Hidden-charm strong decays of charmonium-like states Y(4230) and $X_2(4014)$

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Outline

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    - Isospin symmetry conservation and breaking
    - Strong decay widths $\Gamma(X_2 \to \omega\ J/\Psi)$ and $\Gamma(X_2 \to \varrho^0\ J/\Psi)$
    - **Numerical results**

Introduction

- Heavy meson spectrum -> several 'exotic' states (*XYZ*) with a common feature: *minimal constituent quark-antiquark or three-quark structures do not work*.

Y(4230)

- BaBar (2005) data analysis in production $e^+e^- \to \gamma_{ISR}\, \pi^+\pi^- J/\psi$ revealed a broad resonance ~4.26 GeV with $J^{PC} = 1^{--}$ => *Y*(4230) [aka ψ(4260) or *Y*(4260)]

- Earlier studies: *Y*(4230) is not a conventional charmonium, no mass matching.

Some theoretical interpretations for Y(4230):

- a charmonium-hybrid ($c\bar{c}g$) state in Refs. [3–5]

- 1st orbital excitation of diquark-antidiquark state ($[cs]\,[\bar{c}\bar{s}]$) in Ref. [7]

- a ($\chi_{c1}\,\rho^0$) molecule in Ref. [6]: but $\Gamma(Y\to\pi^+\pi^- J/\psi) > \Gamma(Y\to D\bar{D})$ not observed

- a ($D\bar{D}_1$) weak molecule: $Y\to \pi^- Z_c^+(3900)$ has been studied in Ref. [8,9,15]

$X_2(4014)$

- The first member of the XYZ family, the X(3872) state is located very close to the $(D\overline{D}*)$ threshold -> considered as a shallow bound **J^{PC} = 1^{++}** mesonic molecule.

- Does there exist a possible heavier partner of X(3872) with a similar value for the binding energy, but with **J^{PC} = 2^{++} ?**

⁕ ***Theory***:
- various phenomenological models predict the existence of an isoscalar **2^{++}** $(D^*\overline{D}*)$ molecular partner of the X(3872).
- a compact tetraquark model has been employed to explore the **2^{++}** states.
- alternatively, within the conventional pattern of mesons, a **2^{++}** tensor state with a similar mass could also be a conventional charmonium state in the first radial excitation $\chi_{c2}(3930)$.

⁕ ***Experiment***: the Belle collaboration (2022) has reported a hint of an isoscalar structure with mass M = 4014.3 ± 4.0 ± 1.5 MeV and width Γ = 4 ± 11 ± 6 MeV, seen in the γ ψ(2S) invariant mass distribution via a two-photon process.

Model framework (in short)

- Hadrons *H(x)* interact by *quark exchanges*, with hadron-quark coupling g_H.

$$L_{\text{int}} = g_H H(x) J_H(x)$$

- Interpolating quark current (for 4-quark meson):

$$J_H(x) = \int dx_1 \ldots \int dx_4 \, \delta(x - \omega_1 x_1 - \omega_2 x_2 - \omega_3 x_3 - \omega_4 x_4) \cdot \Phi_H\left(\sum_{i<j}^{4}(x_i - x_j)^2\right) \cdot J^{\mu}_{4q}(x_1, x_2, x_3, x_4)$$

$$\omega_j = m_j / (m_1 + m_2)$$

- Vertex function (Fourier transformation)

$$\Phi_H(-Q^2) = \exp\left(\frac{Q^2}{\Lambda_H^2}\right)$$

Λ_H ~ hadron "size"

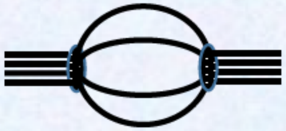
- Quark propagator (in the Schwinger representation):

$$S_m(\hat{p}) = \frac{m + \hat{p}}{m^2 - p^2} = \left(m + \hat{p}\right) \cdot \int_0^{\infty} d\alpha \exp\left[-\alpha\left(m^2 - p^2\right)\right]$$

- The ***compositeness condition*** eliminates the bare fields from consideration.

[A.Salam, S.Weinberg (1962-1964)]

$$Z_H = \left\langle H_{bare} \,|\, H_{phys} \right\rangle^2 = 1 - g_{ren}^2 \, \Pi'_H(M_H^2) = 0$$



• ***Eliminating singularties:*** The integration over loop momenta can be performed for arbitrary Feynman diagram with n-quark propagators. The final multidimensinal integration over an infinite space of the Fock-Schwinger parameters may be **transformed** into an integral over a simplex convoluted with only one-dimensional improper integral:

$$\Pi = \int_0^\infty d^n\alpha \, W(\alpha_1,\ldots,\alpha_n) = \int_0^\infty d^n\alpha \underbrace{\int_0^\infty dt\, \delta\left(t - \sum_{i=1}^n \alpha_i\right)}_{=1} W(\alpha_1,\ldots,\alpha_n)$$
$$= \int_0^\infty dt\, t^{n-1} \int_0^\infty d^n\alpha\, \delta\left(1 - \sum_{i=1}^n \alpha_i\right) W(t\alpha_1,\ldots,t\alpha_n),$$

A cut-off makes the integral to be an analytic function without any singularities:

$$\int_0^\infty dt\, t^{n-1} \ldots \to \int_0^{1/\lambda^2} dt\, t^{n-1} \ldots .$$

• ***Model parameters:*** A hadron in the CCQM is characterized by:
- the global infrared confinement parameter **λ** (universal)
- the constant quark masses **$m_{u,d}$ m_s m_c m_b** (common)
- the hadron size parameter **Λ_H** (adjustable)

I. Strong decays of charmonium-like state Y(4230)

♦ ***Experimental data:*** • PDG-2024

State	J^{PC}	Mass (MeV)	Full width (Γ)	Modes	Fraction (Γ_i / Γ)
Y	**1^{--}**	**4222.1 ± 2.3**	**49 ± 8 MeV**	**$\pi^+\pi^- J/\Psi$, $K^+K^- J/\Psi$**	**seen**

• BES-III Collaboration (2024):

$$M_Y = 4225.3 \pm 2.3 \pm 21.5 \text{ MeV}$$

$$\Gamma_Y = 72.9 \pm 6.1 \pm 30.8 \text{ MeV}$$

$$0.02 < \frac{B(Y \to K^+K^- J/\Psi)}{B(Y \to \pi^+\pi^- J/\Psi)} < 0.26$$

♦ ***Some theoretical interpretations for Y(4230):***

- a charmonium-hybrid ($c\bar{c}g$) state in Refs. [3–5]
- 1^{st} orbital excitation of diquark-antidiquark state ($[cs]\,[\bar{c}\bar{s}]$) in Ref. [7]
- a (χc1 ρ0) molecule in Ref. [6]: but Γ(Y→π+π-J/ψ) > Γ(Y→D̄D) not observed
- a $(D\bar{D}_1)$ weak molecule: Y→ π^- $Z_c^+(3900)$ has been studied in Ref. [8,9,15]

