Threshold amplification of X(a,b)Y reactions: from μCF to EM formfactors of hyperons

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LXXV International conference «NUCLEUS-2025»
St.Peterburg State University, 1-6 July 2025

Problem

Physics highlights at BESIII and STCF

Xiaorong Zhou

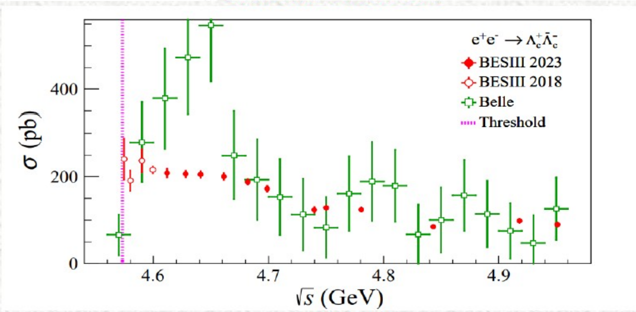
University of Science and technology of China

The XXIV Khariton Topical Scientific Readings on Problems of Accelerator Engineering and High-Energy Physics

25/7/2023, Sarov

Cross section of $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$

M. Ablikim *et al.*
(BESIII Collaboration) PHYSICAL REVIEW LETTERS **131**, 191901 (2023)



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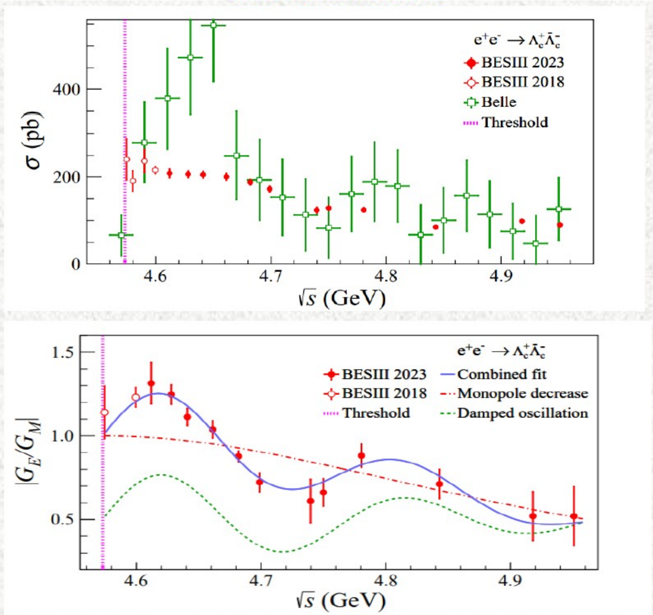

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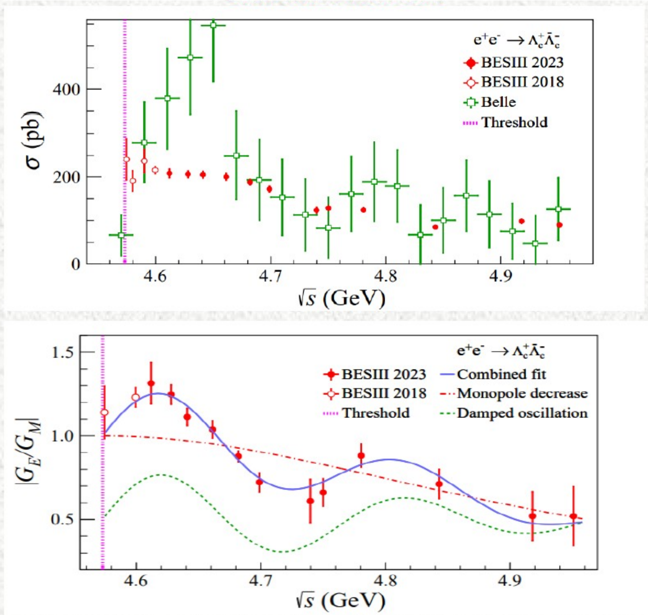

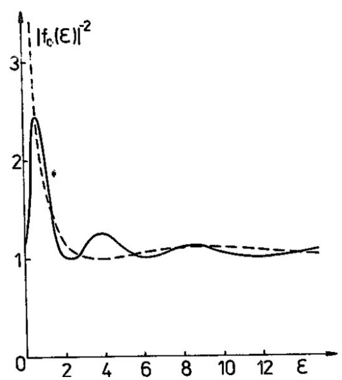

V.S. Melezhik / Resonance amplification ..
Nuclear Physics A550 (1992) 223–234 .

Similar problems ?

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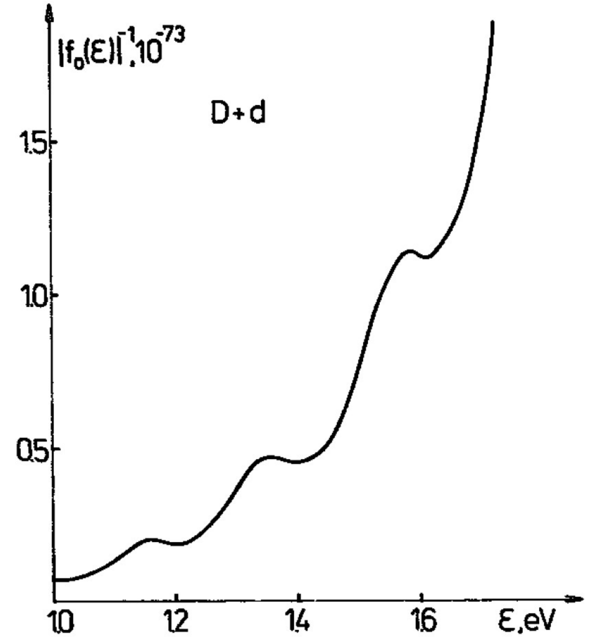
Resonance amplification of the nuclear reaction X(a, b)Y near the a+X channel threshold

V.S. Melezhik

Joint Institute for Nuclear Research, Head P.O. Box 79, Dubna, Moscow Region, Russian Federation

Screening effects in fusion reactions of the type D(d, p)T near the threshold of the channel d+D



