Net-proton high-order cumulants in event-by-event studies in high energy A+A collisions at NICA energies

Grigory Feofilov
St. Petersburg State University
Laboratory of Ultra-High Energy Physics

LXXV International Conference “NUCLEUS-2025”

Saint-Petersburg, Russia, 05.07.2025, 16:30-16:50

Supported by Saint Petersburg State University, project ID: 103821868

Talk layout

1. Some definitions
2. Hot topic: net-proton fluctuations
3. Can we do better with MPD at NICA?

Event-by-event fluctuations of conserved quantities

- Net electric charge Q,
- Net strangeness S,
- Net baryon number B,

…are sensitive to the degrees of freedom that are active in the system,

And the moments (Variance (σ^2), Skewness(S), Kurtosis(k)) of distributions of Q,S,B **are predicted to be sensitive to the correlation length of hot dense matter** created in the collisions [1]**, i.e. sensitive to the QCD critical endpoint** of the first order phase transition between quark-gluon plasma and hadron gas

……

- Net proton number (as a proxy to net-baryon) : $N_{p-\bar{p}} = N_p - N_{\bar{p}} = \Delta N_p$
- We will use **N** below to represent **the net-proton number $N_{p-\bar{p}}$ in one event**

[1] M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).

Fluctuations of conserved quantities and susceptibilities

Fluctuations of conserved quantities (N or Q_i in this slide) relate directly to **the susceptibilities χ_i,** which are quantities that can be calculated for thermodynamic systems, e.g. in lattice QCD.

Susceptibilities are defined as derivatives of the pressure with respect to the chemical potential.

Deviations δQ_i are related to susceptibilities χ_n^{Qi} by[1]:

$$\langle(\delta Q_i)^n\rangle = T^n \frac{\partial^n}{\partial \mu_i^n} \ln Z(T, \mu_i) = V T^3 \chi_n^{Q_i},$$

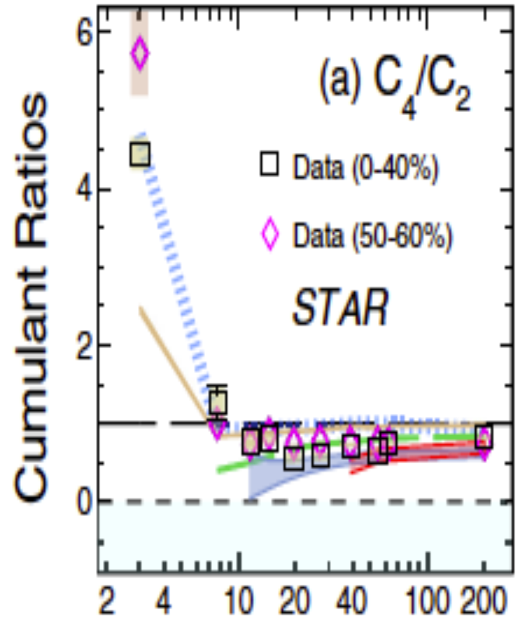
Here -- Q_i is a conserved charge of interest,
T the temperature, μ_i the corresponding chemical potential and Z the partition function [1].

- **Therefore the experimental values (in the Left) could be used to define thermodynamic model parameters (in the Right)**

[1] Hannah Elfner and Berndt Müller 2023, J. Phys. G: Nucl. Part. Phys. 50 103001

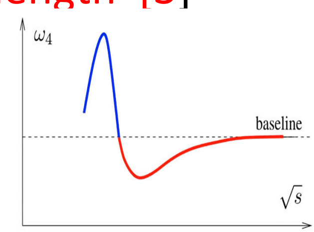
Hot topic! Proton and net-proton High-Order Cumulants and Susceptibilities

DEFINITIONS of central moments and cumulants [1]	
the proton multiplicity in a given event	N
the average over all events	<….>
Mean:	$M = \langle N \rangle = C_1$,
Deviation,	$\delta N = N - \langle N \rangle$
Variance	$\sigma 2 = \langle (\delta N)^2 \rangle = C_2$,
skewness	$S = \langle (\delta N)^3 \rangle / \sigma^3 = C_3 / C_2^{3/2}$,
kurtosis	$k = \langle (\delta N)^4 \rangle / \sigma^4 - 3 = C_4 / C_2^2$



Collision energy dependence of the ratios of cumulants C4/ C2, for net proton (red circles) from 0%–40% and 50-60% Au + Au collisions at RHIC ([2]).