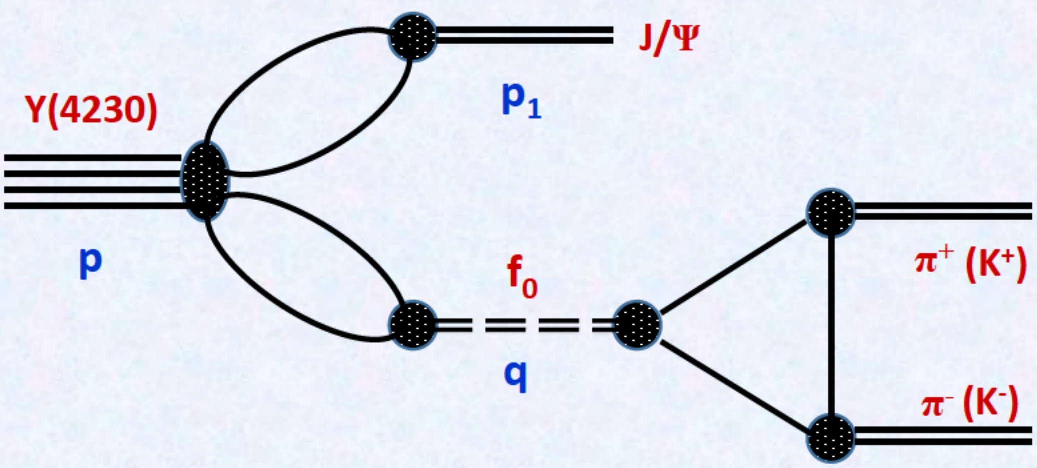
CCQM approach to $Y \to \pi^+\pi^- J/\Psi$ and $Y \to K^+K^- J/\Psi$

in collaboration with M. A. Ivanov (BLTP JINR), Eur.Phys.J. A 60:13 (2024)

- We consider the ***Y(4230)*** as a 4-quark state with a molecular-type interpolating current.

$$J^{\mu}_{4q}(x_1,x_2,x_3,x_4) = \frac{1}{\sqrt{2}}\left\{[\bar{q}(x_3)\gamma_5 c(x_1)]\cdot[\bar{c}(x_2)\gamma^{\mu}\gamma_5 q(x_4)] \; + \; (\gamma_5 \leftrightarrow \gamma^{\mu}\gamma_5)\right\}, \qquad q = \{u,d\}$$

- We suggest the following sequential (cascade) two-body decay scheme:
- first, the ***Y(4230)*** decays into the **J/ψ** and the scalar resonance $f_0 = \{f_0(500),\ f_0(980)\}$,
- then, f_0 decays into two mesons, either $\pi^+\pi^-$, or K^+K^-, wherever it is actual.



- We note that the description of the $c\bar{c}$ - $q\bar{q}$ transitions, which go via gluon exchange, is out of the CCQM scope.

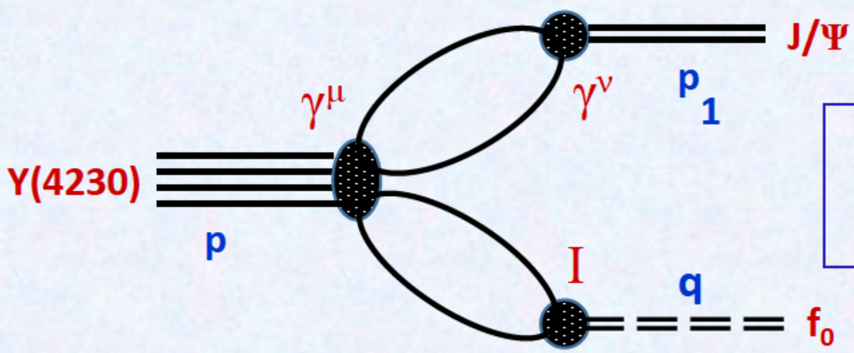
Decay $Y \rightarrow J/\Psi + f_0$

State	J^{PC}	Mass (MeV)	Full width (Γ)	Modes	Fraction (Γ_i / Γ)
f_0 (500)	0^{++}	400 - 550	100 - 800 MeV	$\pi^+\pi^-$	seen
f_0 (980)	0^{++}	980 - 1010	10 - 100 MeV	$\pi^+\pi^-$, K^+K^-	dominant, seen

Further we use the notation $S = \{S_1, S_2\} = \{f_0(500), f_0(980)\}$ for simplicity.

- There is no consensus on the matter of precise f_0 quark structure, whether one deals with a quark-antiquark or rather a four-quark composite.

- We neglect higher Fock contributions to the f_0 bound states and treat the scalar resonances $f_0(500)$ and $f_0(980)$ as quark-antiquark states.

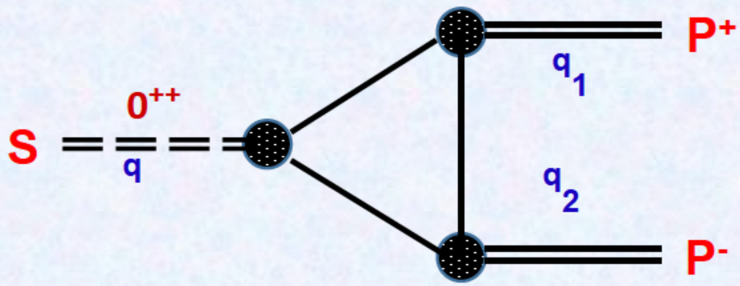


$$\Gamma_{YVS} = \frac{\left|\vec{p}_1\right|^2}{24\pi M_Y^2}\left[\,|H_{00}|^2 + |H_{++}|^2\,|H_{--}|^2\right]$$

- Matrix element:

$$M_{YVS}(Y(p,\varepsilon_p^\mu) \rightarrow J/\Psi(p_1,\varepsilon_{p_1}^\nu) + f_0(q)) = \varepsilon_p^\mu\,\varepsilon_{p_1}^{*\nu}\left(B\cdot g^{\mu\nu} + C\cdot p_1^\mu q^\nu\right)$$