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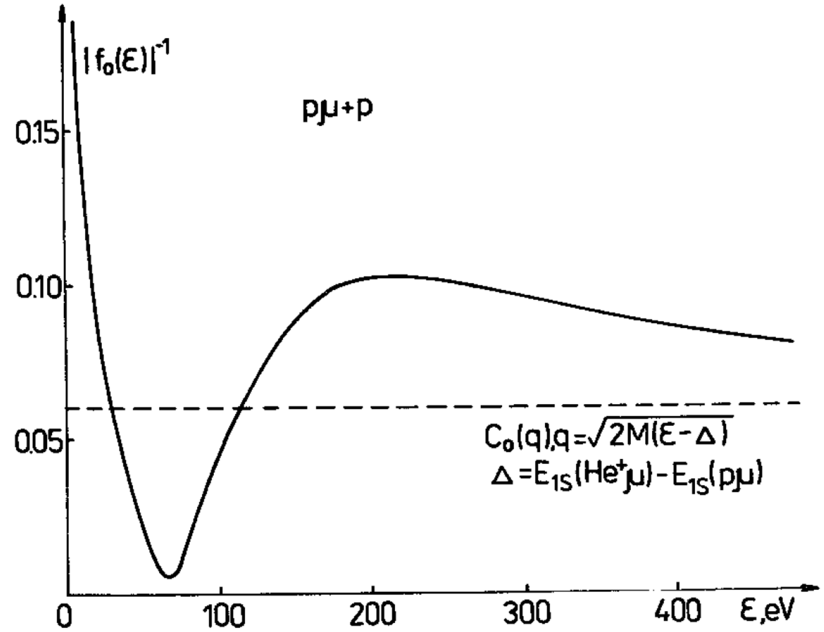
"In flight" fusion reactions in mesic atomic physics

$$t\mu + d \rightarrow {}^4He + n + \mu\,,$$
$$p\mu + p \rightarrow d + e^+ + \mu$$



Simple potential model for X(a,b)Y

V.S. Melezhik, Nucl. Phys. A550, 223 (1992)

It is known that the S-wave cross section of the fusion reaction X(a, b)Y is described near the threshold $\varepsilon \to 0$ of the channel a+X by the formula [1])

$$\sigma^{\text{in}}(\varepsilon) = |\psi_\varepsilon(R=0)|^2 \frac{A}{V}, \qquad A = \text{const}, \tag{1}$$

where R is the distance between the fragments a and X, $\psi_\varepsilon(R)$ is the wave function of relative motion of a and X, and $V = \sqrt{2\varepsilon/M}$ is the relative velocity of their motion.

K.R. Leng, Astrophysical formulae, vol.II (Springer, Berlin 1974
S.Deser, M.L. Goldberger, K. Baumann, W. Thirring, Phys. Rev. 94, 774 (1954)

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If the Coulomb interaction $(1/R)$ occurs between the fragments a and X, formula (1) is reduced to the known Gamow formula

$$\sigma_0^{\text{in}}(\varepsilon) = C_0^2(\varepsilon) \frac{A}{V} \tag{3}$$

$$C_0^2(\varepsilon) = |f_0(\varepsilon)|^{-2} = \frac{2\pi/V}{e^{2\pi/V} - 1}$$

is the Gamow factor), which well describes a great amount of experimental data on fusion-reaction cross sections of the type X(a, b)Y in the range 20–100 keV of the colliding energy ε and is used for the extrapolation of $\sigma_0^{\text{in}}(\varepsilon)$ in the limit $\varepsilon \to 0$

Simple potential model for X(a,b)Y

V.S. Melezhik, Nucl.Phys. A550, 223 (1992)

$$\sigma_J^{in}(\varepsilon) = |f_J(\varepsilon)|^{-2}\frac{A}{V}$$

through the Jost function $f_J(\varepsilon) = |f_J(\varepsilon)|\exp(-i\delta_J)$ of the system a+X

determined from the relation

$$k^J|f_J(\varepsilon)|^{-1}R^J \underset{R\to 0}{\longleftarrow} \psi_\varepsilon^J(R) \underset{R\to\infty}{\longrightarrow} \frac{\sin[kR - \frac{1}{2}J\pi + \delta_J(\varepsilon)]}{kR}$$

and coincident with $|\psi_\varepsilon(R=0)|\exp(-i\delta_J)$ as $J=0$.

L.N. Bogdanova, V.E. Markushin, V.S. Melezhik, L.I. Ponomarev. Sov.J. Nucl. Phys. 34, 662 (1981); Phys. Lett. B115, 171 (1982)

$t + d \rightarrow^4 He + n \quad J = 0$

$d + d \rightarrow^3 He + n, t + p \quad J = 1$

two-channel scattering problem

generalized optical potential

one-channel problem with nonlocal energy-dependent potential

Simple potential model for X(a,b)Y

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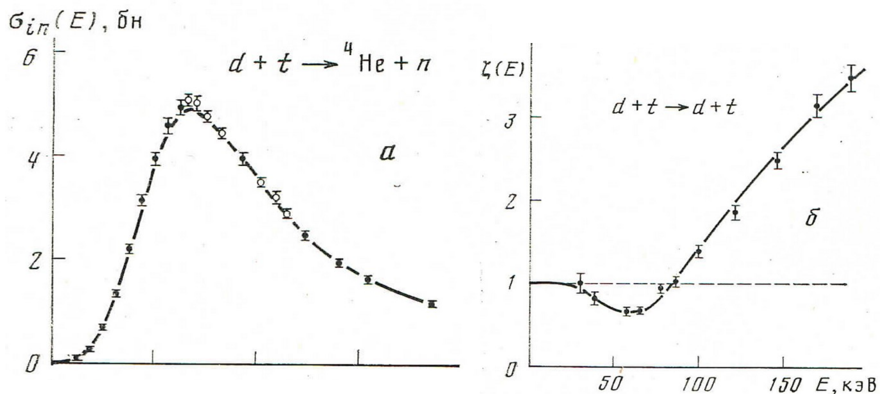

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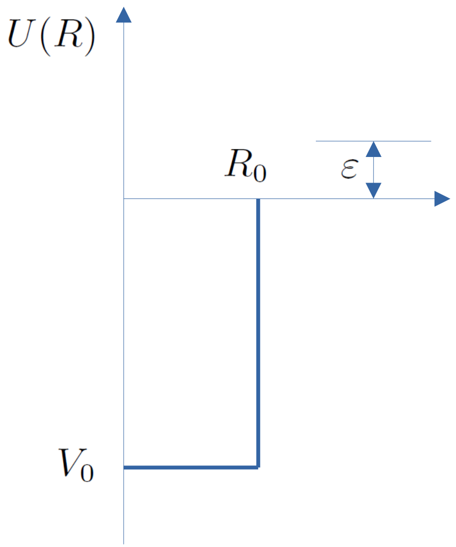


$$\sigma_J^{in}(\varepsilon) = |f_J(\varepsilon)|^{-2}\frac{A}{V}$$

$$U(R) = \begin{cases} -V_0 & R \leqslant R_0 \\ 0 & R > R_0\,, \end{cases}$$

$$\frac{k}{q}|f_0(\varepsilon)|^{-1}\sin qR_0 = \sin(kR_0+\delta_0)\,, \qquad k^2 = 2M\varepsilon$$

$$|f_0(\varepsilon)|^{-1}\cos qR_0 = \cos(kR_0+\delta_0) \qquad q^2 = 2M(\varepsilon+V)\,,$$

