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Higher-order strongly intensive quantities for rapidity correlations in string models

NUCLEUS-2025, 2 JULY 2025, SAINT-PETERSBURG

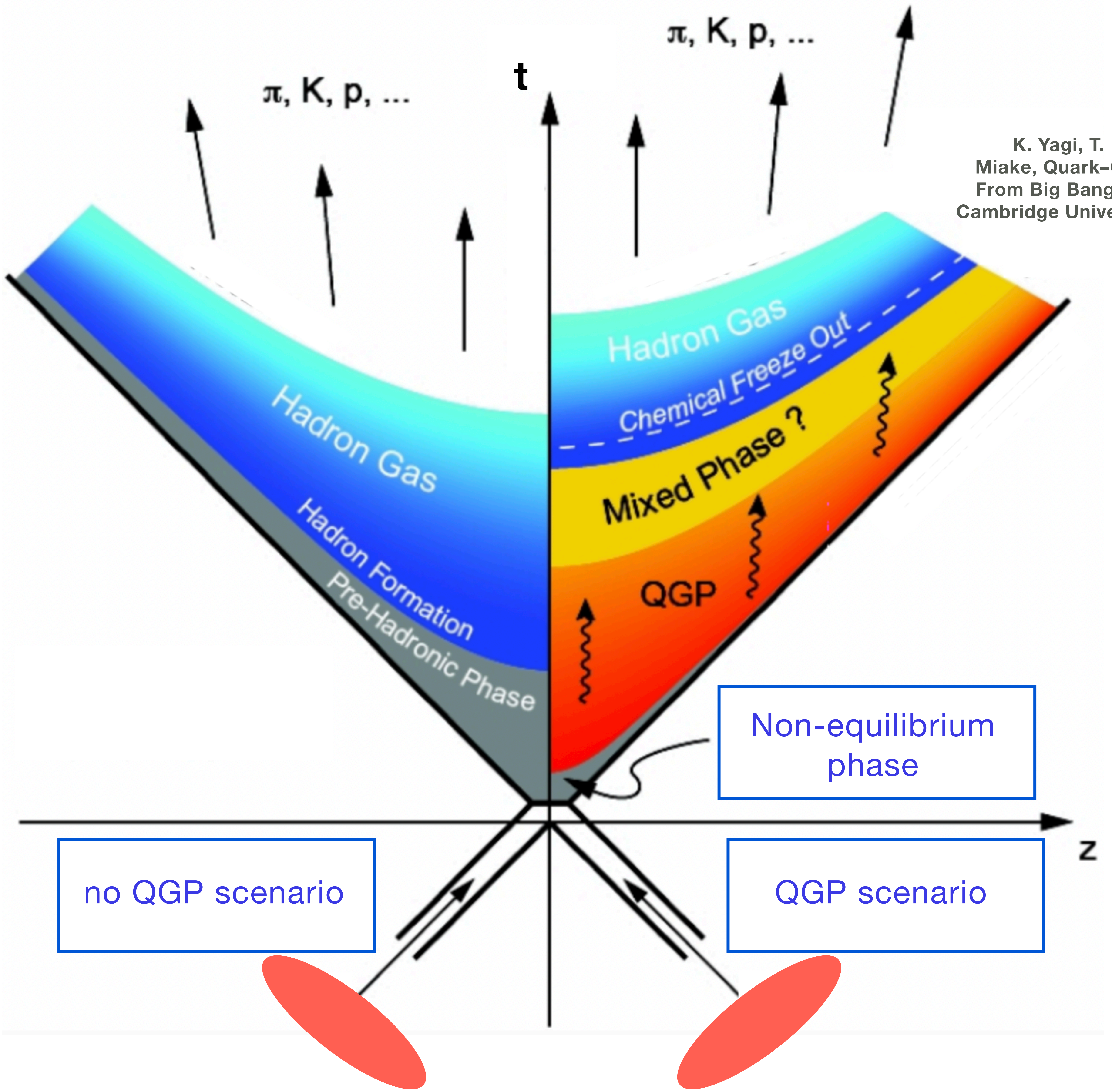
p+p and A+A collisions

Predicted and observed QGP «signals» for A+A:

- azimuthal anisotropy (flow)
- enhanced strange particles yields
- jet quenching

Now, in 2025 we know that some of these effects are present in *p+p* interactions:

- QGP in p+p?
- other source of collectivity?



K. Yagi, T. Hatsuda, Y. Miake, Quark–Gluon Plasma - From Big Bang to Little Bang, Cambridge University Press, 2005

Color string models

Two-stage process:

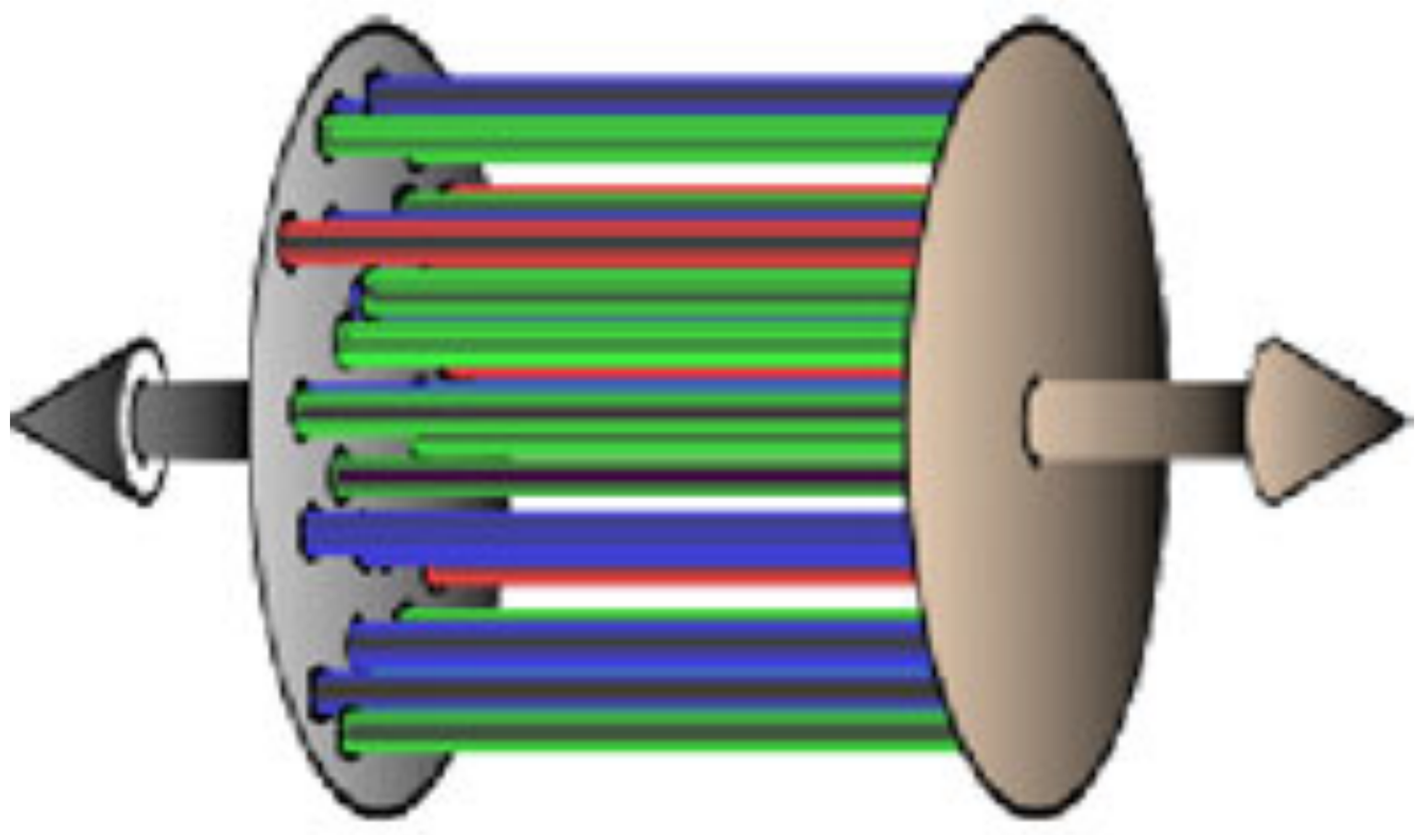
- partons' colour charges form tubes of colour field
- tubes are hadronized via Schwinger mechanism (tunnelling)

but!