Decay $S \to P^+P^-$



$$\Gamma_{SPP}(q^2) = \frac{|\vec{q}_1|}{8\pi q^2} \cdot G^2_{SPP}(q^2)$$

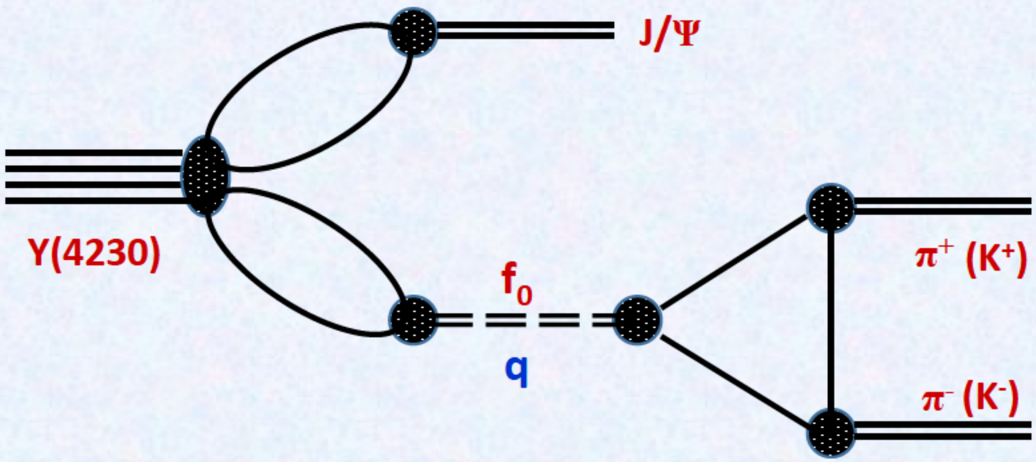
- A rough estimate of the ratio of decay widths:

$$\frac{\Gamma_{S_1\pi^+\pi^-}(M^2_{S_1})}{\Gamma_{S_2\pi^+\pi^-}(M^2_{S_2})} \approx \frac{g^2_{S_1} M^3_{S_2}}{g^2_{S_2} M^3_{S_1}} \frac{\lambda^{1/2}(M^2_{S_1}, M^2_\pi, M^2_\pi)}{\lambda^{1/2}(M^2_{S_2}, M^2_\pi, M^2_\pi)} \approx 59.$$

- A rough model-independent estimate of the ratio:

For a model-independent case (if $g^2_{S_1} = g^2_{S_2}$), the approximate ratio is ≈ 3. The two-body decay channel $f_0(500) \to \pi^+\pi^-$ may strongly dominate over $f_0(980) \to \pi^+\pi^-$.

Cascade Decays: $Y \rightarrow V + f_0(P^+ P^-)$



- $f_0(500) + f_0(980)$: for $\pi^+\pi^-$
- $f_0(980)$ only : for K^+K^-

- The conventional Breit-Wigner amplitude (propagator) for the scalar $S = f_0$

$$D_S(q^2) = \frac{1}{q^2 - M_S^2 + i\, M_S\, \Gamma_S(q^2)} \doteq \frac{1}{A_S(q^2) + i\, B_S(q^2)}$$

- The Breit-Wigner distribution for a solely scalar resonance **S**:

$$BW_S(q^2) = \frac{\sqrt{q^2}}{\pi} \frac{1}{A_S^2(q^2) + B_S^2(q^2)}$$

- A reasonable approximation: $\Gamma_S(q^2) \simeq \Gamma_{SPP}(q^2)$

♦ We adopt a modified Breit-Wigner form for the intermediate ***f*₀**- propagators:

$$BW_S(q^2) = \frac{\sqrt{q^2}}{\pi} \frac{1}{(q^2 - M_S^2)^2 + (M_S \Gamma_S)^2 \left[R_S^P(q^2) \right]^2}$$

- where a correction function is defined:

$$R_S^P(q^2) \doteq \frac{M_S^2}{q^2} \left(\frac{q^2 - 4M_P^2}{M_S^2 - 4M_P^2} \right)^{1/2}, \quad R_S^P(4M_P^2) = 0, \quad R_S^P(4M_S^2) = 1.$$

- The behaviour of the correction function:

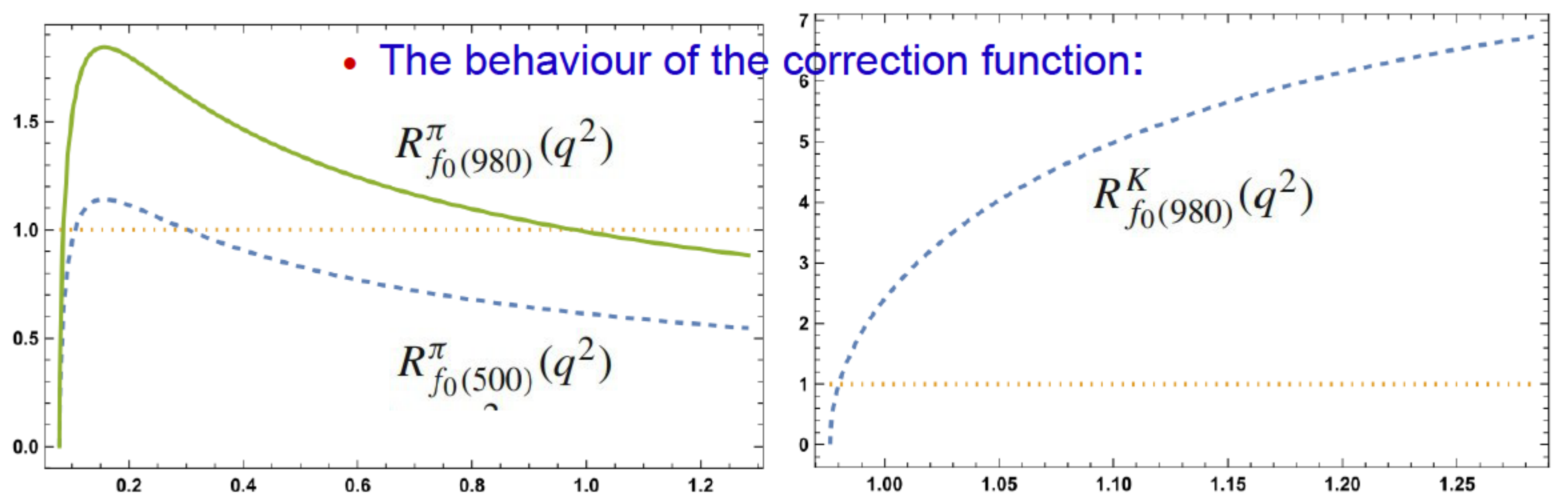


Fig. 5 The correction functions $R^{\pi}_{f0(500)}(q^2)$ (dashed, first plot), $R^{\pi}_{f0(980)}(q^2)$ (solid, first plot) and $R^{K}_{f0(980)}(q^2)$ (dashed, second plot). The horizontal axes are bound by $4M_P^2 < q^2 < (M_Y - M_{J/\psi})^2$