- Fluctuation of conserved quantities were predicted to be sensitive near CEP to the correlation length [3]



[1] Xiaofeng Luo, Nu Xu, NUCL SCI TECH (2017) 28:112 DOI 10.1007/s41365-017-0257-0;

[2] B.E.Aboona et al. *STAR Collaboration), "**Beam Energy Dependence of Fifth- a Order Net-Proton Number Fluctuations in Au+Au Collisions at RHIC**", Phys. Rev. Lett. **130**, 082301,(2023) https://doi.org/10.1103/PhysRevLett.130.082301

[3] C. Athanasiou, K. Rajagopal, M. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).doi:10.1103/ PhysRevD.82.074008

In 2021: Non-monotonic energy dependence of net-proton number fluctuations[1]

The first evidence of a nonmonotonic variation in kurtosis times variance of the net-proton number (proxy for net-baryon number) distribution as a function of $\sqrt{s_{NN}}$ with 3.1s significance, for central Au+A collisions measured using the STAR detector at RHIC [1]

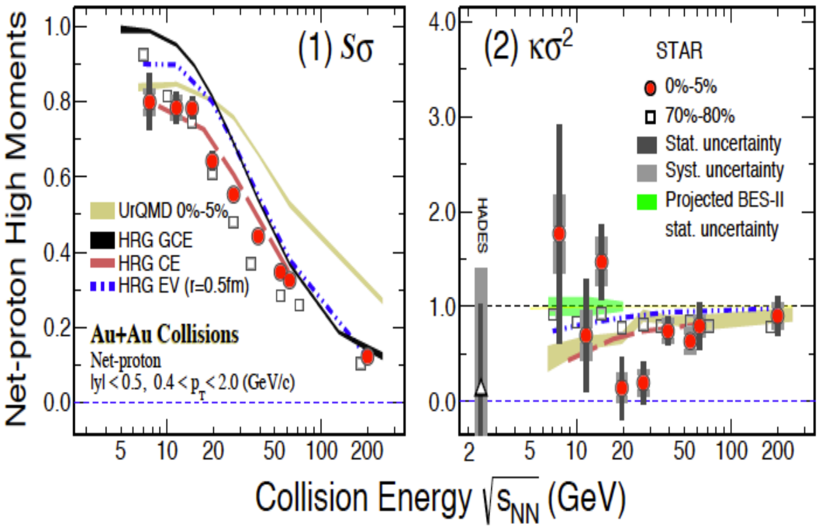


FIG. 4. Sσ (1) and kσ² (2) as a function of $\sqrt{s_{NN}}$ for net-proton distributions measured in Au+Au. Central (0-5%, filled circles), peripheral (70-80%, open squares) collisions 0.4 < pT (GeV/c) < 2.0 and jyj < 0.5 [1].

- **Higher moments of distributions are more interesting due to their stronger dependence on the correlation length**
- **Centrality bin width correction (CBWC) is applied[3]**

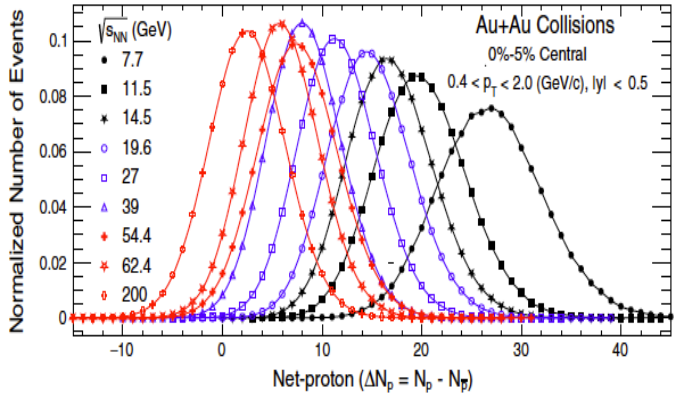
[1] J. Adam et al. (STAR Collaboration), Phys. Rev. Lett. 126, 092301 (2021).

[2] M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).

[3] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013).

In 2021: Non-monotonic energy dependence of net-proton number (N) fluctuations[1]

DEFINITIONS of central moments and cumulants [1]	
the proton multiplicity in a given event	N
the average over all events	<….>
Mean:	$M = \langle N \rangle = C_1$,
Deviation,	$\delta N = N - \langle N \rangle$
Variance	$\sigma^2 = \langle (\delta N)^2 \rangle = C_2$,
skewness	$S = \langle (\delta N)^3 \rangle / \sigma^3 = C_3 / C_2^{3/2}$,
kurtosis	$k = \langle (\delta N)^4 \rangle / \sigma^4 - 3 = C_4 / C_2^2$



$$C_1 = M, \quad C_2 = \sigma^2, \quad C_3 = S\sigma^3, \quad C_4 = \kappa\sigma^4$$

