^{2+n} R^n$$

If

$$M_{Pl(4+n)} \sim m_{EW}$$

Then

$$R \sim 10^{\frac{30}{n}-17} \mathrm{cm} \times \left(\frac{1\mathrm{TeV}}{m_{\mathrm{EW}}}\right)^{1+\frac{2}{n}}$$

Kaluza-Klein excitations of the graviton

G.F. Giudice, R. Rattazzi, J.D. Well [Nucl. Phys. B 544 (1999) 3-38]

let's denote $M_{Pl(4+n)} = M_D$

$$\mathcal{R}_{AB} - \frac{1}{2} g_{AB} \mathcal{R} = - \frac{T_{AB}}{M_D^{2+n}}$$

$$g_{AB} = \eta_{AB} + 2 M_D^{1-\frac{n}{2}} h_{AB}$$

$$\Box h_{AB} - \partial_A \partial^C h_{CB} - \partial_B \partial^C h_{CA} + \partial_A \partial_B h_C^C - \eta_{AB} \Box h_C^C + \eta_{AB} \partial^C \partial^D h_{CD} = - M_D^{-1-\frac{n}{2}} T_{AB}$$

If

$$z(x, y) \to (x_0, \mathbf{x}, y_1, \ldots, y_n); \qquad y_j = y_j + 2\pi R, \qquad j = 1, \ldots, n$$

Then

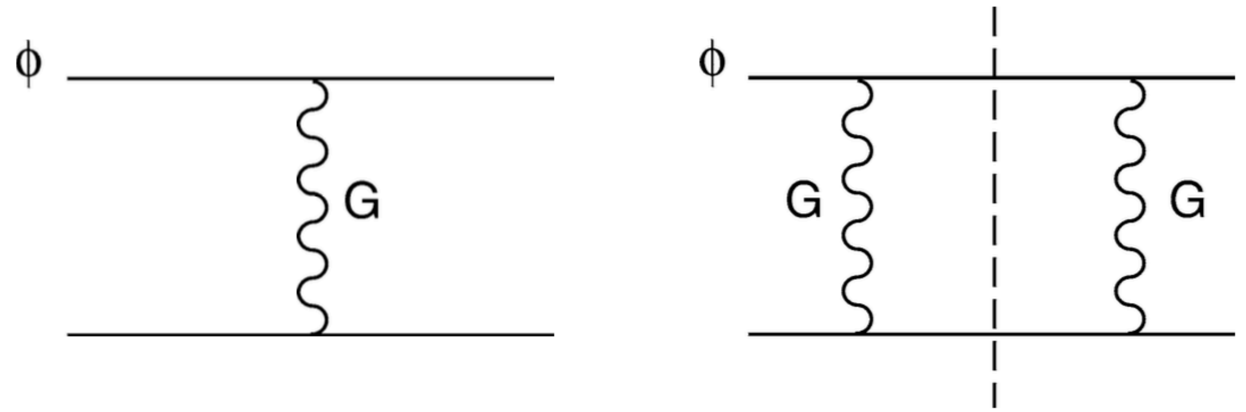
$$h_{AB}(z) = \sum_{n_1=-\infty}^{\infty} \cdots \sum_{n_n=-\infty}^{\infty} \frac{h_{AB}^{(n)}(x)}{\sqrt{(2\pi R)^n}} e^{i\frac{n_j y_j}{R}} \qquad T_{AB}(z) = \eta_A^\mu \eta_B^\nu T_{\mu\nu} \delta(y)$$

Transplanckian eikonal regime (1)

G.F. Giudice, R. Rattazzi, J.D. Well [Nucl. Phys. B 630 (2002) 293-325]

$s \gg M_D^2$ - Transplanckian

$s \gg |t|$ - eikonal



$$\mathcal{A}_{\mathrm{Born}}(-t) = \frac{s^2}{M_D^{n+2}} \int \frac{d^n q_T}{t - q_T^2} = \pi^{n/2} \Gamma(1 - n/2) \left(\frac{-t}{M_D^2} \right)^{\frac{n}{2}-1} \left(\frac{s}{M_D^2} \right)^2$$

$$\mathcal{A}_{1-\mathrm{loop}}(-q^2) = \frac{i}{4s} \int \frac{d2k_\perp}{(2\pi)^2} \mathcal{A}_{\mathrm{Born}}[k_\perp^2] \mathcal{A}_{\mathrm{Born}}[(q_\perp - k_\perp)^2]$$