- The Breit-Wigner distributions – the conventional versus a modified:

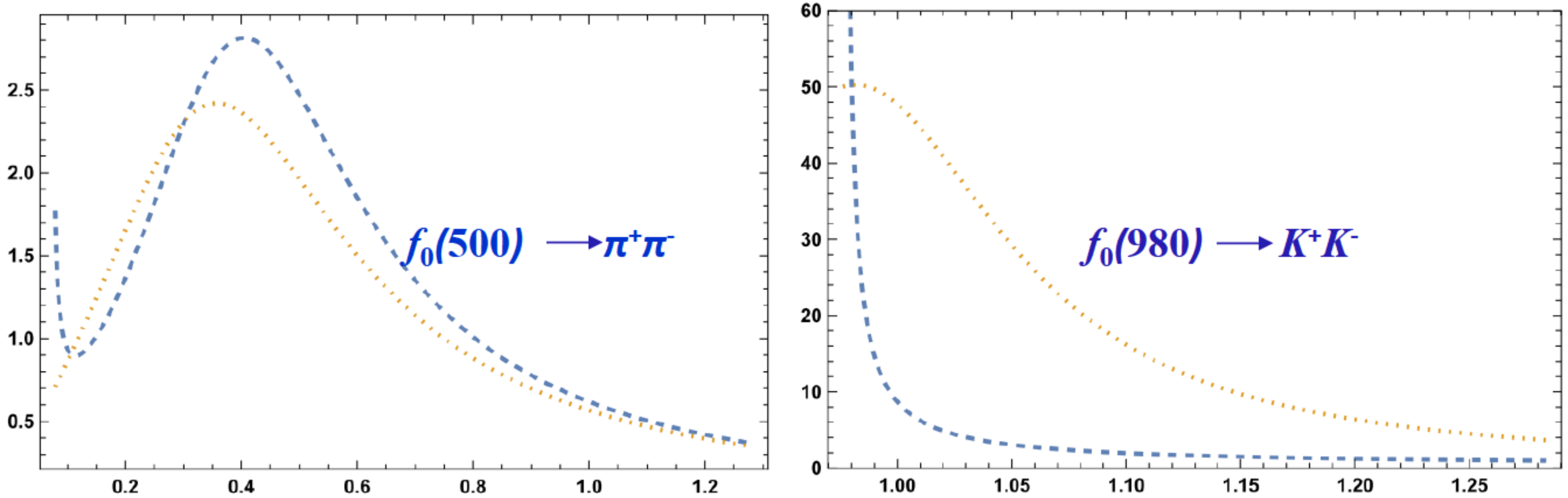


Fig. 6 The Breit-Wigner distributions for decays $f_0(500) \to \pi^+\pi^-$ (first plot) and $f_0(980) \to K^+K^-$ (second plot). The dashed (blue) curves correspond to the versions with nontrivial correction functions defined in Eq. (26), while the dotted (yellow) curves simulate the conventional approximations with $R_S^P(q^2) \doteq 1$

- Integration limits:

$$4M_P^2 \doteq q_{\min}^2 < q^2 < q_{\max}^2 \doteq (M_\Upsilon - M_{J/\Psi})^2$$

Numerical Integration over q^2

$$\Gamma(Y \to \pi^+\pi^- J/\psi)$$

$$= \int_{4M_\pi^2}^{q^2_{\max}} dq^2 \Big\{ \Gamma_{YJS_1}(q^2)\, BW_{S_1}(q^2) \Gamma_{S_1\pi^+\pi^-}(q^2)$$

$$+ BW_{X_{12}}(q^2) \Big[\Gamma_{YJS_1}(q^2)\, \Gamma_{S_2\pi^+\pi^-}(q^2)$$

$$+ \Gamma_{YJS_2}(q^2)\, \Gamma_{S_1\pi^+\pi^-}(q^2) \Big]$$

$$+ \Gamma_{YJS_2}(q^2)\, BW_{S_2}(q^2) \Gamma_{S_2\pi^+\pi^-}(q^2) \Big\},$$

$$q^2_{\min(\pi\pi)} = 4M_\pi^2,$$

$$q^2_{\max} = (M_Y - M_{J/\Psi})^2.$$

$$\Gamma(Y \to K^+K^- J/\psi)$$

$$= \int_{4M_K^2}^{q^2_{\max}} dq^2\, \Gamma_{YJS_2}(q^2)\, BW_{S_2}(q^2) \Gamma_{S_2K^+K^-}(q^2).$$

$$q^2_{\min(KK)} = 4M_K^2,$$

$$q^2_{\max} = (M_Y - M_{J/\Psi})^2.$$

Numerical results

- $\Gamma_1(Y \to f_0 J/\Psi)$, $\Gamma_4(Y \to \pi^+\pi^- J/\Psi)$ and $\Gamma_5(Y \to K^+K^- J/\Psi)$ depend strongly on Λ_Y :