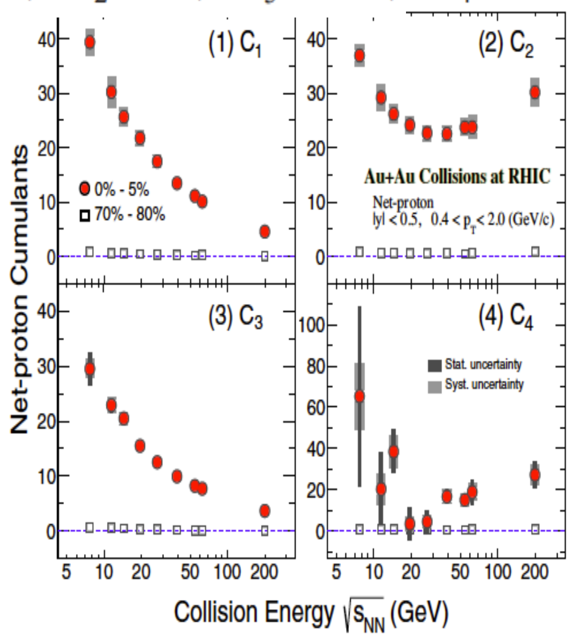


FIG. 1. Net-proton number distributions for (0%–5% central) Au +Au collisions for nine $\sqrt{s_{NN}}$ values measured by STAR [2].

FIG. 2. Cumulants (Cn) of the net-proton distributions for central (0%–5%) and peripheral (70%–80%) Au +Au collisions as a function of collision energy[2].

- Corrected for Centrality Bin Width Effect[3] CBWC
- Non-gaussian shape of net-baryon distribution is expected near the CEP (M.Stepanov - 2008)
- **Non-zero C3 and C4 and non-gaussian shape here?**

[1] Xiaofeng Luo, Nu Xu, NUCL SCI TECH (2017), 28:112, DOI 10.1007/s41365-017-0257-0;
[2] (STAR Collaboration), PHYSICAL REVIEW LETTERS, 126, 092301 (2021)
[3]] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013)

Problems of using δN=N-<N>

$\sigma^2 = \langle(\delta N)^2\rangle$

skewness: $S = \langle(\delta N)^3\rangle/\sigma^3 = C_3/C_2^{3/2}$

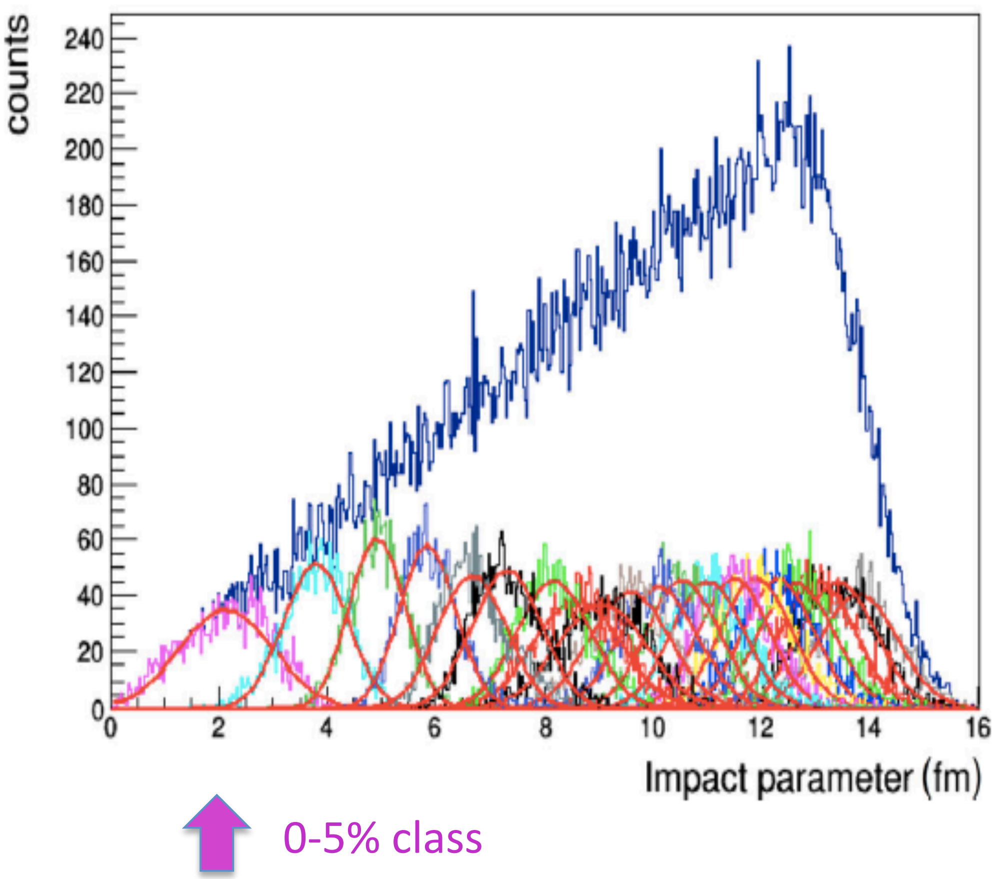
kurtosis	**$k = \langle(\delta N)^4\rangle / \sigma^4 - 3 = C_4/C_2^2$**