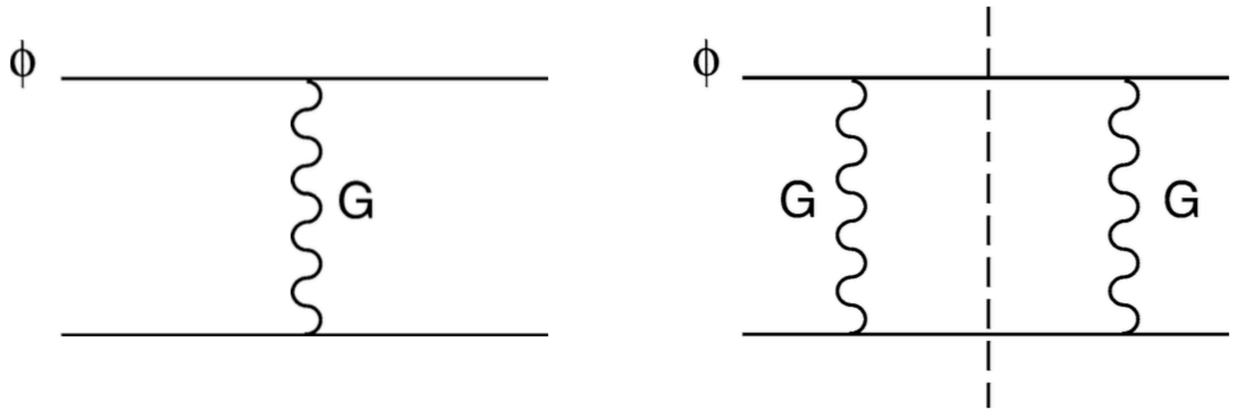
$$\mathcal{A}_{\mathrm{eik}} = \mathcal{A}_{\mathrm{Born}} + \mathcal{A}_{1-\mathrm{loop}} + \ldots = -2is \int d^2 b_\perp e^{iq_\perp b_\perp} (i\chi - \frac{1}{2}\chi^2 + \ldots) = -2is \int d^2 b_\perp e^{iq_\perp b_\perp} \left(e^{i\chi} - 1 \right)$$

$$\chi(b_\perp) = \frac{1}{2s} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-iq_\perp b_\perp} \mathcal{A}_{\mathrm{Born}}(q_\perp^2)$$

$$\frac{d\sigma_{\mathrm{eik}}}{dt} = \frac{|\mathcal{A}_{\mathrm{eik}}|^2}{16\pi s^2}$$

Transplanckian eikonal regime (2)

G.F. Giudice, R. Rattazzi, J.D. Well [Nucl. Phys. B 630 (2002) 293-325]