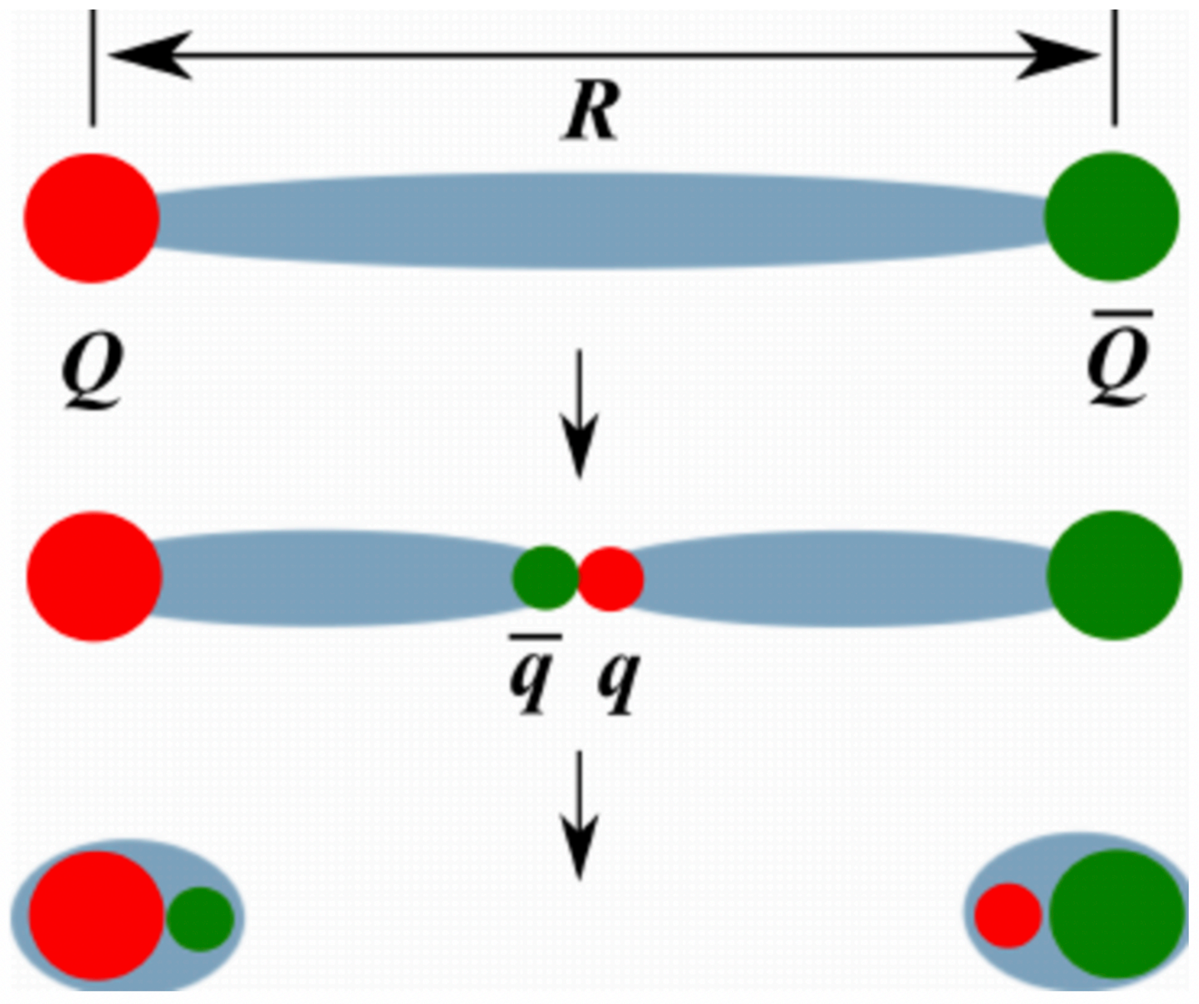
- string may interact before hadronization
  [M.A. Braun, C. Pajares, Phys.Lett.B 287, 154 (1992)]
- how can we distinguish between different scenarios of string-string interactions?

Goal:

- find observables that are sensitive to details of string fragmentation



F. Gelis et al., Ann.Rev.Nucl.Part.Phys. 60, 463 (2010)



M.N. Chernodub, Mod.Phys.Lett.A 29, 1450162 (2014)

Strongly intensive quantities

Extensive quantities:

- proportional to the system's volume
- e.g. mean multiplicity $\langle N \rangle$

Intensive quantities:

- independent of the expectation value of the system's volume
- e.g. event-mean transverse momentum $\langle \overline{p_T} \rangle$

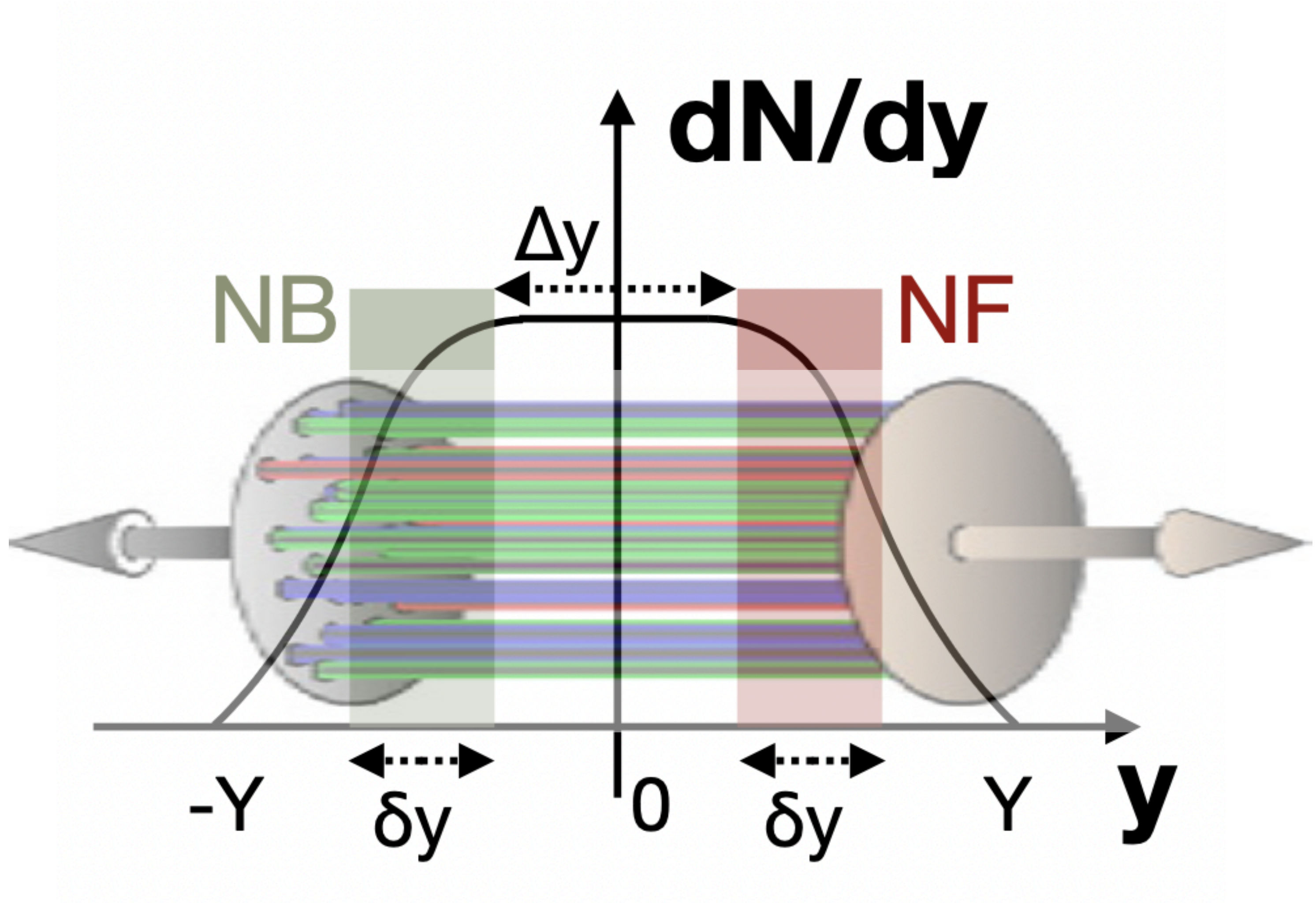
Strongly intensive quantities: **M.I. Gorenstein, M. Gazdzicki Phys.Rev.C 84, 014904 (2011)**