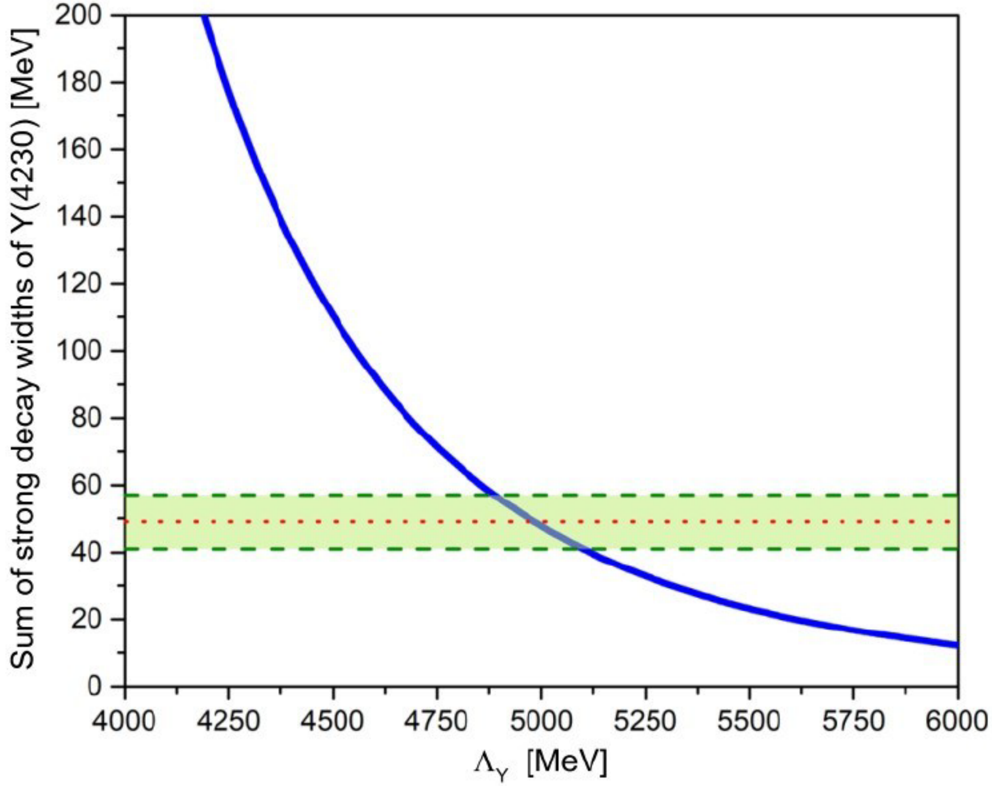


Fig. 7 Dependence of the sum of partial decay widths $\Gamma(Y \to J/\psi\ f_0(500)) + \Gamma(Y \to J/\psi\ f_0(980)) + \Gamma(Y \to K^+K^- J/\psi) + \Gamma(Y \to \pi^+\pi^- J/\psi)$ on size parameter Λ_Y. The horizontal (green) band depicts the full width data $\Gamma_Y = 49 \pm 8$MeV [1]

$\Gamma_Y \approx 49 \pm 8$ MeV

We fix:

$\mathbf{\Lambda_Y = 5.15 \pm 0.26}$ **GeV**

Numerical results on the decay widths (MeV) and the branching ratio

Decays	CCQM	CCQM (R=1)	PDG (2024)
$\Gamma(Y \to J/\Psi + f_0(500))$	0.32 ± 0.03	0.32 ± 0.03	$\Gamma_Y = 49 \pm 8$
$\Gamma(Y \to J/\Psi + f_0(980))$	0.47 ± 0.05	0.47 ± 0.05	$\Gamma_Y = 49 \pm 8$
$\Gamma(f_0(980) \to K^+ + K^-)$	30.2 ± 3.1	30.2 ± 3.1	$\Gamma_{f0(980)} = 10 - 100$
$\Gamma(f_0(500) \to \pi^+ + \pi^-)$	10.7 ± 1.2	10.7 ± 1.2	$\Gamma_{f0(500)} = 400 - 700$
$\Gamma(f_0(980) \to \pi^+ + \pi^-)$	34.6 ± 3.5	34.6 ± 3.5	$\Gamma_{f0(980)} = 10 - 100$
$\Gamma(Y \to J/\Psi + K^+ + K^-)$	1.2 ± 0.1	5.1 ± 0.6	$\Gamma_Y = 49 \pm 8$
$\Gamma(Y \to J/\Psi + S_1(\pi^+ + \pi^-)$	24.5 ± 2.5	23.1 ± 2.4	$\Gamma_Y = 49 \pm 8$
$\Gamma(Y \to J/\Psi + S_1(\pi^+ + \pi^-)$	1.1 ± 0.1	1.1 ± 0.1	$\Gamma_Y = 49 \pm 8$
$\Gamma(Y \to J/\Psi + S_1(\pi^+ + \pi^-)$	9.5 ± 1.0	9.3 ± 1.0	$\Gamma_Y = 49 \pm 8$
$\Gamma(Y \to J/\Psi + \pi^+ + \pi^-$	35.1 ± 3.6	33.4 ± 3.5	$\Gamma_Y = 49 \pm 8$
$\left[\frac{B(Y \to K^+K^- J/\Psi)}{B(Y \to \pi^+\pi^- J/\Psi)}\right]_{INTEG}$	0.033 ± 0.005	0.15 ± 0.03	0.02 - 0.26 (BESIII, 2024)

Our findings:

- The two-body decay of the wide resonance **f_0(500)** into the $\pi^+\pi^-$ pair is dominated by the same decay of the narrow resonance **f_0(980)**.

- The decay widths $\Gamma(Y \to J/\psi\ f_0(500))$ and $\Gamma(Y \to J/\psi\ f_0(980))$ differ insignificantly despite the large mass difference between the final scalar mesons.

- The ratio $\Gamma(f_0(980) \to \pi^+\pi^-) / [\Gamma(f_0(980) \to \pi^+\pi^-) + \Gamma(f_0(980) \to K^+K)] \approx 0.53$ does not contradict the BaBar (2006) and BESIII (2005) data.

- Taking into account the modified Breit-Wigner distribution function instead of its conventional form considerably decreases the decay width $\Gamma(Y \to K^+K^- J/\psi)$.

- The estimated branching ratio

$$\left[\frac{B(Y \to K^+K^- J/\Psi)}{B(Y \to \pi^+\pi^- J/\Psi)}\right]_{INTEG} \approx 0.033 \pm 0.005$$

is located is near the experimental lower bound reported by the BESIII.

II. Strong decays of the spin-2 partner of charmonium-like state X(3872)

- The first member of the XYZ family, the X(3872) state is located very close to the $(D\overline{D}*)$ threshold -> considered as a shallow bound **J^{PC} = 1^{++}** mesonic molecule.

- Does there exist a possible heavier partner of X(3872) with a similar value for the binding energy, but with **J^{PC} = 2^{++} ?**

⁎ ***Theory***:
- various phenomenological models predict the existence of an isoscalar **2^{++}** $(D^{*}\overline{D}*)$ molecular partner of the X(3872).
- a compact tetraquark model has been employed to explore the **2^{++}** states.
- alternatively, within the conventional pattern of mesons, a **2^{++}** tensor state with a similar mass could also be a conventional charmonium state in the first radial excitation $\chi_{c2}(3930)$.

⁎ ***Experiment***: the Belle collaboration (2022) has reported a hint of an isoscalar structure with mass M = 4014.3 ± 4.0 ± 1.5 MeV and width Γ = 4 ± 11 ± 6 MeV, seen in the $\gamma\ \psi(2S)$ invariant mass distribution via a two-photon process.

How to discriminate the various multiquark configurations of the X2(4014) ?