In proton-proton collisions

$\hat{s} = M_{jj} \gg M_D^2$ - Transplanckian

$\hat{s} \gg |\hat{t}|$ - eikonal

$$\exp\{\Delta y\} \approx \frac{\hat{s}}{|\hat{t}|}$$

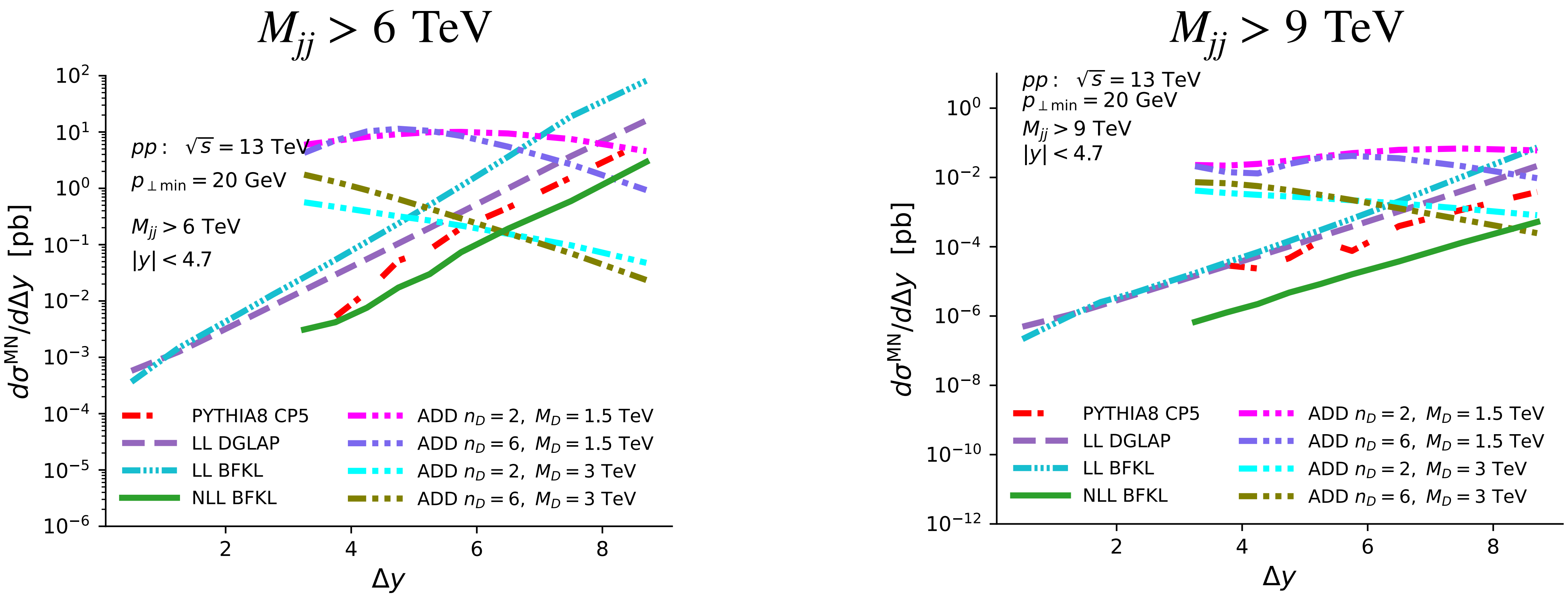
- High-M_{jj} dijets with large rapidity separation are the signal of Large Extra Dimension Gravity

- QCD is the main source of the background

- NLL BFKL is a right tool for the background estimation

Dijets with large Δy @ $\sqrt{s}$ = 13 TeV

$p_{\perp \min} > 20$ GeV
$|y| < 4.7$



$$\frac{\hat{t}}{|\hat{s}|}(\Delta y = 4) \approx 0.018$$

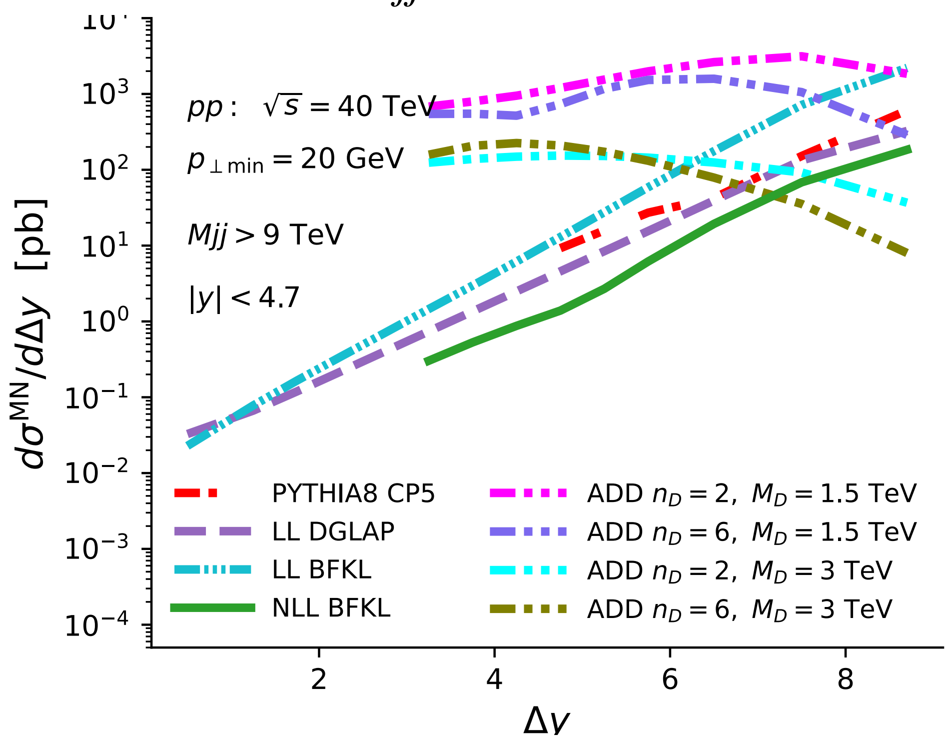
$$\frac{\hat{t}}{|\hat{s}|}(\Delta y = 8) \approx 0.00034$$