$$|f_0(\varepsilon)|^{-2} = \frac{\varepsilon+V_0}{\varepsilon+V_0\cos^2 qR_0}$$

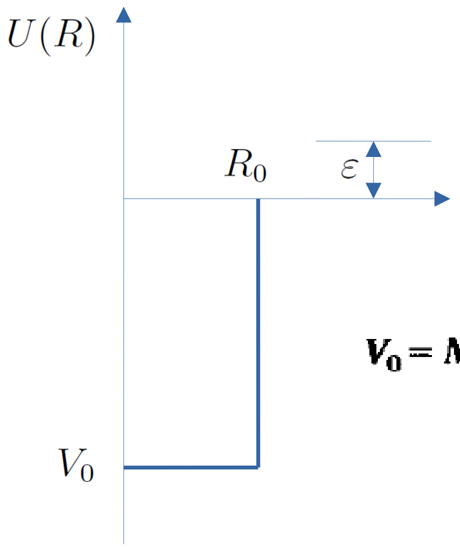
$$|f_0(\varepsilon)|^{-2} = \begin{cases} 1+\dfrac{V_0}{\varepsilon} & \text{max} \\ 1 & \text{min} \end{cases}$$

$$n = \nu, \nu+1, \ldots \qquad \varepsilon_n = \frac{\pi^2(1+2n)^2}{8MR_0^2} - V_0 \qquad \varepsilon_{n+1}-\varepsilon_n = \frac{(n+1)\pi^2}{MR_0^2}$$

$$\frac{\varepsilon_{n+2}-\varepsilon_{n+1}}{\varepsilon_{n+1}-\varepsilon_n} = \frac{n+2}{n+1}$$

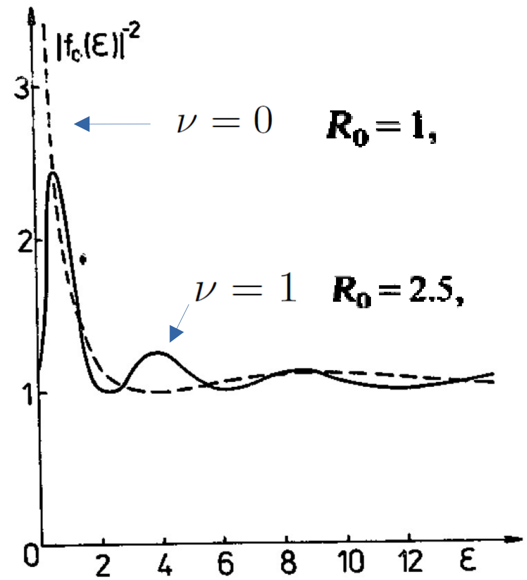
Simple potential model for X(a,b)Y

V.S. Melezhik, Nucl.Phys. A550, 223 (1992)



$$\sigma_J^{in}(\varepsilon) = |f_J(\varepsilon)|^{-2}\frac{A}{V}$$

$V_0 = M = 1.$



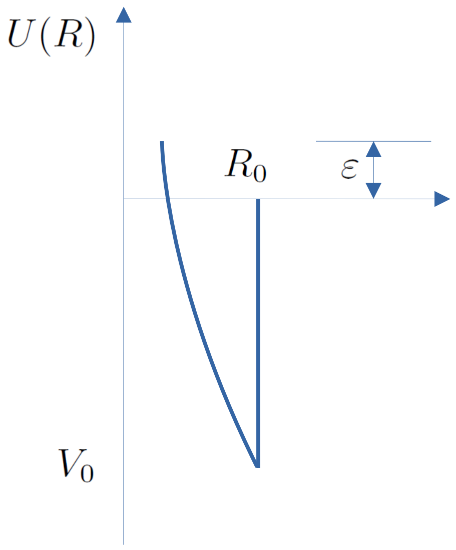
$$U(R) = \begin{cases} -V_0 & R \leq R_0 \\ 0 & R > R_0 , \end{cases}$$

$\nu = 1$

$$\frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_2 - \varepsilon_1} = \frac{\nu + 2}{\nu + 1} = \frac{3}{2} \rightarrow \frac{8.66 - 3.93}{3.93 - 0.78} = \frac{4.73}{3.15} = 1.50$$

Simple potential model for X(a,b)Y

V.S. Melezhik, Nucl.Phys. A550, 223 (1992)



$$\sigma_J^{in}(\varepsilon) = |f_J(\varepsilon)|^{-2} \frac{A}{V}$$

D(d, p)T $\quad R_0 \sim a_B = 0.525 \times 10^{-8} cm$

$p\mu + p \to d + e^+ + \mu \quad R_0 \sim a_B/206$

$$|f_0(\varepsilon)|^{-2} = \frac{C_0^2(q)(\varepsilon + V_0)}{\varepsilon + [V_0 - R_0^{-1} + (\varepsilon + V_0)^{-1} R_0^{-2}] \cos^2 \beta},$$

$$U(R) = \begin{cases} (Z_1 Z_2/R) - V_0 & R \leq R_0 \\ 0 & R > R_0 \end{cases}$$

$$\beta = qR_0 - \frac{M}{q} \ln 2qR_0 + \arg \Gamma\left(1 + i\frac{M}{q}\right), \qquad q = \sqrt{2M(\varepsilon + V_0)}$$

$$f_{max}^{-2} = C_0^2(q_n)\left(1 + \frac{V_0}{\varepsilon_n}\right);$$

$$q_n R_0 - \frac{M}{q_n} \ln 2q_n R_0 + \arg \Gamma\left(1 + i\frac{M}{q_n}\right) = (\tfrac{1}{2} + n)\pi$$

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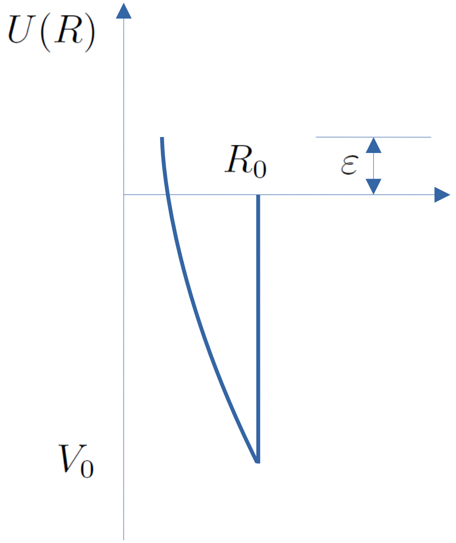