- independent of the expectation value of the system's volume and of the volume fluctuations
- e.g. $\Sigma[A,B] = \frac{1}{C_\Sigma}\left(\omega[B]\langle A \rangle + \omega[A]\langle B \rangle - 2cov(A,B)\right)$

$\langle N_F \rangle$ - mean A, $\omega[A] = \frac{\langle A^2 \rangle - \langle A \rangle^2}{\langle A \rangle}$ - scaled variance

Rapidity correlations

E.V. Andronov, Theor. Math. Phys. 185(1), 1383 (2015)



Strings - source of correlated particle production

- **Q**: how can one quantify degree of collectivity_
- **A**: joint particle number fluctuations in separated rapidity intervals
- Let us construct SIQ for multiplicities in two windows

$$\Sigma[N_F, N_B] = \frac{\omega[N_B] \cdot \langle N_F \rangle + \omega[N_F] \cdot \langle N_B \rangle - 2 \cdot cov(N_F, N_B)}{\langle N_B + N_F \rangle}$$

$\langle N_F \rangle$ - mean NF, $\omega[N_F] = \frac{\langle N_F^2 \rangle - \langle N_F \rangle^2}{\langle N_F \rangle}$ - scaled variance

In case of symmetric windows we have

$$\Sigma[N_F, N_B] = \omega[N_B] - \frac{cov(N_F, N_B)}{\langle N_B \rangle}$$

Independent strings

E. Andronov, V. Vechernin, Eur.Phys.J.A 55(1), 14 (2019).

V. Vechernin, **E. Andronov**, Universe 5(1), 15 (2019).

E. Andronov, V. Vechernin, Phys.Part.Nucl. 51(3), 337 (2020).

Model:

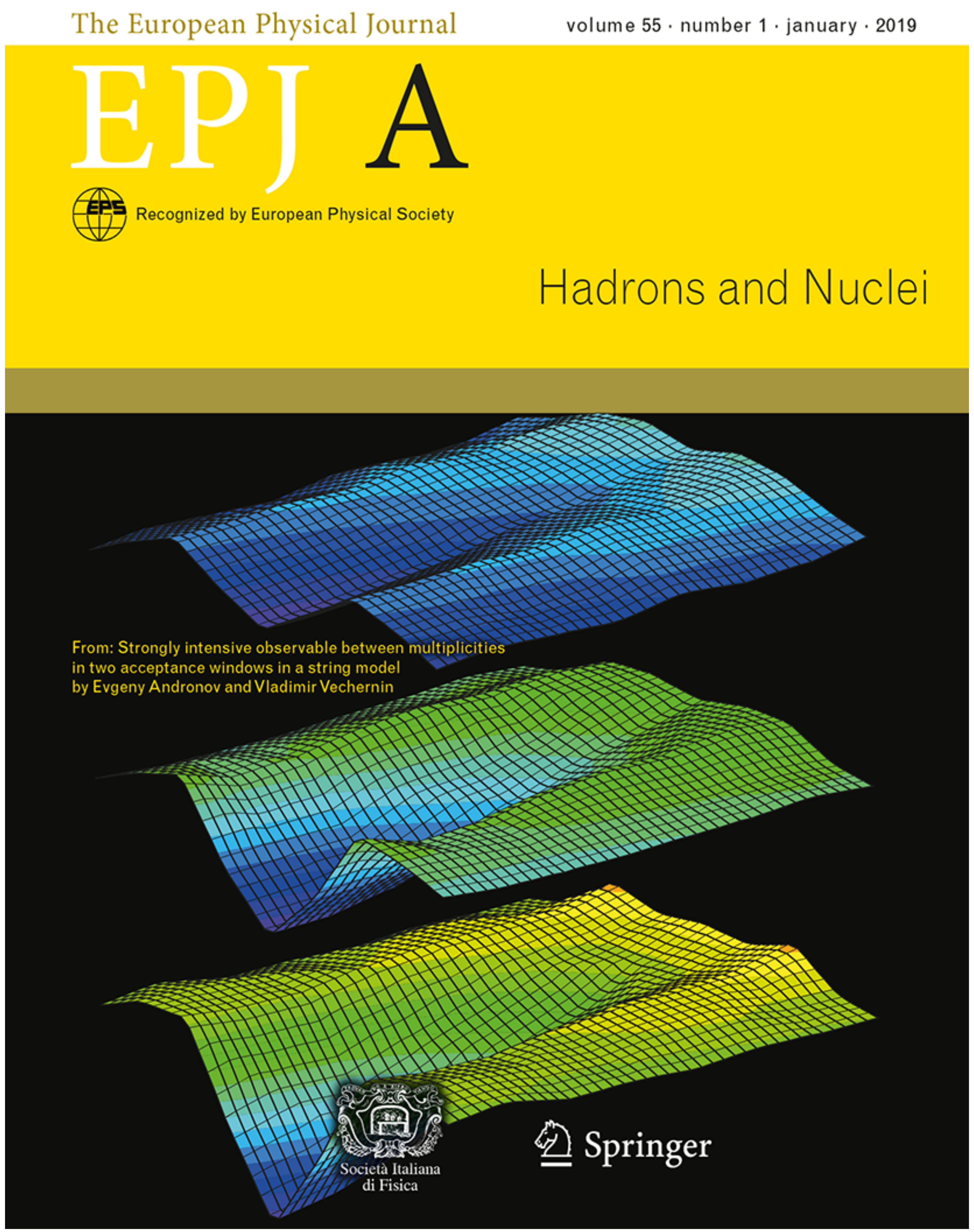
- Infinitely long non-interacting strings

Datasets:

- p+p interactions at 900, 2760 and 7000 GeV

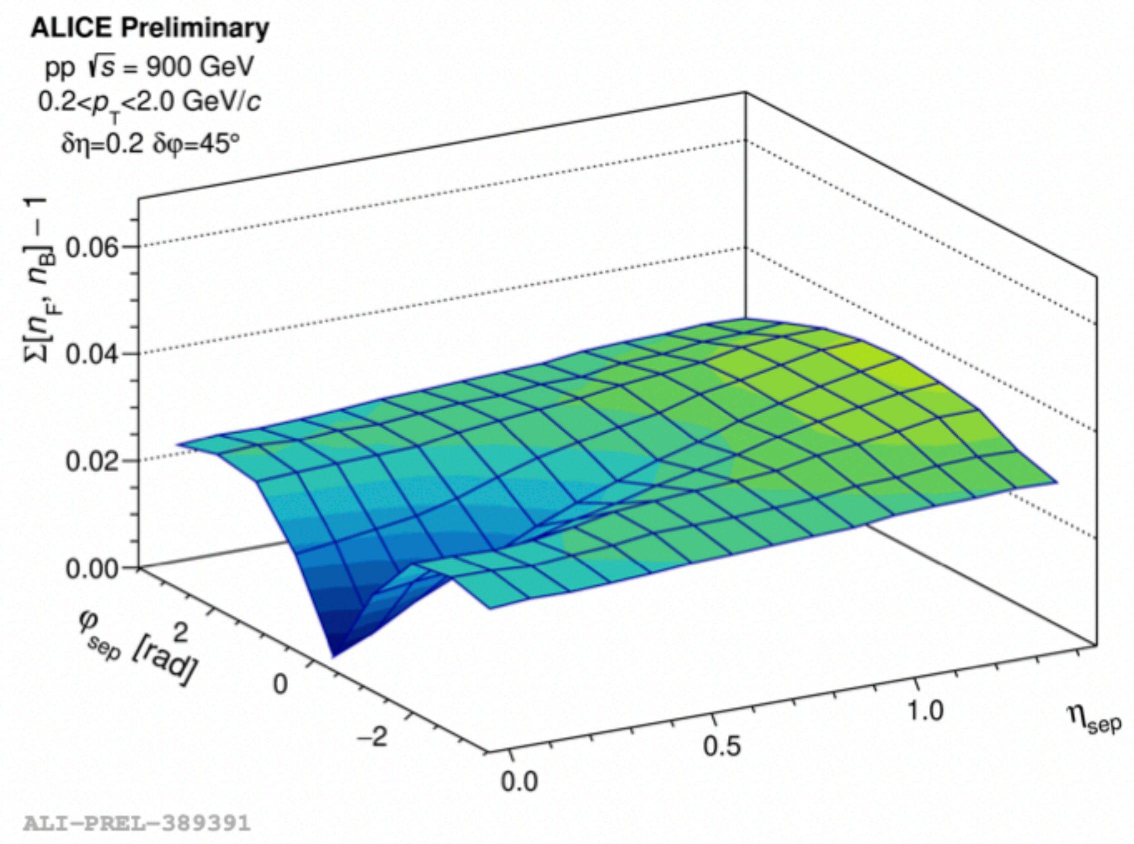
Results:

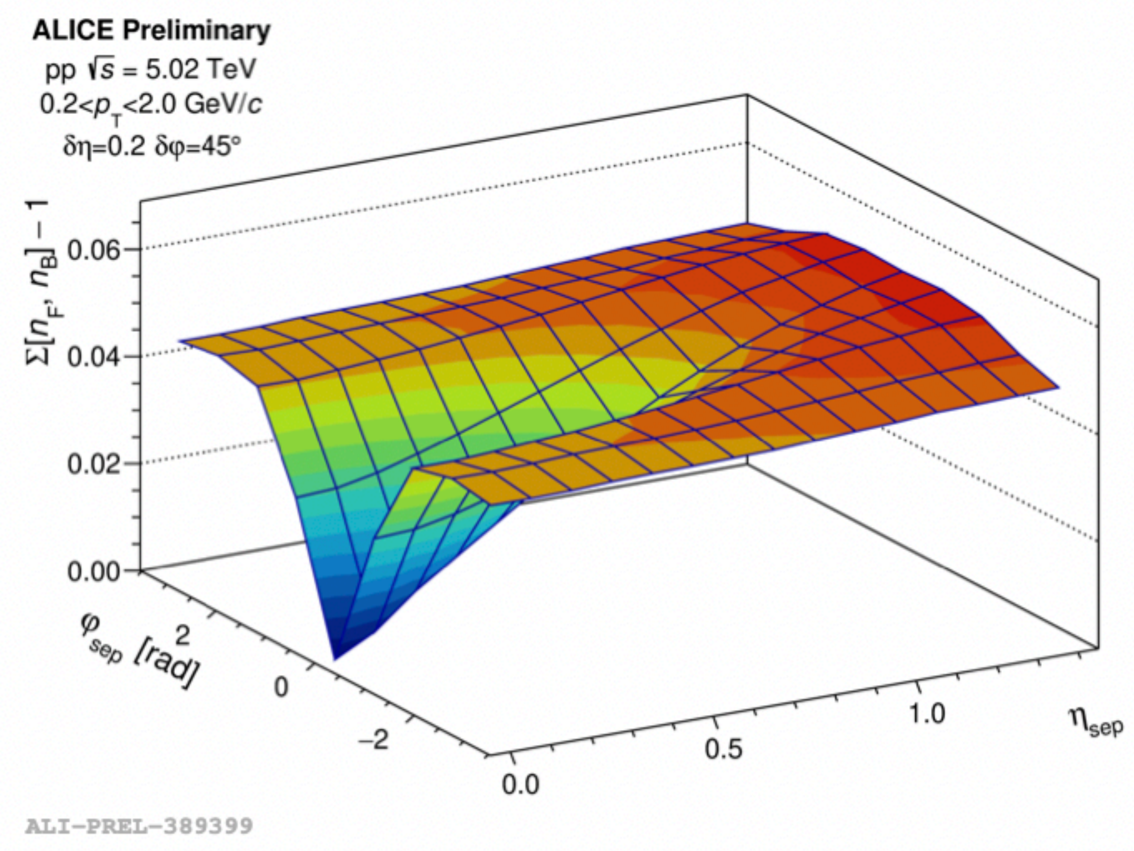
- rapidity and azimuthal dependence of Σ -> string fragmentation function