- Large statistics is needed for large Δy (Experiment difficultly)
- GLAPD overestimates background x10 times at large Δy
- BFKL predicts stronger suppression under M_{jj} selection than GLAPD
- Sensitivity to ADD gravity up to $M_D \sim 3$ TeV

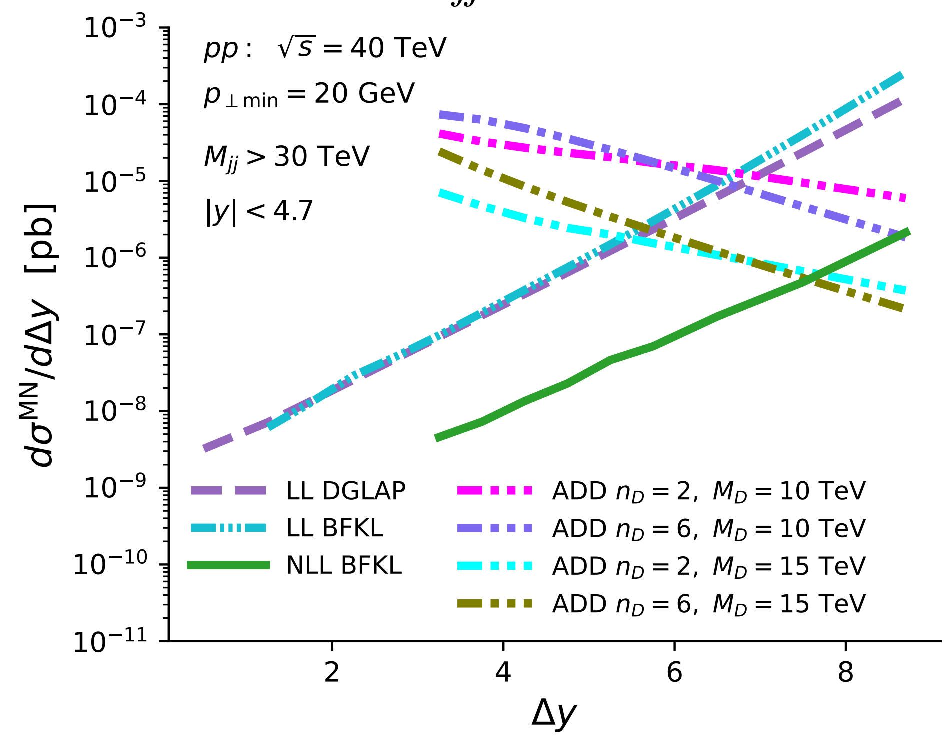
Dijets with large Δy @ $\sqrt{s}$ = 40 TeV