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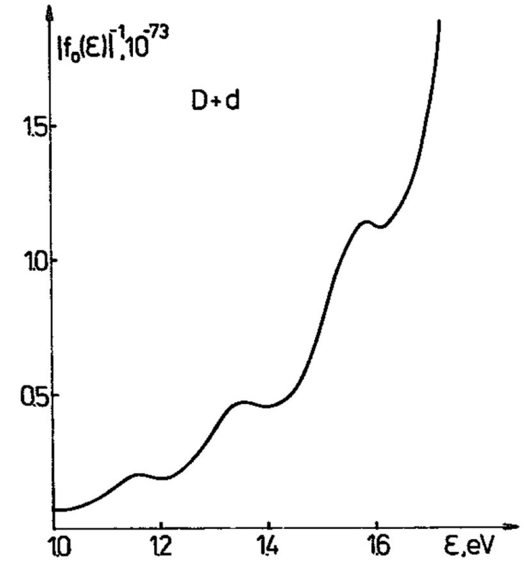

$$U(R) = \begin{cases} (Z_1 Z_2/R) - V_0 & R \leq R_0 \\ 0 & R > R_0 \end{cases}$$

$$\varepsilon_{n+1} - \varepsilon_n \underset{n \simeq \nu}{\simeq} \frac{\pi^2(\nu+1)}{MR_0^2} \simeq \frac{\pi^2 \times 30}{2\times 10^3 \times 10} \times 27\text{ eV} \simeq 0.4\text{ eV}$$

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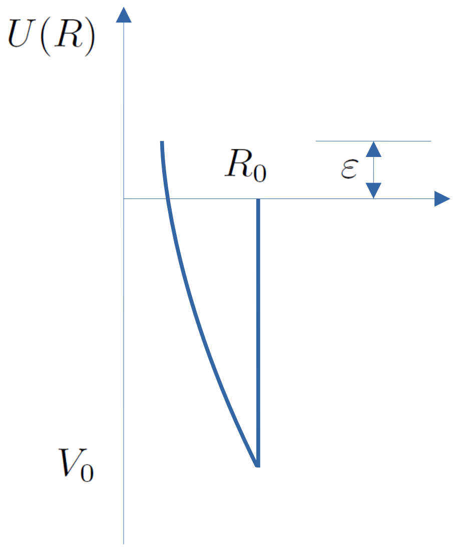

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$$\mathrm{p}\mu + \mathrm{p} \to \mathrm{d} + \mathrm{e}^+ + \mu \qquad R_0 \sim a_B/206$$

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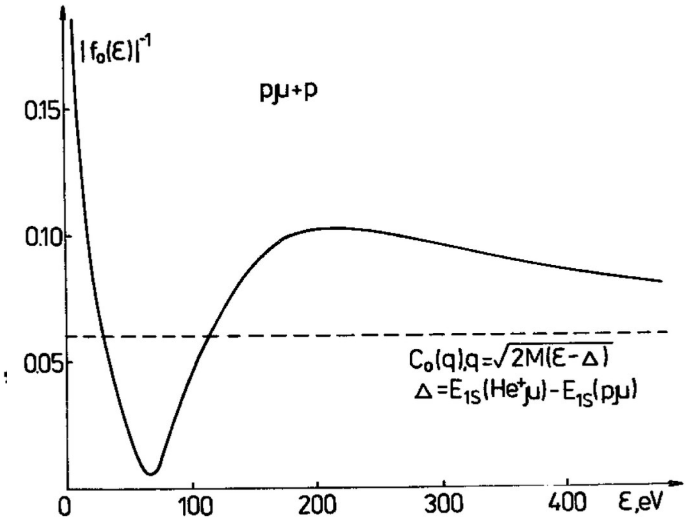
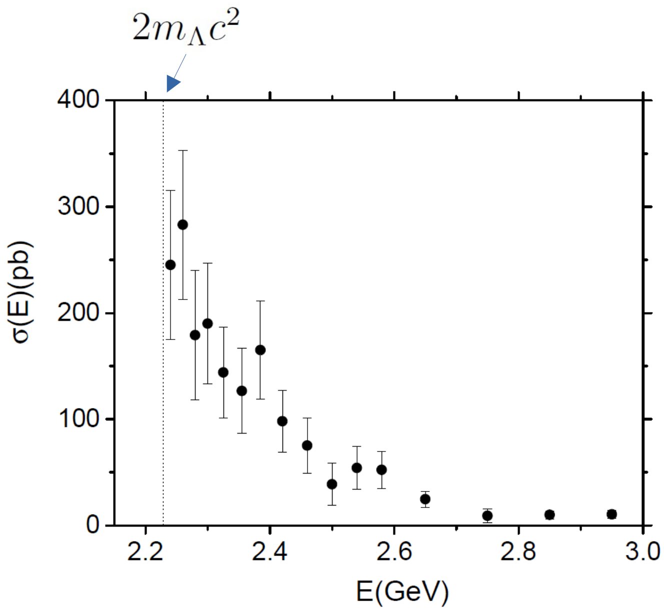
$$U(R) = \begin{cases} (Z_1 Z_2/R) - V_0 & R \le R_0 \\ 0 & R > R_0 \end{cases}$$

$$\varepsilon_{\nu+1} - \varepsilon_\nu \simeq \frac{\pi^2(\nu+1)}{MR_0^2} \simeq \frac{10\times 3}{10\times 10}\times 5\times 10^3 \simeq 10^2\text{–}10^3\ \text{eV},$$



Simple potential model for $e^+ + e^- \to \Lambda + \bar{\Lambda}$

M. Ablikim et. al (BESIII Collaboration) PHYSICAL REVIEW D **107,** 072005 (2023)



detailed balance principle:

$$\sigma(e^+e^- \to \Lambda\bar{\Lambda}) = \frac{k_\Lambda^2}{k_e^2}\sigma(\Lambda\bar{\Lambda} \to e^+e^-) = \frac{k_\Lambda^2}{k_e^2}\frac{A}{v_\Lambda}|f_J(E)|^{-2} = \frac{k_\Lambda}{k_e^2}A|f_J(E)|^{-2}$$

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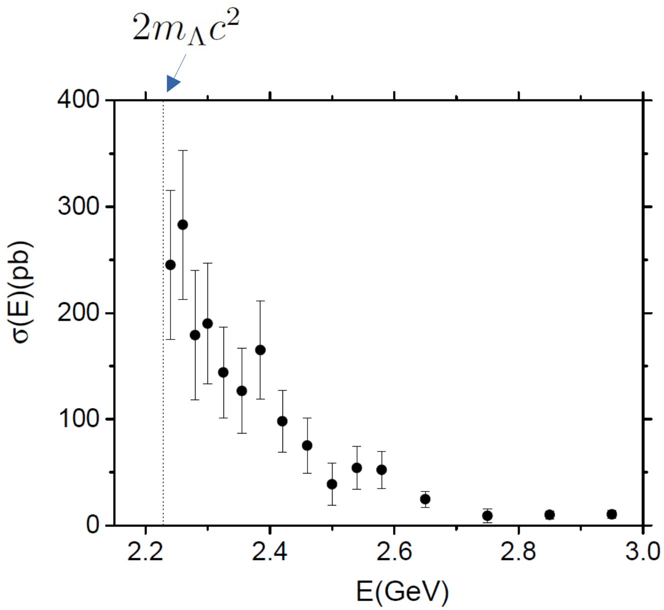