- δN =N − <N>brings an added weight to the outliers
- The 2nd,3rd or the 4th power of <δN> gives more weight
- Volume (V) dependence of baryon number susceptibilities.
- V is also a strongly fluctuating quantity

- ”Therefore, we conclude that fluctuations of conserved charges in heavy ion collisions can provide robust probes of the chiral phase boundary **if a good control of volume fluctuations can be achieved.**” (see in [1]).

[1] V.Skokov, B. Friman and K. Redlich, Volume fluctuations and higher order cumulants of the net baryon number, arXiv:1205.4756;

Problems of using $\delta N = N - \langle N \rangle$



- V is a strongly fluctuating quantity,

see , for example, impact parameter distribution **for 0-5% class** in MC simulations for Au+Au collisions at $\sqrt{s_{NN}}$ =7.7 GeV (Fig.44 from ref.[1]).

- Event-by-event fluctuations of impact parameter *b* in the given centrality class selected for analysis

[1]The MPD Collaboration, Status and initial physics performance studies of the MPD experiment at NICA,
Eur. Phys. J. A (2022) 58:140, https://doi.org/10.1140/epja/s10050-022-00750-6

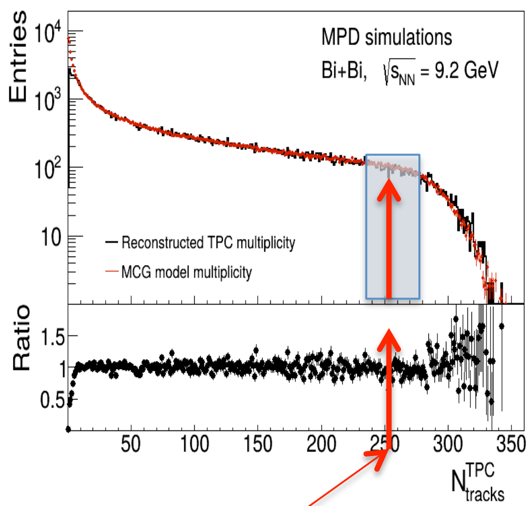
Centrality Bin Width Correction procedure (CBWC) [1]

$$\sigma = \frac{\sum_r n_r \sigma_r}{\sum_r n_r} = \sum_r \omega_r \sigma_r$$

$$S = \frac{\sum_r n_r S_r}{\sum_r n_r} = \sum_r \omega_r S_r$$

$$\kappa = \frac{\sum_r n_r \kappa_r}{\sum_r n_r} = \sum_r \omega_r \kappa_r$$

Selection **of very narrow** multiplicity class from multiplicity distribution of charged particles measured in some pseudorapidity interval