$p_{\perp\min} > 20\ \mathrm{GeV}$
$|y| < 4.7$

$M_{jj} > 9\ \mathrm{TeV}$



$M_{jj} > 30\ \mathrm{TeV}$

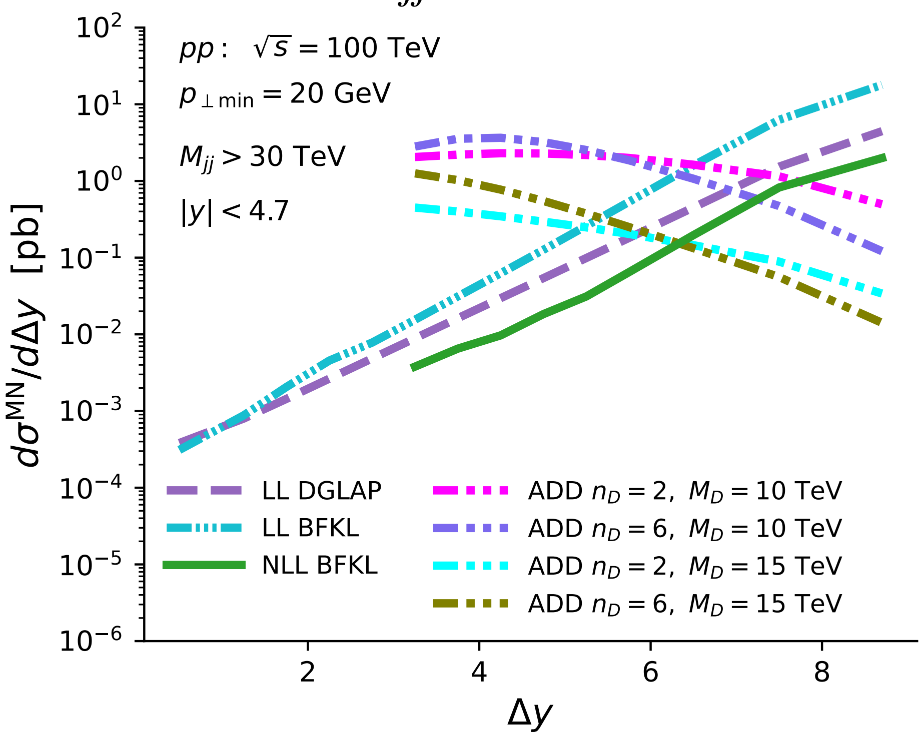


- PYTHIA 8 is about LL GLAPD analytical calculation
- PYTHIA 8: It is difficult to get statistics for large M_{jj}
- GLAPD overestimates background 100 times at large Δy and $M_{jj} > 30\ \mathrm{GeV}$
- BFKL predicts stronger suppression under M_{jj} selection than GLAPD
- Sensitivity to ADD gravity up to $M_D \sim 10\ \mathrm{TeV}$

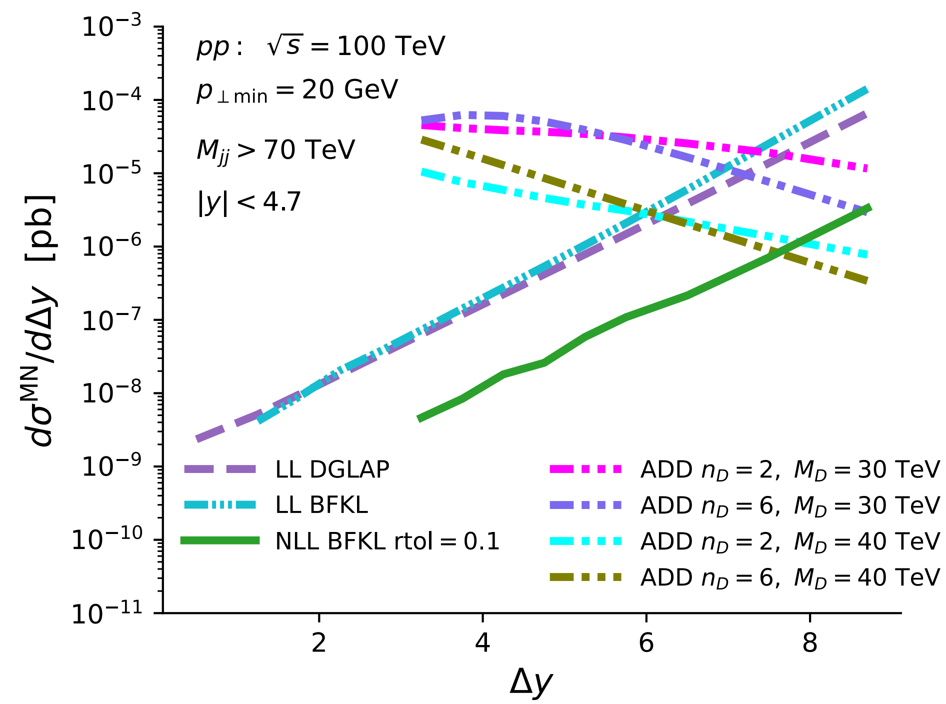
Dijets with large Δy @ $\sqrt{s}$ = 100 TeV