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$$S^{11}(E) = e^{2i\bar{\delta}(E)}\frac{1-2m_1 k\Delta|f(E)|^{-2}+iF(E)}{1+2m_1 k\Delta|f(E)|^{-2}+iF(E)}$$

$$\Delta = \beta|\langle\varphi_E|\xi\rangle|^2; \quad F(E) = \frac{(2m_1)^{3/2}}{\pi}\int_0^\infty \frac{\beta|\langle\varphi_E|\xi\rangle|^2|f(\varepsilon)|^{-2}}{E-\varepsilon}\sqrt{\varepsilon}\,d\varepsilon$$

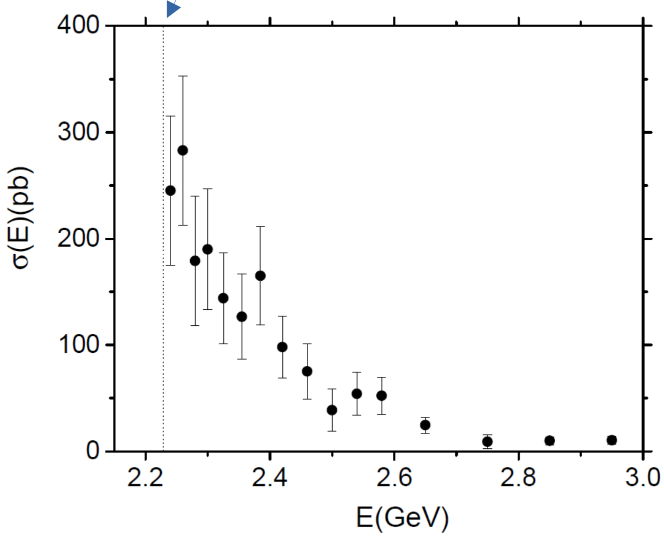
$$V_a = -i\beta|\xi\rangle\langle\xi|$$

$$V_a = -i(2m_2)^{3/2}(E+\Delta)^{1/2}V_{12}|E+\Delta\rangle\langle E+\Delta|V_{21}$$

Simple potential model for $e^+ + e^- \to \Lambda + \bar{\Lambda}$

M. Ablikim et. al (BESIII Collaboration) PHYSICAL REVIEW D **107,** 072005 (2023)

$2m_\Lambda c^2$



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$$A = A'F_D^2(k_e^2) \leftarrow \Delta = \beta|\langle\varphi_E|\xi\rangle|^2; \quad F(E) = \frac{(2m_1)^{3/2}}{\pi}\int_0^\infty \frac{\beta|\langle\varphi_E|\xi\rangle|^2|f(\varepsilon)|^{-2}}{E-\varepsilon}\sqrt{\varepsilon}\,d\varepsilon$$

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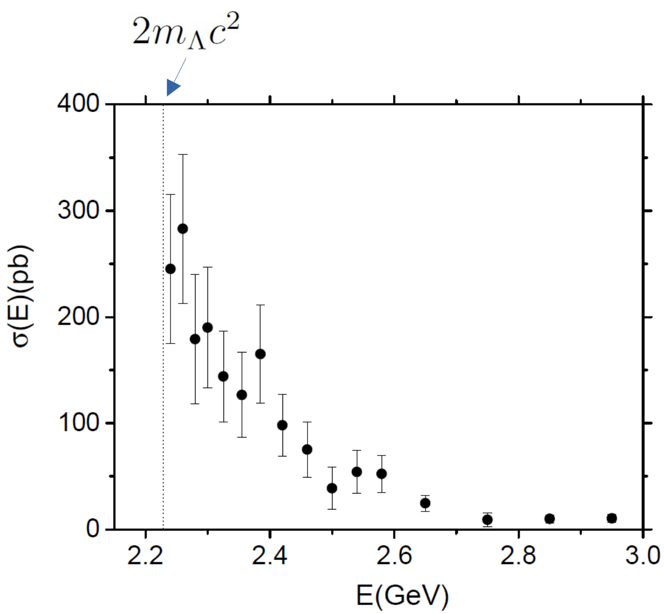
M. Ablikim et. al (BESIII Collaboration) PHYSICAL REVIEW D **107,** 072005 (2023)



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$$\sigma(e^+e^- \to \Lambda\bar{\Lambda}) = \frac{k_\Lambda^2}{k_e^2}\sigma(\Lambda\bar{\Lambda} \to e^+e^-) = \frac{k_\Lambda^2}{k_e^2}\frac{A}{v_\Lambda}|f_J(E)|^{-2} = \frac{k_\Lambda}{k_e^2}A|f_J(E)| = \frac{k_\Lambda}{k_e^2}AF_D^2(k_e^2)|f_J(E)|^{-2}$$

$\Lambda\bar{\Lambda}$ interaction in final state (S-wave): $\sigma(e^+e^- \to \Lambda\bar{\Lambda}) = \frac{k_\Lambda}{k_e^2}F_D^2(k_e^2)g|\psi(R=0)|^2$

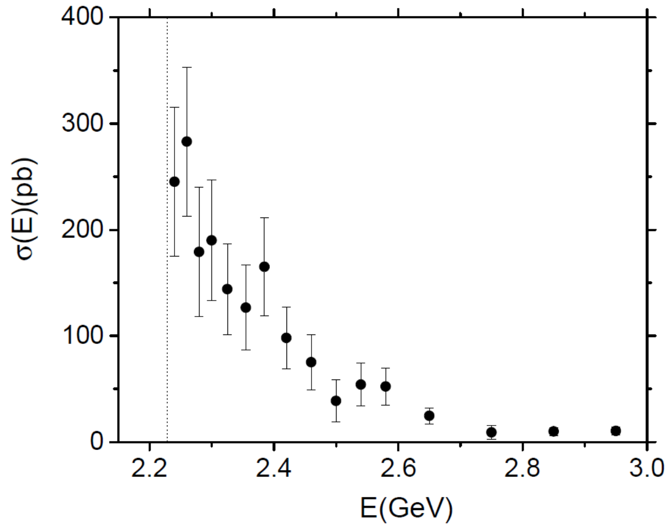
dipole formfactor $F_D = (1 - \frac{k_e^4}{\Delta^2})^{-2}$ $\Delta \sim 1\text{GeV}$