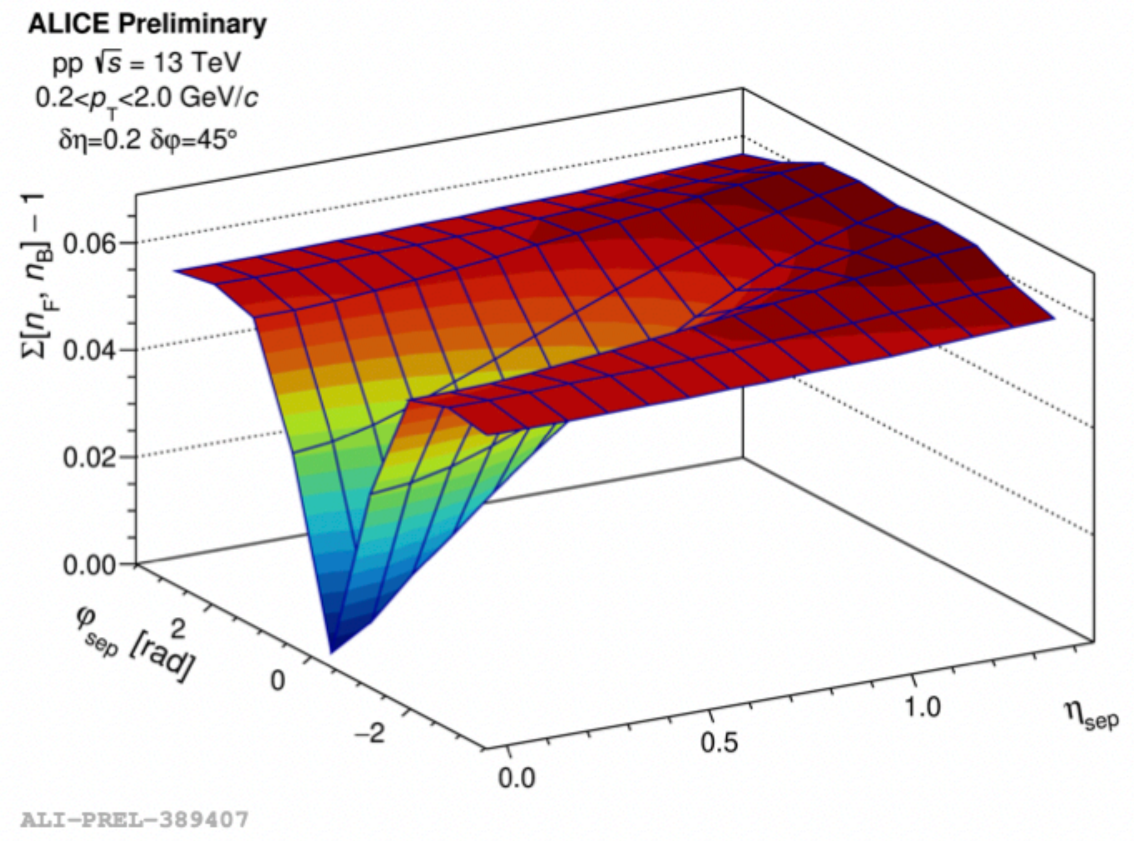


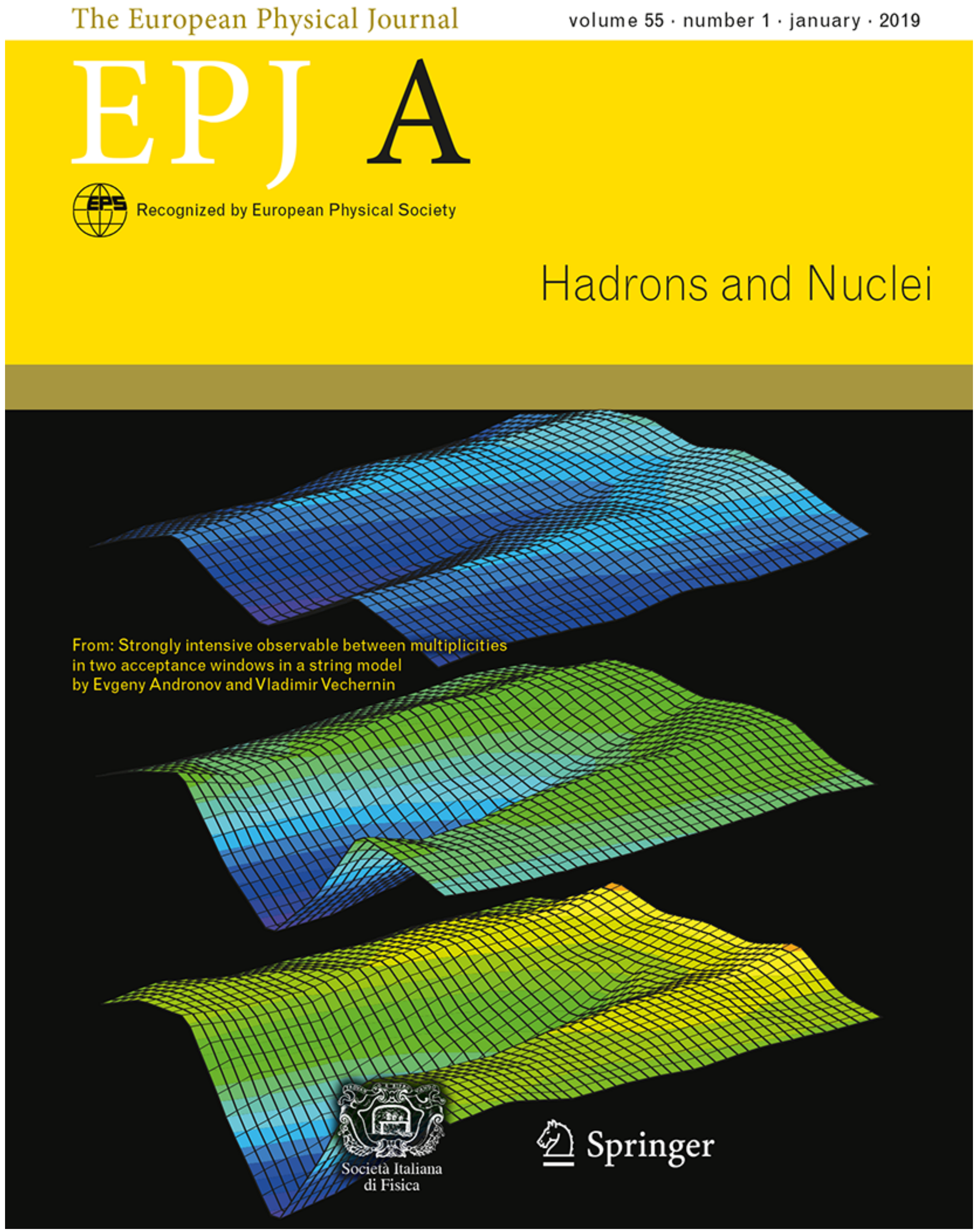
ALICE experiment in two years:

ALICE, *PoS* CPOD2021 (2022) 027









Interacting strings

ALICE, *PoS* CPOD2021 (2022) 027

E. Andronov, V. Vechernin, Eur.Phys.J.A 55(1), 14 (2019).

V. Vechernin, **E. Andronov**, Universe 5(1), 15 (2019).

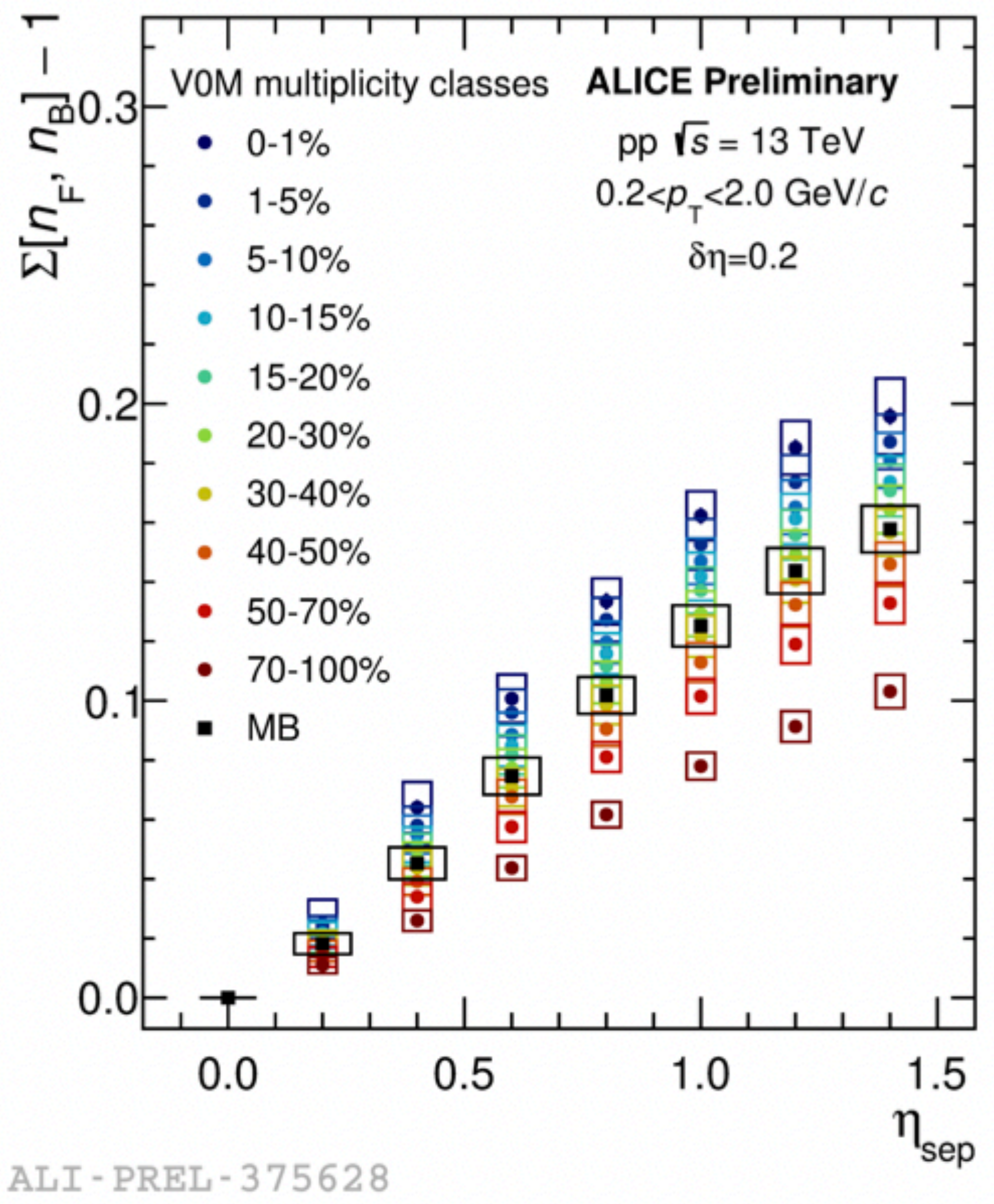
E. Andronov, V. Vechernin, Phys.Part.Nucl. 51(3), 337 (2020).

One type of strings:

- $\Sigma[N_F, N_B] = \Sigma[\mu_F, \mu_B]$ - SIQ for a single string

Two types of strings:

- $$\Sigma[N_F, N_B] = \frac{\overline{n_{str,1}}}{\overline{n_{str,1}} + \overline{n_{str,2}}}\Sigma^1[\mu_F, \mu_B] + \frac{\overline{n_{str,2}}}{\overline{n_{str,1}} + \overline{n_{str,2}}}\Sigma^2[\mu_F, \mu_B]$$
- no longer strongly intensive
- fraction of strings of different types unknown



Breaking of strong intensity?