$p_{\perp\min} > 20$ GeV
$|y| < 4.7$

$M_{jj} > 30$ TeV



$M_{jj} > 70$ TeV



- GLAPD overestimates background x100 times at large Δy and large M_{jj}
- BFKL predicts stronger suppression under M_{jj} selection than GLAPD
- Sensitivity to ADD gravity up to $M_D \sim 30$ TeV

Summary

- NLL BFKL evolution is the right tool for QCD at large Δy. So possible new physics is the signal above BFKL QCD background at large Δy
- Dijet production with large Δy and M_{jj} is sensitive to signals of gravity with large extra dimensions in transplanckian eikonal regime
- NLL BFKL predicts less background QCD events than more conventional GLAPD evolution at large Δy and M_{jj} (up to two orders in amount of events). The BFKL evolution is more sensitive to M_{jj} selection than the GLAPD evolution.
- large experimental luminosity is needed (up to ~1000 fb) for searches of new physics at large Δy. So high-granularity detectors which are able to disentangle large pileup events at large rapidities are needed

THANK YOU!

NLL BFKL for MN dijets

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \sum_{ij} \int f_i(x_1,\mu_F) f_j(x_2,\mu_F) \frac{d\hat{\sigma}_{ij}(x_1x_2s,\mu_F,\mu_R)}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}}$$

Large Δy: $$f^{\text{eff}}(x,\mu_F) = \frac{C_A}{C_F} f_g(x,\mu_F) + \sum_{i=q,\bar{q}} f_i(x,\mu_F),$$

NLL BFKL

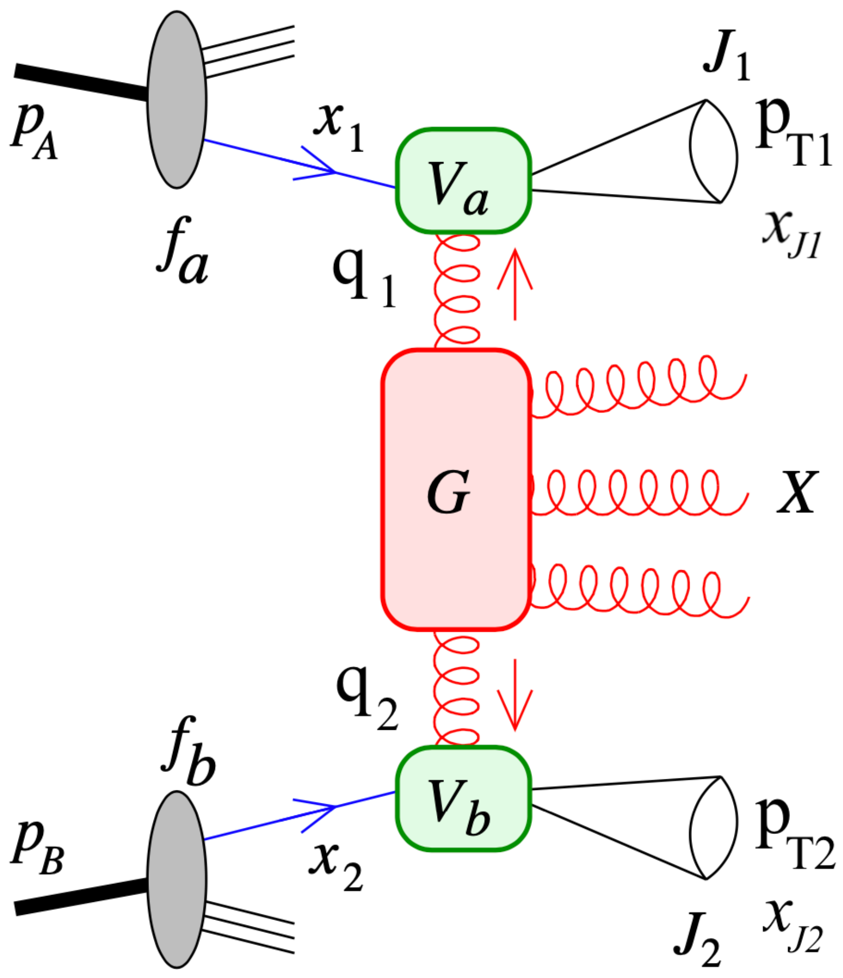
$$\frac{d\hat{\sigma}_{gg}}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \frac{x_{J1}x_{J2}}{(2\pi)^2} \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} V_1(\vec{q}_1, x_1, \vec{p}_{T1}, x_{J1}) \times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} V_2(-\vec{q}_2, x_2, \vec{p}_{T2}, x_{J2}) \times \int_C \frac{d\omega}{2\pi i} \left(\frac{x_1x_2s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1,\vec{q}_2),$$