- An easiest way to disentangle these different multiquark configurations is to analyze the mass splitting $\triangle M = M_{X2(4014)} - M_{X(3872)}$ between the **1**$^{++}$ and **2**$^{++}$ states:
  - molecular approach: $\triangle M \approx 140$ MeV (Refs[14,32])
  - tetraquark model: $\triangle M \approx 80$ MeV (Refs[33])
  - excited charmonium scheme: $\triangle M \approx 30$ - 40 MeV (Refs[27,28])

- Another more complicated, but more reliable way is to investigate the decay properties of the $X_2(4014)$ tensor state:
  - molecular approach: $\boldsymbol{\Gamma(X_2 \to D\overline{D} + D\overline{D}^*)} \approx$ a few MeV (Refs[25])

    $\boldsymbol{\Gamma(X_2 \to D\overline{D} + D\overline{D}^*)} \approx 50$ MeV (Refs[26])
  - excited charmonium scheme: $\boldsymbol{\Gamma(X_2 \to \text{charmed})} \approx$ tens of MeV (Refs[34])

Isospin violation in strong decays of X(3872)

♦ By using the relevant data from the recent LHCb experiment (2023):

$$\boldsymbol{B}(X(3872) \to \rho^0 J/\Psi) = (2.8 \pm 0.7)\% \ \text{ and } \ \boldsymbol{B}(X(3872) \to \omega J/\Psi) = (4.1 \pm 1.4)\% :$$

$$BR_X \approx 1.5 \, .$$

- These data indicate **a severe isospin violation**.
- The decay X(3872) ⟶ 3π J/Ψ via ω ⟶ 3π is suppressed kinematically (see Fig.2).

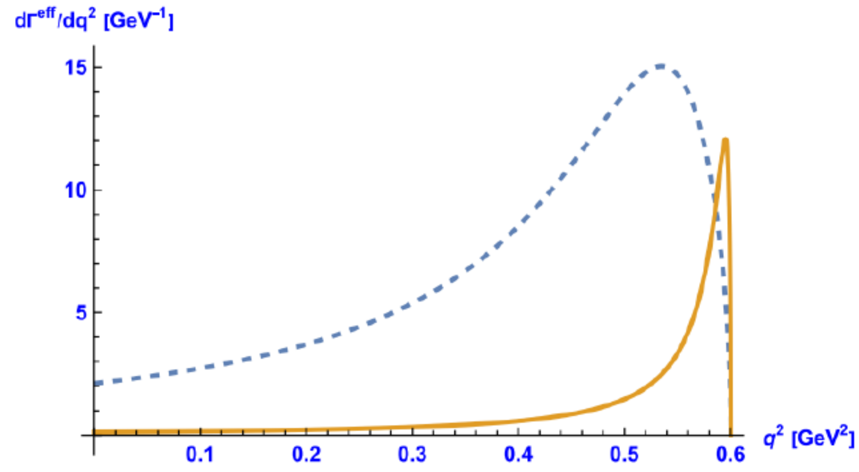


FIG. 2. The behaviors of the effective differential widths of the decays $X(3872) \to \omega(\to 3\pi)J/\Psi$ (the yellow solid curve) and $X(3872) \to \rho(\to 2\pi)J/\Psi$ (the blue dashed line). Hereby, we

♦ The strong decay $X_2 \to \omega\, J/\psi$ conserves isospin, while the other process $X_2 \to \rho^0\, J/\psi$ breaks isospin symmetry.

- The new observed $X_2(4014)$ is considering as the spin-2 partner of X(3872). One may expect that the decays of X_2 into $\rho^0\, J/\psi$ will proceed with strong isospin violation.
- Hereby, since the mass of X_2 is above the $\rho^0(\omega)\, J/\psi$ thresholds the decay into $\omega(\to 3\pi)\, J/\Psi$ is not suppressed, that may be seen below in Fig.3.

♦ Sizeable isospin violating decays of X_2 may be expected.

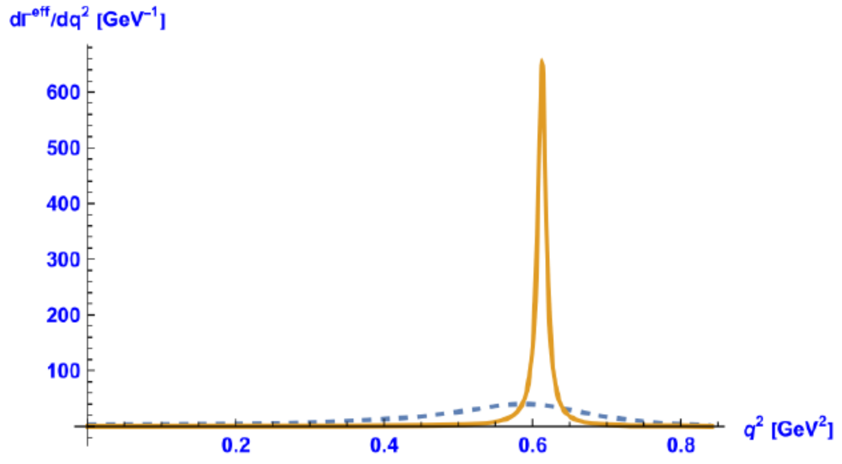


FIG. 3. The behaviors of the effective differential widths of the decays $X(4014) \to \omega(\to 3\pi)J/\Psi$ (the yellow solid curve) and $X(4014) \to \rho(\to 2\pi)J/\Psi$ (the blue dashed line). Hereby, we

Isospin violation in strong decays of $X_2(4014)$

- Consider the ratio of the branching fractions:

$$BR_{X_2} \doteq \frac{\Gamma(X_2 \to \omega J/\Psi)}{\Gamma(X_2 \to \rho^0 J/\Psi)}$$

- It has recently been investigated using the an effective Lagrangian approach by assuming the X_2 as a molecular state of $(D^*\overline{D}*)$. The only contributions from the triangle hadron loops made of the charmed mesons D^* and $\overline{D}*$ have been considered.
- It has been found that the decay widths are quite sensitive to the X_2 mass. At the present center mass M_{X2} = 4.0143 GeV, the widths were estimated:

$\Gamma(X_2 \to \rho^0 J/\Psi) \approx 10^1$ keV

and

$\Gamma(X_2 \to \omega J/\Psi) \approx 10^2 - 10^3$ keV:

BR_{X2} ≈ 15 !

(one order of magnitude larger than ***BR_X*** ≈ 1.5)

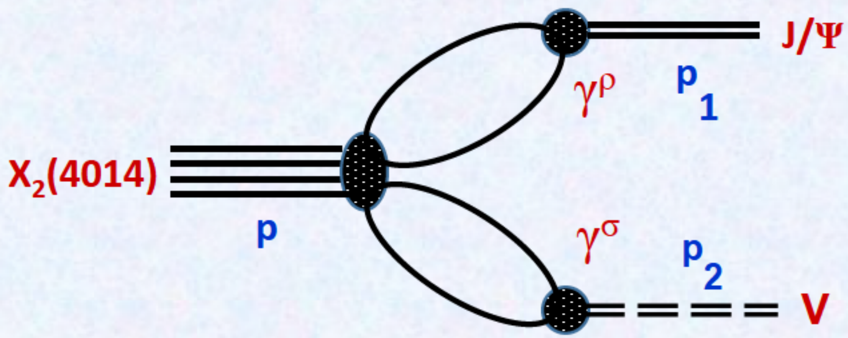
CCQM approach to the strong decays of $X_2(4014)$

in collaboration with M. A. Ivanov (BLTP JINR), Phys.Rev. D 111, 014007 (2025)