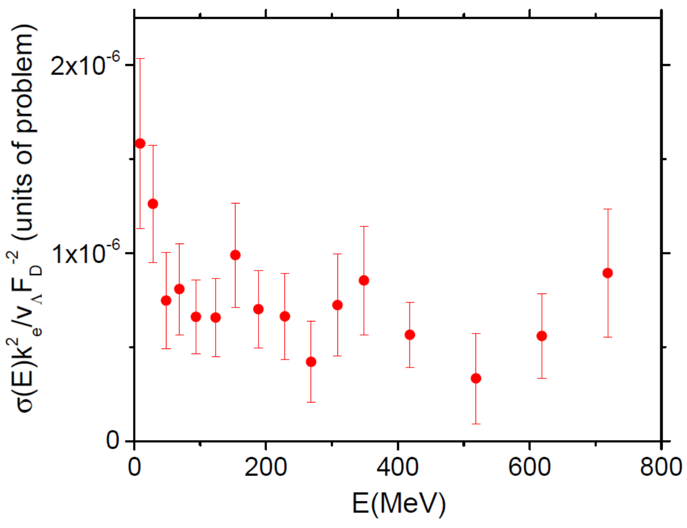
A.I.Milstein, S.G. Salnikov, JETP Letters 117 (2023); Phys Rev D106 (2022)

J. Haidenbauer, X-G. Kang, U-G. Meissner, Nucl Phys A929 (2016)

Simple potential model for $e^+ + e^- \rightarrow \Lambda + \bar{\Lambda}$

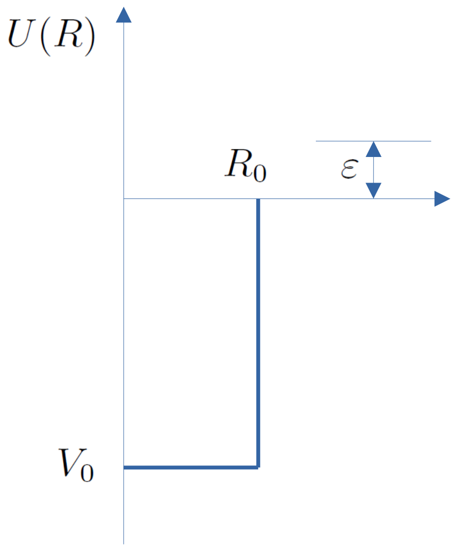
$$A|f_J(E)|^{-2} = \sigma(e^+e^- \rightarrow \Lambda\bar{\Lambda})\frac{k_e^2}{k_\Lambda F_D^2}$$





$$\Delta = 1.8\text{GeV}$$

Simple potential model for $e^+ + e^- \to \Lambda + \bar{\Lambda}$



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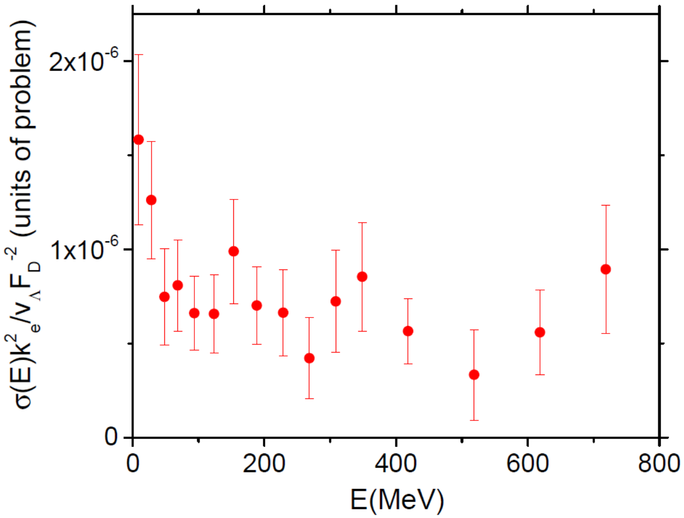
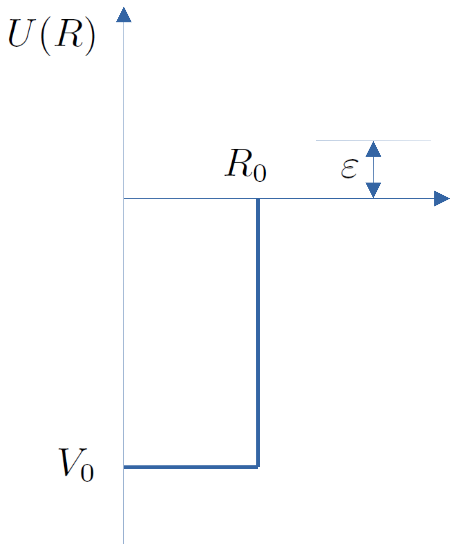


$$U(R) = \begin{cases} -V_0 & R \leq R_0 \\ 0 & R > R_0 , \end{cases}$$

$$\varepsilon_n = \frac{\pi^2(1+2n)^2}{8MR_0^2} - V_0 \qquad n = \nu, \nu+1, \ldots$$

$$\varepsilon_{n+1} - \varepsilon_n = \frac{\pi^2(n+1)}{MR_0^2}$$

Simple potential model for $e^+ + e^- \to \Lambda + \bar{\Lambda}$



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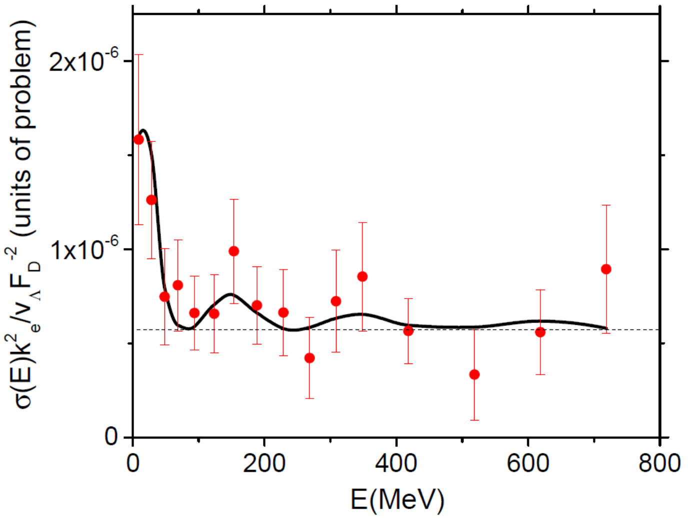


$V_0 = 72\text{MeV}$ $R_0 = 3.26\text{fm}$

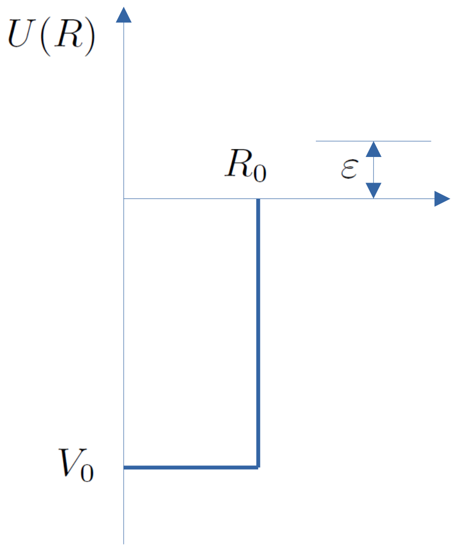
$A = 0.57 \times 10^{-6}$

$\chi^2 = 12/16$

$$\varepsilon_n = \frac{\pi^2(1+2n)^2}{8MR_0^2} - V_0 \qquad n = \nu, \nu+1, \ldots$$