$$\Phi(\vec{q},\vec{p}_T,x_J,\omega) \equiv \sum_i \int_0^1 dx f_i(x,\mu_F) \left(\frac{x}{x_J}\right)^{\omega} V_i(\vec{q},x,\vec{p}_T,x_J),$$

$$\Phi_{1,2}(n,\nu,\vec{p}_{T1,2},x_{J1,2},\omega) = \alpha_s(\mu_R)[c_{1,2}(n,\nu) + \bar{\alpha}_s(\mu_R) c^{(1)}_{1,2}(n,\nu)]$$

$$\frac{d\sigma}{dy_1 dy_2 |\vec{p}_{T1}| d|\vec{p}_{T2}| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2} \left[\mathscr{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\mathscr{C}_n\right]$$

F. Caporale, D. Yu. Ivanov, B. Murdaca & A. Papa
[Phys. Rev. D 91 (2015) 114009]
D. Colferai, F. Schwennsen, L. Szymanowski & S. Wallon
[JHEP12(2010)026]



Generalized BLM optimal scale setting
Brodsky-Fadin-Kim-Lipatov-Pivovarov (BFKLP)
[JETP Lett. 70 (1999) 155-160]

NLL BFKL for MN dijets

NLL BFKL

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{p}_{T1}|d|\vec{p}_{T2}|d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathscr{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\mathscr{C}_n\right]$$

$$\mathscr{C}_n = \frac{x_{J1}x_{J2}}{|\vec{p}_{T1}||\vec{p}_{T2}|}\int_{-\infty}^{+\infty} d\nu e^{(Y-Y_0)\bar{\alpha}_s(\mu_R)\chi(n,\nu)}\alpha_s^2(\mu_R)c_1(n,\nu)c_1(n,\nu)\left[1+\bar{\alpha}_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)}+\frac{\bar{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right.\right.$$
$$\left.\left.+\frac{\beta_0}{2N_c}\left(\frac{5}{3}+\ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}\right)\right)+\bar{\alpha}_s^2(\mu_r)\ln\frac{x_{J1}x_{J2}s}{s_0}\left\{\bar{\chi}(n,\nu)+\frac{\beta_0}{4N_c}\chi(n,\nu)\left(-\frac{\chi(n,\nu)}{2}+\frac{5}{3}+\ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}\right)\right\}\right]$$

where

$Y = y_1 - y_2 = \ln\frac{x_{J1}x_{J2}s}{|\vec{p}_{T1}||\vec{p}_{T2}|}$ and $Y_0 = \ln\frac{s_0}{|\vec{p}_{T1}||\vec{p}_{T2}|}$

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{p}_{T1}|d|\vec{p}_{T2}|} = \mathscr{C}_0$$

- NLL BFKL contains both renormalization scheme and renormalization scale ambiguities

BFKLP prescription

This is the generalisation of Brodsky-Lepage-Mackenzie (BLM) optimal scale setting procedure [Phys. Rev. D 28 (1983) 229].