- We consider the ***X_2(4014)*** as a 4-quark state with a molecular-type interpolating current:

$$J^{\mu}_{X_2}(x_1,x_2,x_3,x_4)=\frac{1}{\sqrt{2}}\Big\{[\bar{q}(x_3)\gamma^{\mu}c(x_1)]\cdot[\bar{c}(x_2)\gamma^{\nu}q(x_4)]\ +\ (\gamma^{\mu}\leftrightarrow\gamma^{\nu})\Big\},\qquad q=\{u,d\}$$

- ***Decay schemes: $X_2\to\rho^0 J/\Psi$ and $X_2\to\omega J/\Psi$***



- Matrix element:

$$\mathrm{M}_{X_2JV}=i(2\pi)^4\delta^{(4)}(p-p-p)\,\varepsilon_{\mu\nu}(p)\,\varepsilon^*_{\rho}(p_1)\,\varepsilon^*_{\sigma}(p_2)\cdot T^{\mu\nu\rho\sigma}_{X_2JV}(p_1,p_2)$$

- We calculate:

$$|\mathcal{M}_{X_2JV}|^2 \sim |\varepsilon_{\mu\nu}(p)\varepsilon^*_\rho(p_1)\varepsilon^*_\nu(p_2)T^{\mu\nu\rho\sigma}_{X_2JV}|^2 = M^4_{X_2}(C^V_A\cdot A^2_V + C^V_{AB}\cdot A_V\cdot B_V + C^V_B\cdot B^2_V),$$

- Coefficients are completely defined through the meson masses as follows:

$$\begin{aligned} C^V_A &\doteq [(3+2\xi_V)\xi_J^4 + (1-\xi_V)^4(3+2\xi_V) + (28+20\xi_V - 8\xi_V^2)\xi_J^3 \\ &\quad + 4\xi_J(1-\xi_V)^2(7+5\xi_V-2\xi_V^2) - 2\xi_J^2(31-52\xi_V+27\xi_V^2-6\xi_V^3)]/(12\xi_J), \\ C^V_{AB} &\doteq [\xi_J^4(7-2\xi_V) - (1-\xi_V)^4(3+2\xi_V) - 2\xi_J^3(9+5\xi_V-4\xi_V^2) \\ &\quad + 2\xi_J(1-\xi_V)^2(1+5\xi_V+4\xi_V^2) + 4\xi_J^2(3-\xi_V+\xi_V^2-3\xi_V^3)]/(6\xi_J), \\ C^V_B &\doteq [3+4\xi_J+2\xi_V)(\xi_J^2+(1-\xi_V)^2-2\xi_J(1+\xi_V)]^2/(12\xi_J), \end{aligned}$$

$\xi_J \doteq M^2_{J/\psi}/M^2_{X_2}$ and $\xi_V \doteq M^2_V/M^2_{X_2}$ for $V = \{\rho^0, \omega\}$.

- The two-body strong decay width reads:

$$\Gamma_{X_2JV} = \frac{1}{2S+1}\frac{|\vec{p}_1|}{8\pi M^2_{X_2}}\sum_{polariz}|\mathrm{M}_{X_2JV}|^2$$

where S=2 is the spin of X_2, final state three-momentum and the Kallen kinematical function:

$$|\vec{p}_1| \doteq \frac{1}{2M_{X_2}}\lambda^{1/2}(M^2_{X_2}, M^2_{J\Psi}, M^2_V), \qquad \lambda(x,y,z) \doteq x^2+y^2+z^2-2(xy+xz+yz).$$

Numerical results

1. Early experimental data and rough theoretical estimates:

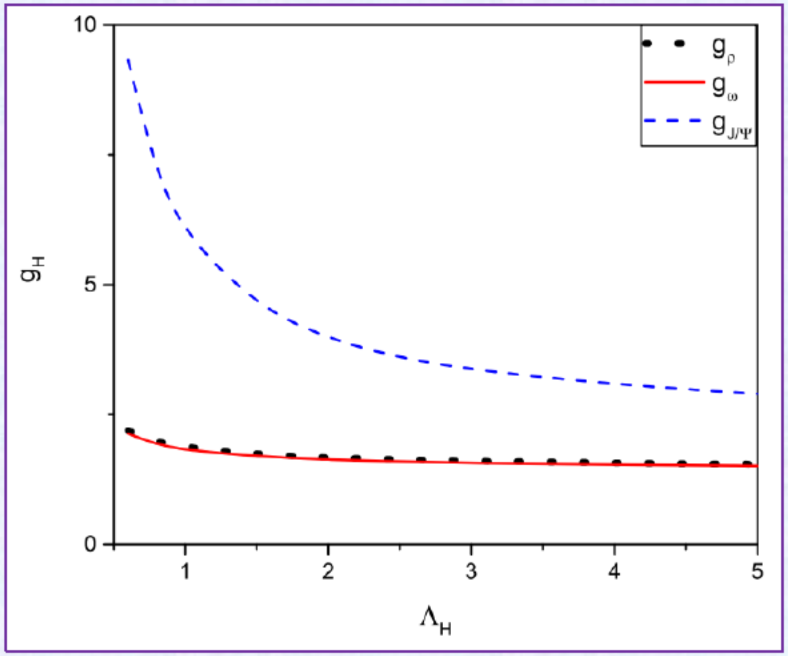
TABLE III. The known data from early experiments of *BABAR* [65], Belle [66], and BESIII [67] for the strong-decay width ratio $\mathrm{BR}_X^{\mathrm{expr}} \doteq \Gamma(X(3872) \to \omega J/\psi)/\Gamma(X(3872) \to \rho^0 J/\psi)$ and corresponding rough recalculations for $\mathrm{BR}_X^{\mathrm{rough}}$ [59].

	BABAR [65]	Belle [66]	BESIII [67]
$\mathrm{BR}_X^{\mathrm{expr}}$	0.8	1.0	1.43
$\mathrm{BR}_X^{\mathrm{rough}}$	0.9	1.1	1.6

2. Central values of the CCQM parameters of the CCQM are determined by minimizing χ^2 in a fit to the available experimental data and some lattice results. The fitted parameters may vary around their central value by about ±10%.