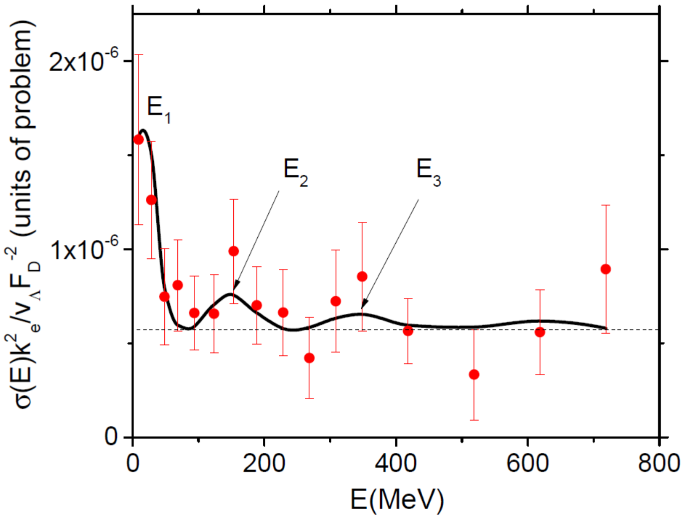
$$\varepsilon_{n+1} - \varepsilon_n = \frac{\pi^2(n+1)}{MR_0^2}$$

Simple potential model for $e^+ + e^- \to \Lambda + \bar{\Lambda}$



$$U(R) = \begin{cases} -V_0 & R \leqslant R_0 \\ 0 & R > R_0, \end{cases}$$

$$A|f_J(E)|^{-2} = \sigma(e^+e^- \to \Lambda\bar{\Lambda}) \frac{k_e^2}{k_\Lambda F_D^2}$$



$V_0 = 72\text{MeV} \quad R_0 = 3.26\text{fm}$

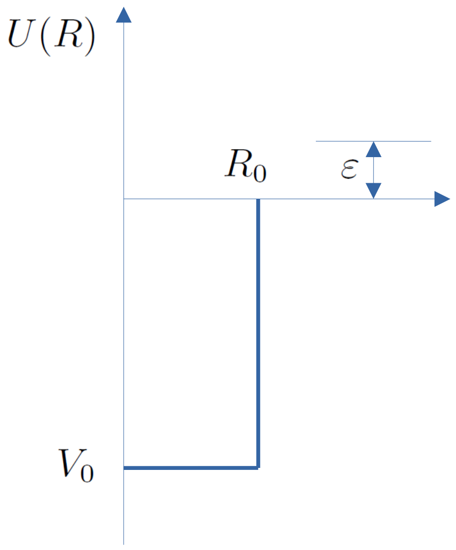
$A = 0.57 \times 10^{-6}$

$\chi^2 = 7/16$

$\nu = 1 \qquad \frac{\nu+2}{\nu+1} = 1.5$

$\frac{E_3 - E_2}{E_2 - E_1} = 1.4$

Simple potential model for $e^+ + e^- \to \Lambda + \bar{\Lambda}$



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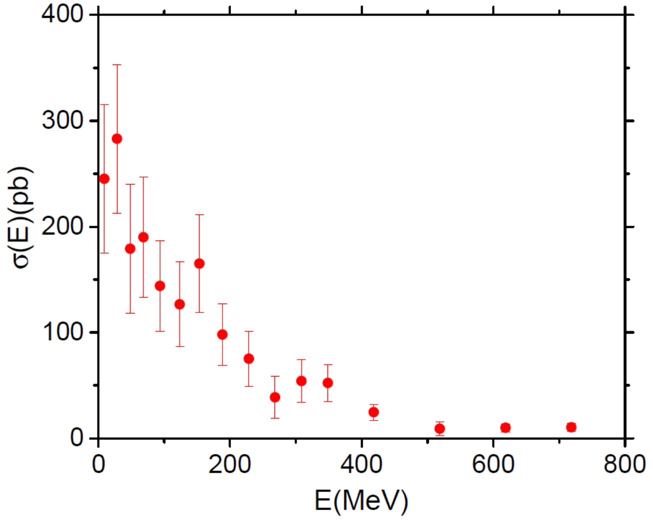

Simple potential model for $e^+ + e^- \to \Lambda + \bar{\Lambda}$

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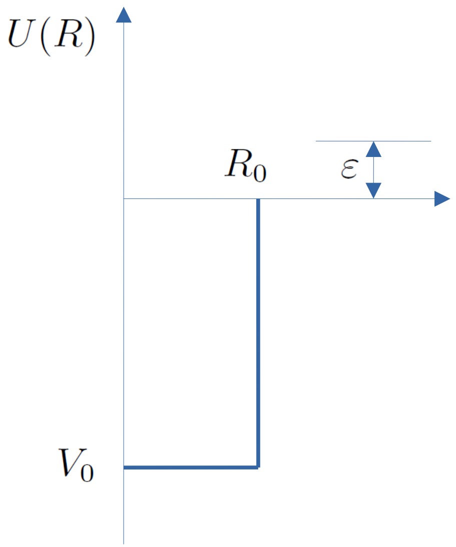

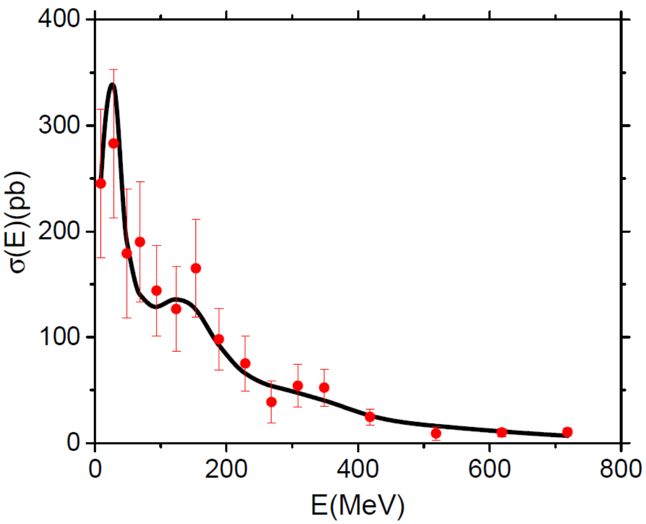

$$U(R) = \begin{cases} -V_0 & R \leqslant R_0 \\ 0 & R > R_0 , \end{cases}$$

$$\Delta = 1.8\text{GeV} \;\; A = 0.57 \times 10^{-6} \;\; V_0 = 72\text{MeV} \;\; R_0 = 3.26\text{fm}$$