Interacting strings

$$\Sigma[N_F, N_B] = \frac{\overline{n_{str,1}}}{\overline{n_{str,1}} + \overline{n_{str,2}}}\Sigma^1[\mu_F, \mu_B] + \frac{\overline{n_{str,2}}}{\overline{n_{str,1}} + \overline{n_{str,2}}}\Sigma^2[\mu_F, \mu_B]$$

- one can extract info on ratios with additional measurements
- let's look at rapidity interval (c) separated from both F and B windows
  - $\langle N_C \rangle = \overline{n_{str,1}} \cdot \overline{\mu_{C,1}} + \overline{n_{str,2}} \cdot \overline{\mu_{C,2}}$
  - $\langle P_{T,C} \rangle = \overline{n_{str,1}} \cdot \overline{\mu_{C,1}} \cdot \overline{p_{T,C,1}} + \overline{n_{str,2}} \cdot \overline{\mu_{C,2}} \cdot \overline{p_{T,C,1}}$ - mean sum of transverse momenta

Clearly:

$$\frac{\overline{n_{str,1}}}{\overline{n_{str,1}} + \overline{n_{str,2}}} = \frac{1}{\overline{\mu_{C,2}}} \cdot \frac{\overline{p_{T,C,2}}\langle N_C \rangle - \langle P_{T,C} \rangle}{(\overline{p_{T,C,2}}\,\overline{\mu_{C,2}} - \overline{p_{T,C,1}}\,\overline{\mu_{C,1}})\langle N_C \rangle - (\overline{\mu_{C,2}} - \overline{\mu_{C,1}})\langle P_{T,C} \rangle}$$

$$\frac{\overline{n_{str,2}}}{\overline{n_{str,1}} + \overline{n_{str,2}}} = \frac{1}{\overline{\mu_{C,1}}} \cdot \frac{-\overline{p_{T,C,1}}\langle N_C \rangle + \langle P_{T,C} \rangle}{(\overline{p_{T,C,2}}\,\overline{\mu_{C,2}} - \overline{p_{T,C,1}}\,\overline{\mu_{C,1}})\langle N_C \rangle - (\overline{\mu_{C,2}} - \overline{\mu_{C,1}})\langle P_{T,C} \rangle}$$

Independent strings - higher-order SIQs

In W. Broniowski, A. Olszewski, Phys. Rev. C 95, 064910 (2017) explicit relations of higher-order multiplicity cumulants were presented for the model of independent sources

P - cumulants of final observables, Q - cumulants in number of sources, R -cumulants from a single source

$$\langle N_F \rangle = \overline{n_{str}} \cdot \overline{\mu_F} = P_{01} = Q_1 \cdot R_{01} \qquad \langle N_B \rangle = \overline{n_{str}} \cdot \overline{\mu_B} = P_{10} = Q_1 \cdot R_{10}$$

$$\langle N_F^2 \rangle - \langle N_F \rangle^2 = P_{02} = Q_2 \cdot R_{01}^2 + Q_1 \cdot R_{02} \qquad \langle N_B^2 \rangle - \langle N_B \rangle^2 = P_{20} = Q_2 \cdot R_{10}^2 + Q_1 \cdot R_{20}$$

$$\langle N_F N_B \rangle - \langle N_F \rangle \langle N_B \rangle = P_{11} = Q_2 \cdot R_{01} R_{10} + Q_1 \cdot R_{11}$$

Clearly, $\Sigma[N_F, N_B] = \dfrac{R_{20} - R_{11}}{R_{10}}$ for symmetric windows - independent of Q

Independent strings - higher-order SIQs

From W. Broniowski, A. Olszewski, Phys. Rev. C 95, 064910 (2017):

P - cumulants of final observables, Q - cumulants in number of sources, R -cumulants from a single source

$$\langle N_F^3\rangle - 3\langle N_F^2\rangle\langle N_F\rangle + 2\langle N_F\rangle^3 = P_{03} = Q_3 \cdot R_{01}^3 + 3Q_2R_{01}R_{02} + Q_1R_{03}$$

$$\langle N_F^2N_B\rangle - 2\langle N_FN_B\rangle\langle N_F\rangle - \langle N_F^2\rangle\langle N_B\rangle + 2\langle N_F\rangle^2\langle N_B\rangle = P_{12} = Q_3 \cdot R_{01}^2R_{10} + Q_2(2R_{01}R_{11} + R_{02}R_{10}) + Q_1R_{12}$$

$$\langle N_B^2N_F\rangle - 2\langle N_FN_B\rangle\langle N_B\rangle - \langle N_B^2\rangle\langle N_F\rangle + 2\langle N_B\rangle^2\langle N_F\rangle = P_{21} = Q_3 \cdot R_{10}^2R_{01} + Q_2(2R_{10}R_{11} + R_{20}R_{01}) + Q_1R_{21}$$

$$\langle N_B^3\rangle - 3\langle N_B^2\rangle\langle N_B\rangle + 2\langle N_B\rangle^3 = P_{30} = Q_3 \cdot R_{10}^3 + 3Q_2R_{10}R_{20} + Q_1R_{30}$$

One can define:

$$\Gamma[N_F, N_B] = \frac{P_{01}P_{30}}{P_{10}^2} - 3\frac{P_{21}}{P_{10}} + 3\frac{P_{12}}{P_{01}} - \frac{P_{10}P_{03}}{P_{01}^2} = \frac{R_{01}R_{30}}{R_{10}^2} - 3\frac{R_{21}}{R_{10}} + 3\frac{R_{12}}{R_{01}} - \frac{R_{10}R_{03}}{R_{01}^2}$$

$\Gamma[N_F, N_B] = 0$ for symmetric windows

Independent strings - higher-order SIQs

For small (->Poissonian distribution in a windows) far-separated (->not correlated) windows:

$$R_{01} = \overline{\mu_F} = \delta y_F \cdot \mu_0 \qquad R_{10} = \overline{\mu_B} = \delta y_B \cdot \mu_0$$

$$R_{12} = \overline{\mu_F^2 \mu_B} - 2\overline{\mu_F \mu_B} \cdot \overline{\mu_F} - \overline{\mu_F^2} \cdot \overline{\mu_B} + 2\overline{\mu_F}^2 \cdot \overline{\mu_B} \approx \overline{\mu_F} \cdot \overline{\mu_B} = \delta y_F \cdot \delta y_B \cdot \overline{\mu_0}^2 = R_{21}$$

$$R_{03} = \overline{\mu_F^3} - 3\overline{\mu_F^2} \cdot \overline{\mu_F} + 2\overline{\mu_F}^3 \approx \overline{\mu_F} = \delta y_F \cdot \mu_0$$