λ	$m_{u/d}$	m_s	m_c	$\Lambda_{J/\Psi}$
0.181 GeV	**0.241**	**0.428**	**1.67**	**1.55**

3. The renormalized couplings g_H of the mesons ($H=J/\psi$, ω, ρ^0) in dependence on Λ_H:

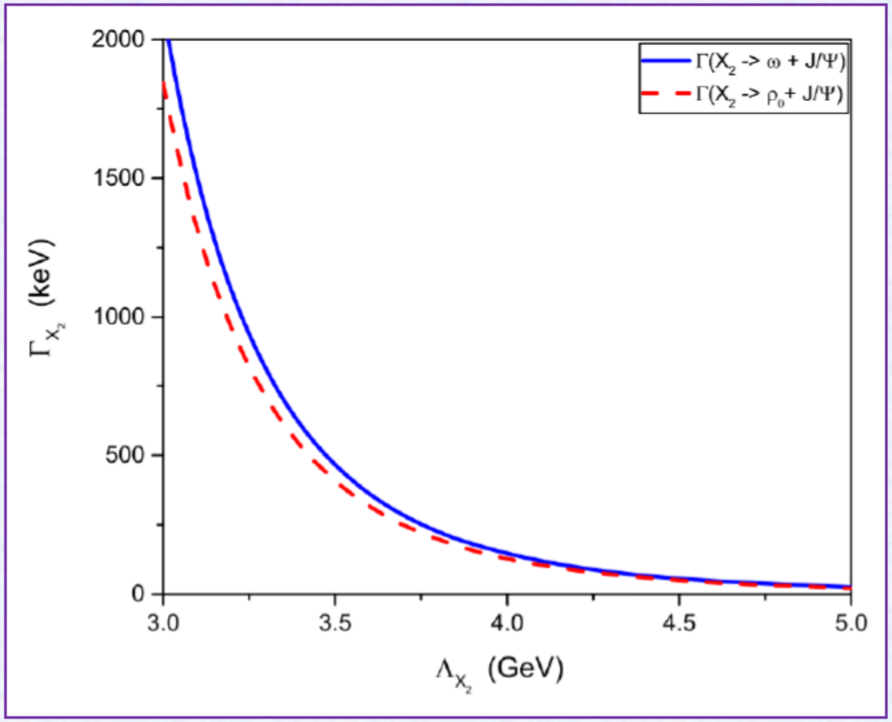


4. A simple and rough approximation to the desired branching ratio BR_{X2} by neglecting the difference in matrix elements :

$$\frac{\lambda^{1/2}(M_{X_2}^2, M_{J/\psi}^2, M_\omega^2)}{\lambda^{1/2}(M_{X_2}^2, M_{J/\psi}^2, M_{\rho^0}^2)} \approx 0.976, \qquad \frac{g_\omega^2}{g_{\rho^0}^2} \approx 0.780. \quad \longrightarrow \quad BR_{X_2}^{approx} \approx 0.762$$

5. *Accurate calculations*: The partial decay widths in dependence on Λ_{X2}:

Λ_{X_2} [GeV]	3.0	3.5	4.0	4.5	5.0
$\Gamma(X_2 \to \omega J/\psi)$ [keV]	1825.3	430.4	138.4	54.4	24.6
$\Gamma(X_2 \to \rho^0 J/\psi)$ [keV]	1600.9	376.7	120.9	47.5	21.5
$\mathrm{BR}_{X_2}^{\mathrm{CCQM}}$	1.140	1.143	1.145	1.146	1.146



Almost cancellation or at least weakening of the size-dependence!

6. ***Accurate calculations***: The partial decay widths in dependence on M_{X2}:

- Recently (Ref[59](2024)), the decays $X_2 \to \omega\, J/\psi$ and $X_2 \to \rho^0\, J/\psi$ via the intermediate meson loops have been estimated in within the effective field theory within a molecular picture.
  The decay widths were strongly dependent on the X_2 mass.

TABLE IV. Comparison of some theoretical predictions for the strong decay width ratio $\mathrm{BR}_{X_2} \doteq \Gamma(X_2(4014) \to \omega J/\psi)/\Gamma(X_2(4014) \to \rho^0 J/\psi)$.

M_{X_2} [GeV]	4.0137	4.0143	4.0167
$\mathrm{BR}_{X_2}^{(\mathrm{EFT})}$ [59]	~ 3.0	~ 15.0	$\sim 10^3$
$\mathrm{BR}_{X_2}^{(\mathrm{CCQM})}$	~ 1.143	~ 1.144	~ 1.144

- Our numerical results show that the decay widths monotonically increase with no peaks and drops in the mass interval from 4.010 to 4.020 GeV;
  the width ranges from 260 to 274 keV
  for $X_2 \to \omega\, J/\psi$,
  while
  for $X_2 \to \rho^0\, J/\psi$,
  it is between 227 and 240 keV, by keeping the branching ratio almost constant:
  $(\mathrm{BR}_{X2})^{\mathrm{CCQM}} \approx 1.14$

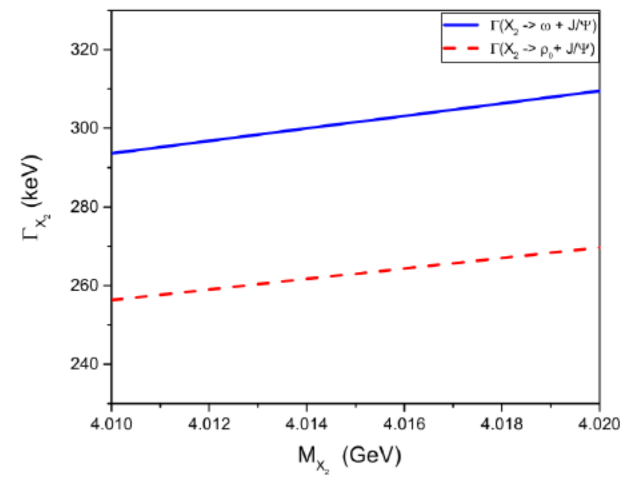


FIG. 7. The dependencies of the partial decay widths $\Gamma(X_2 \to \omega J/\psi)$ and $\Gamma(X_2 \to \rho^0 J/\psi)$ on the exotic state mass M_{X_2} for a fixed size parameter value $\Lambda_{X_2} = 3.70$ GeV.

Summary - our findings:

- A simple and rough approximation by taking into account only effects due to the phase space factors and renormalized couplings results in the branching ratio:

$$BR_{X2} \approx 0.762.$$

- Our numerical results show that the decay widths monotonically increase with no peaks and drops in the mass interval from 4.010 to 4.020 GeV; the width ranges from 260 to 274 keV for $X_2 \to \omega\, J/\psi$, while for $X_2 \to \rho^0\, J/\psi$, it is between 227 and 240 keV, by keeping the branching ratio almost constant:

$$(BR_{X2})^{CCQM} \approx 1.14$$

- The reason is simple: the hidden charm decays of the X_2 in our approach occur via the confined quark loops but not the charmed D^* and $\overline{D}^*$ meson loops, so with no threshold effects.