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Conclusion & Outlook

- simple potential model $$\sigma(e^+e^- \to \Lambda\bar{\Lambda}) = \frac{k_\Lambda}{k_e^2} F_D^2 A |f_0(E)|^{-2}$$

fitting parameters: A V_0 R_0

Conclusion & Outlook

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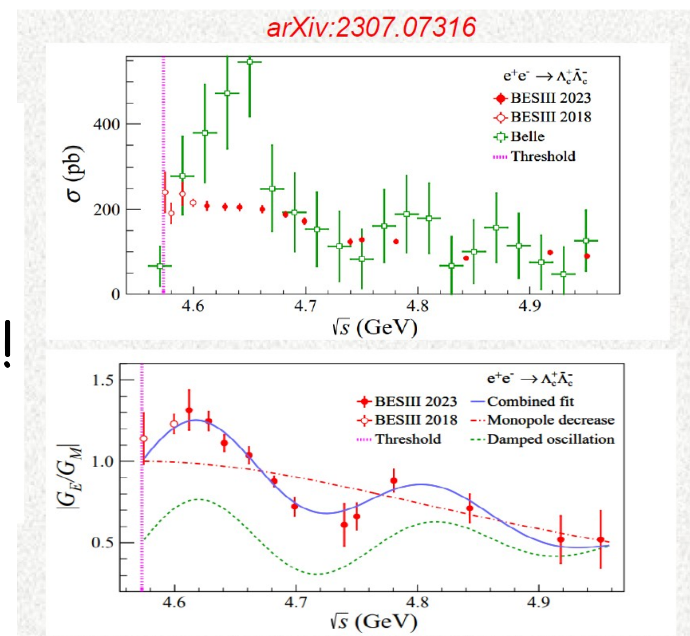
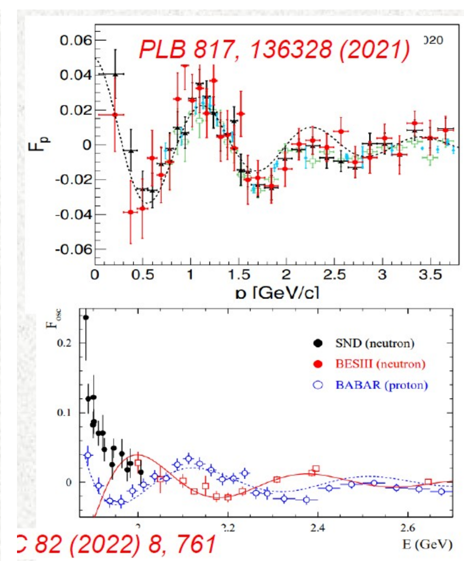
fitting parameters: A V_0 R_0

- $e^+e^- \to \Lambda_c\bar{\Lambda}_c$

formfactors ?!

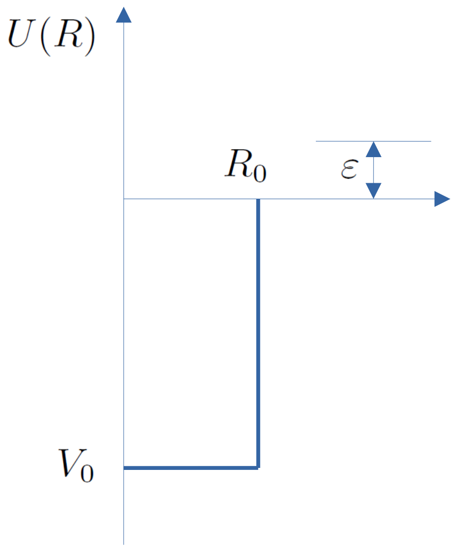
S, D-waves
Coulomb

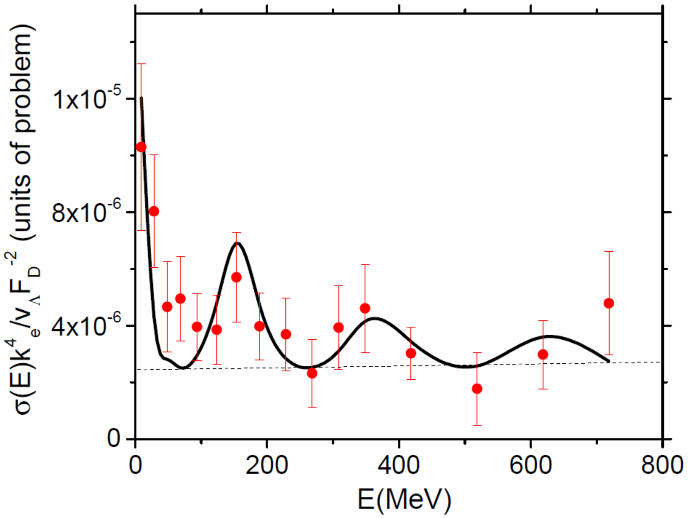




Simple potential model for $e^+ + e^- \to \Lambda + \bar{\Lambda}$

$$\sigma_J^{\text{in}}(\varepsilon) = |f_J(\varepsilon)|^{-2} \frac{A}{V}$$





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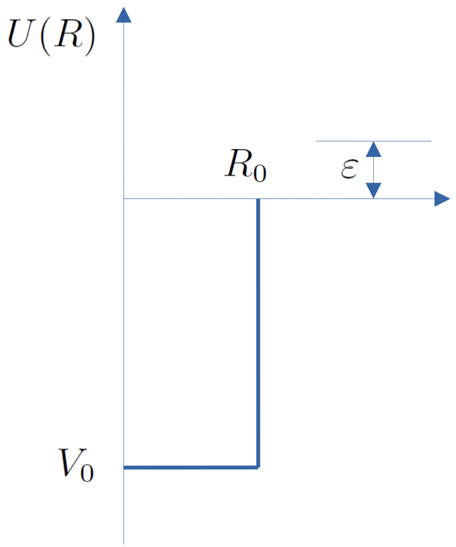

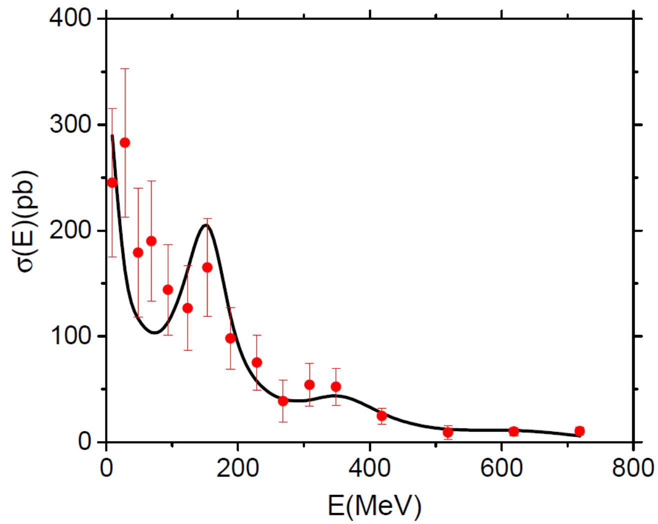

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