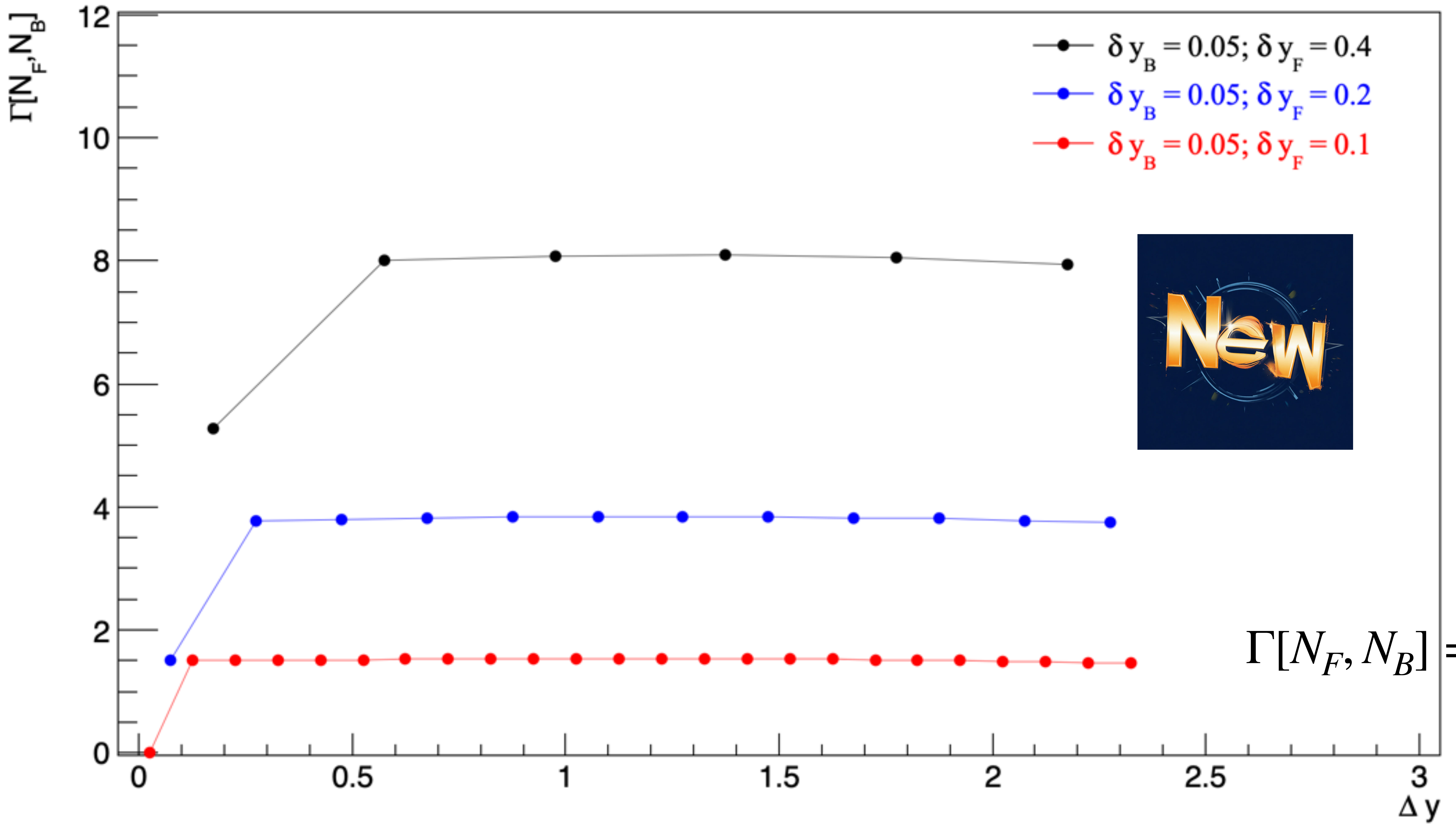
$$R_{30} = \overline{\mu_B^3} - 3\overline{\mu_B^2} \cdot \overline{\mu_B} + 2\overline{\mu_B}^3 \approx \overline{\mu_B} = \delta y_B \cdot \mu_0$$

$$\Gamma[N_F, N_B] = \frac{R_{01}R_{30}}{R_{10}^2} - 3\frac{R_{21}}{R_{10}} + 3\frac{R_{12}}{R_{01}} - \frac{R_{10}R_{03}}{R_{01}^2} \approx \frac{\overline{\mu_F} \cdot \overline{\mu_B}}{\overline{\mu_B}^2} - 3\frac{\overline{\mu_F} \cdot \overline{\mu_B}}{\overline{\mu_B}} + 3\frac{\overline{\mu_F} \cdot \overline{\mu_B}}{\overline{\mu_F}} - \frac{\overline{\mu_B} \cdot \overline{\mu_F}}{\overline{\mu_F}^2}$$

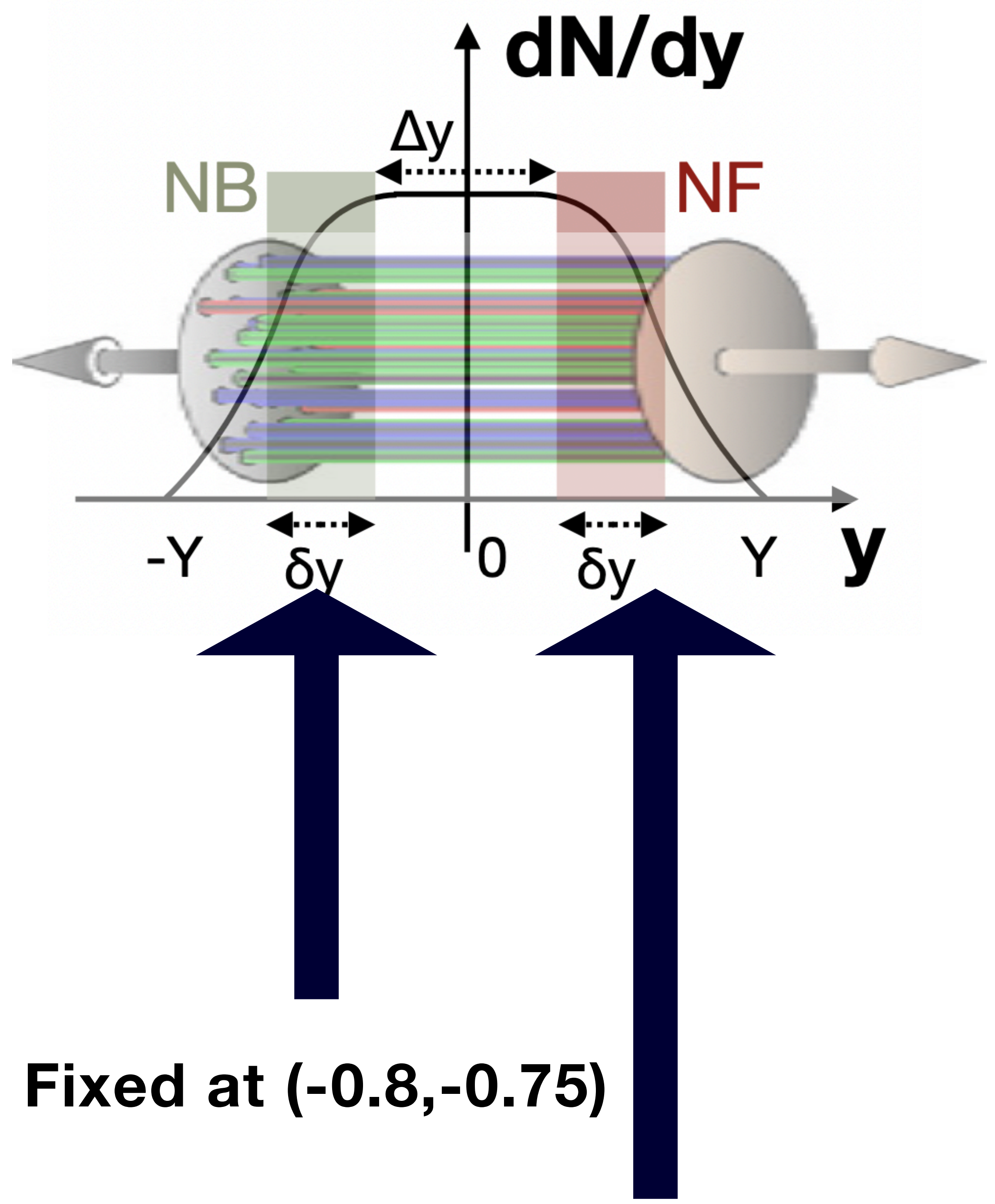
Finally, $\Gamma[N_F, N_B] = 3 \cdot \mu_0 \cdot \left(\delta y_B - \delta y_F\right) + \dfrac{\delta y_F^2 - \delta y_B^2}{\delta y_B \delta y_F}$

Independent strings - higher-order SIQs

PYTHIA8.3, NSD p+p @ $\sqrt{s} = 900$GeV, $0.3 < p_T < 1.5$GeV/c



Distance between centers of windows



Fixed at (-0.8,-0.75)

Moving

$$\Gamma[N_F, N_B] = 3 \cdot \mu_0 \cdot \left(\delta y_B - \delta y_F\right) + \frac{\delta y_F^2 - \delta y_B^2}{\delta y_B \delta y_F}$$

Summary

Second-order strongly intensive quantities:

- in fact - not so strongly intensive!
- can be disentangled by additional measurements

Third-order strongly intensive quantities:

- new observable $\Gamma[N_F, N_B]$ was analyzed in simplified string model and within PYTHIA
- at large separations $\Gamma[N_F, N_B]$ reaches its plateau values

Thank you for your attention!