Brodsky-Fadin-Kim-Lipatov-Pivovarov (BFKLP) [JETP Lett. 70 (1999) 155-160]

The ambiguity is related to large running coupling effects and non-Abelian nature of the QCD

$\Rightarrow$ one needs to use physical renormalization scheme, in which the non-Abelian contributions presented in the first order, for example physical momentum subtraction scheme (MOM).

MOM and $\overline{\mathrm{MS}}$ schemes are related:

$$\alpha_s^{\overline{\mathrm{MS}}} = \alpha_s^{\mathrm{MOM}}\left(1 + \frac{\alpha_s^{\mathrm{MOM}}}{\pi}(T^\beta + T^{\mathrm{conf}})\right),$$

$$T^\beta = -\frac{\beta_0}{2}\left(1 + \frac{2}{3}I\right),$$

$$T^{\mathrm{conf}} = \frac{C_A}{8}\left[\frac{17}{2}I + \frac{3}{2}(I-1)\xi + \left(1 - \frac{1}{3}I\right)\xi^2 - \frac{1}{6}\xi^3\right],$$

where $I \approx 2.3439$, ξ - gauge parameter

BFKLP prescription (2)

Transform to MOM scheme, and separate conformal (β_0 -independent) and non-conformal (β_0 -dependent) parts:

$$\mathscr{C}_n^{\mathrm{MOM}} = \frac{x_{J1}x_{J2}}{|\vec{p}_{T1}||\vec{p}_{T2}|}\int_{-\infty}^{+\infty} d\nu e^{(Y-Y_0)\bar{\alpha}_s^{\mathrm{MOM}}(\mu_R)\chi(n,\nu)}\left(\alpha_s^{\mathrm{MOM}}(\mu_R)\right)^2 c_1(n,\nu)c_2(n,\nu)$$
$$\times\left[1+\bar{\alpha}_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)}+\frac{\bar{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)}+\frac{2T^{\mathrm{conf}}}{N_c}+\frac{\beta_0}{2N_c}\left(\frac{5}{3}+\ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}-2\left(1+\frac{2}{3}I\right)\right)\right)\right.$$
$$\left.+\left(\bar{\alpha}_s^{\mathrm{MOM}}(\mu_R)\right)^2\ln\frac{x_{J1}x_{J2}s}{s_0}\left\{\bar{\chi}(n,\nu)+\frac{T^{\mathrm{conf}}}{N_c}\chi(n,\nu)+\frac{\beta_0}{4N_c}\chi(n,\nu)\left(-\frac{\chi(n,\nu)}{2}+\frac{5}{3}+\ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}-2\left(1+\frac{2}{3}I\right)\right)\right\}\right],$$

choose μ_R scale so that the non-conformal part vanishes

$$\mathscr{C}_n^{\beta} = \frac{x_{J1}x_{J2}}{|\vec{p}_{T1}||\vec{p}_{T2}|}\int_{-\infty}^{+\infty} d\nu e^{(Y-Y_0)\bar{\alpha}_s^{\mathrm{MOM}}(\mu^{\mathrm{BFKLP}})\chi(n,\nu)}\left(\alpha_s^{\mathrm{MOM}}(\mu^{\mathrm{BFKLP}})\right)^3 c_1(n,\nu)c_2(n,\nu)$$
$$\times\frac{\beta_0}{2N_c}\left[\frac{5}{3}+\ln\frac{(\mu^{\mathrm{BFKLP}})^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}-2\left(1+\frac{2}{3}I\right)+\bar{\alpha}_s^{\mathrm{MOM}}(\mu^{\mathrm{BFKLP}})\ln\frac{x_{J1}x_{J2}s}{s_0}\frac{\chi(n,\nu)}{2}\right.$$
$$\left.\times\left(-\frac{\chi(n,\nu)}{2}+\frac{5}{3}+\ln\frac{(\mu^{\mathrm{BFKLP}})^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}-2\left(1+\frac{2}{3}I\right)\right)\right]=0,$$