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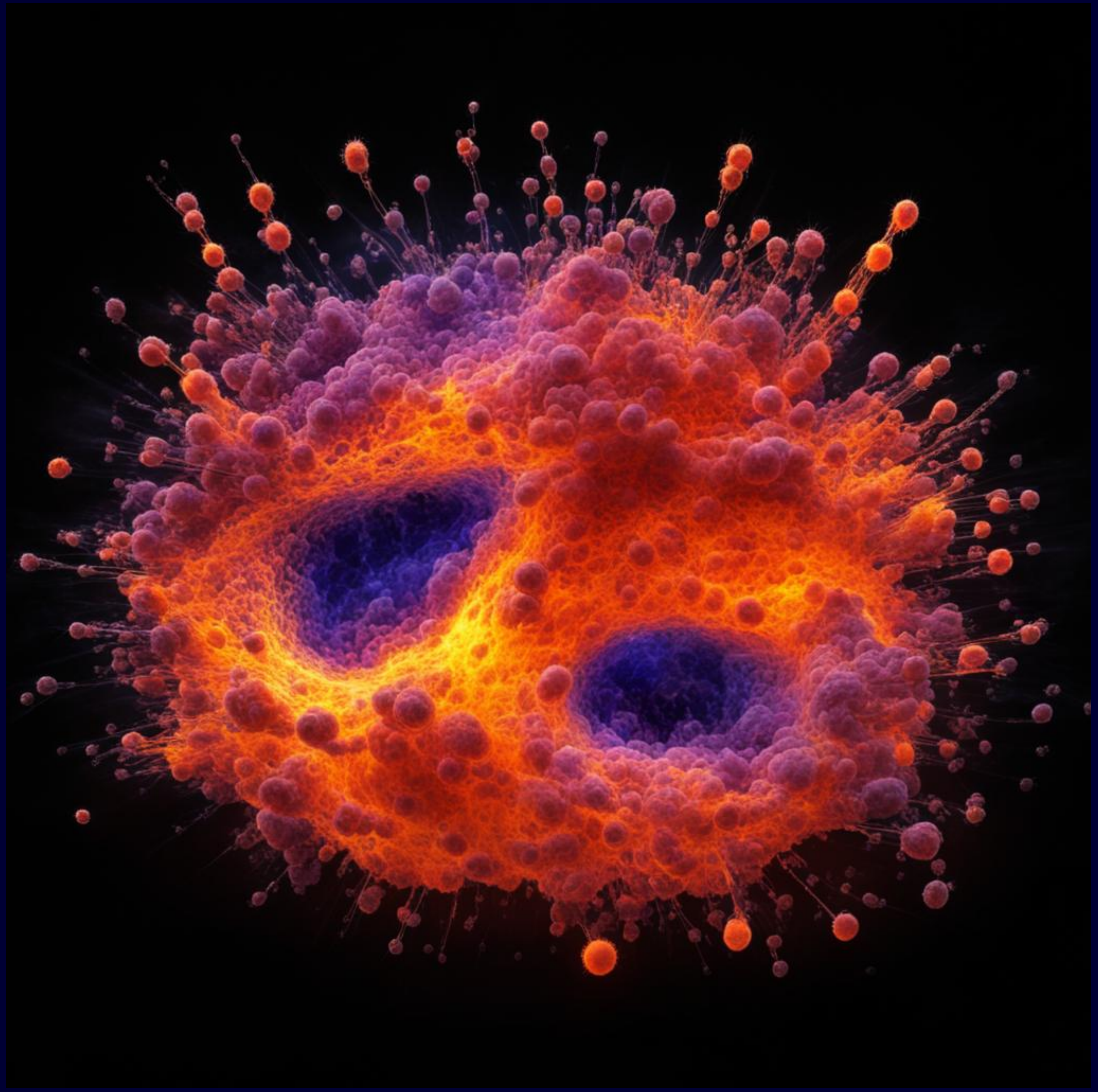


Kandinsky bot: «Quark-gluon plasma»

EXTRA

Independent strings

For independent strings:

$$\Sigma[N_F, N_B] = \frac{\omega[N_B] \cdot \langle N_F \rangle + \omega[N_F] \cdot \langle N_B \rangle - 2 \cdot cov(N_F, N_B)}{\langle N_B + N_F \rangle} = \Sigma[\mu_F, \mu_B]$$

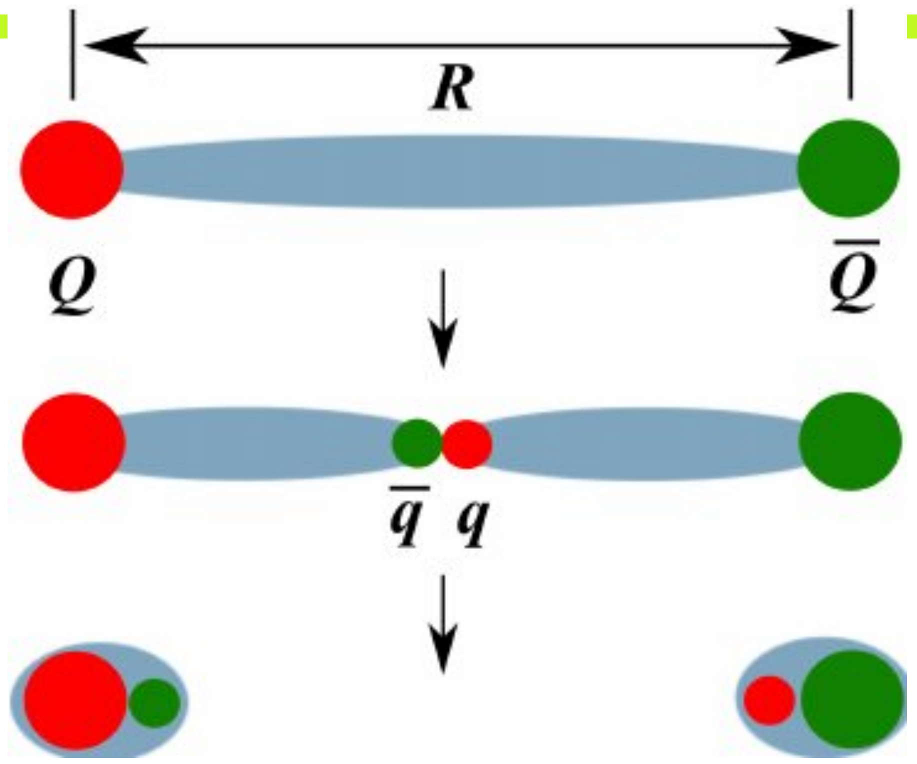
$$\Delta[N_F, N_B] = \frac{\omega[N_B] \cdot \langle N_F \rangle - \omega[N_F] \cdot \langle N_B \rangle}{\langle N_F - N_B \rangle} = \Delta[\mu_F, \mu_B]$$

For small (->Poissonian distribution in a windows) far-separated (->not correlated) windows:

$$\Sigma[N_F, N_B] \approx 1$$

$$\Delta[N_F, N_B] \approx 1$$

Color string models



M.N. Chernodub *Mod.Phys.Lett.A* 29, 1450162 (2014)

- PYTHIA/FRITIOF/QGSM/PHSD/EPOS are among the most successful MC event generators that are able to describe p+p and A+A data (Color strings as particle emitting sources)
- With an increase of the collision energy multi-string configurations start to play a bigger role, ideas: rope formation, string fusion, string repulsion/shoving - useful for description of strangeness enhancement, correlations etc.

V.A. Abramovsky, O.V. Kanchely, JETP Lett. 31, 566 (1980)
T.S. Biro, H.B. Nielsen, J. Knoll, Nucl.Phys.B 245, 449 (1984)
M.A. Braun, C. Pajares, Phys.Lett.B 287, 154 (1992)
I. Altsybeev, AIP Conf. Proc. 1701, 100002 (2016)
I. Altsybeev, G. Feofilov, EPJ Web Conf 125, 04011 (2016)
C. Bierlich, G. Gustafson, L. Lonnblad, Phys.Lett.B 779, 58 (2018)

- Anisotropy in string model can be produced due to the quenching of partons/hadrons momenta due to the presence of the gluon field of the stretched strings (NB: field changes due to interaction of strings) [M.A.Braun,C.Pajares, Eur.Phys.J.C 71, 1558 (2011)] - description of elliptic and triangular flow in A+A collisions