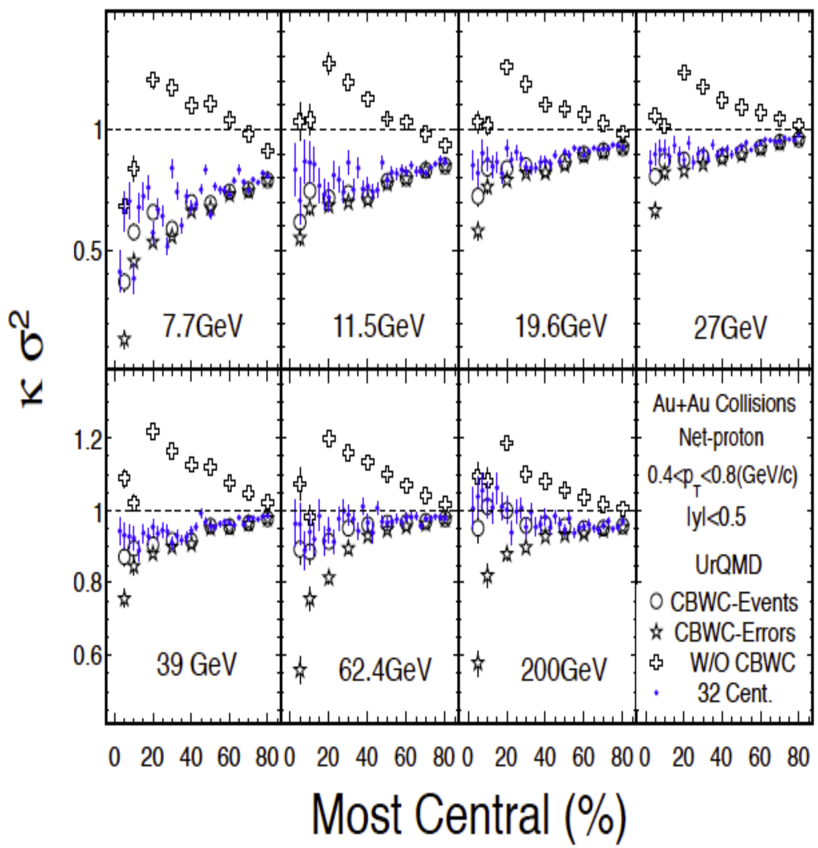


Some r^{th} multiplicity bin in the selected multiplicity class

Here n_r -- is the number of events in r^{th} multiplicity , **σ_r, S_r and k_r --** the standard deviation, skewness and kurtosis of particle number distributions at r^{th} multiplicity, $\omega_r = n_r/\Sigma n_r$ -- the corresponding weight for the r^{th} multiplicity .

[1] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013).

Some examples of CBWC results in UrQMD model obtained in [1]



- FIG. 3. The centrality dependence of the moments products ($k\sigma^2$) of net-proton multiplicity distributions for Au+Au collisions at $\sqrt{s_{NN}}$=7.7, 11.5, 19.6, 27, 39, 62.4, 200GeV in UrQMD model. See in [1].

➢ **CBWC works and it is more important for the central collisions**

➢ **However, it is not sufficient… -->**

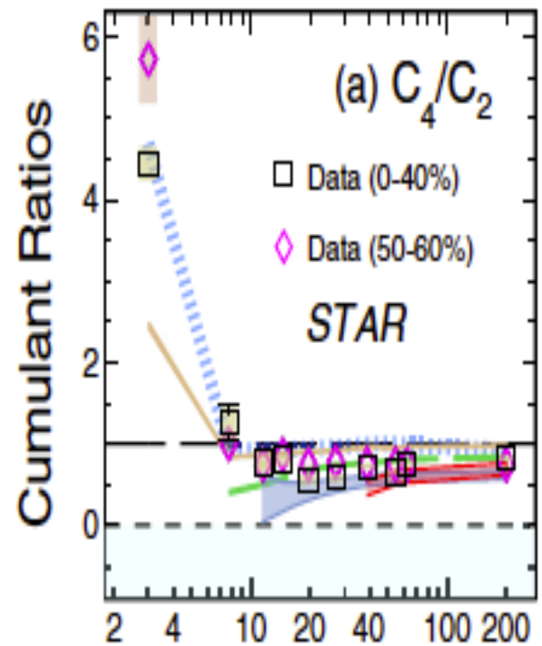
[1] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013); arXiv: 1302.2332v2

Hot topic :
Proton and net-proton High-Order Cumulants

DEFINITIONS of central moments and cumulants [1]	
the proton multiplicity in a given event	N
the average over all events	<….>
Mean:	$M = \langle N \rangle = C_1$,
Deviation,	$\delta N = N - \langle N \rangle$
Variance	$\sigma^2 = \langle(\delta N)^2\rangle = C_2$,
skewness	$S = \langle(\delta N)^3\rangle/\sigma^3 = C_3/C_2^{3/2}$,
kurtosis	$k = \langle(\delta N)^4\rangle/\sigma^4 - 3 = C_4/C_2^2$

[1] Xiaofeng Luo, Nu Xu, NUCL SCI TECH (2017) 28:112 DOI 10.1007/s41365-017-0257-0;
[2] C. Athanasiou, K. Rajagopal, M. Stephanov, Phys. Rev. Lett. 102, 032301 (2009). doi:10.1103/PhysRevD.82.074008
[3] J. Adam et al. (STAR Collaboration), Phys. Rev. Lett. 126, 092301 (2021)

Experiment [3] :



- …so, where is the CEP ---?
- **Can we do better with MPD at NICA?**

Problems of centrality class selection

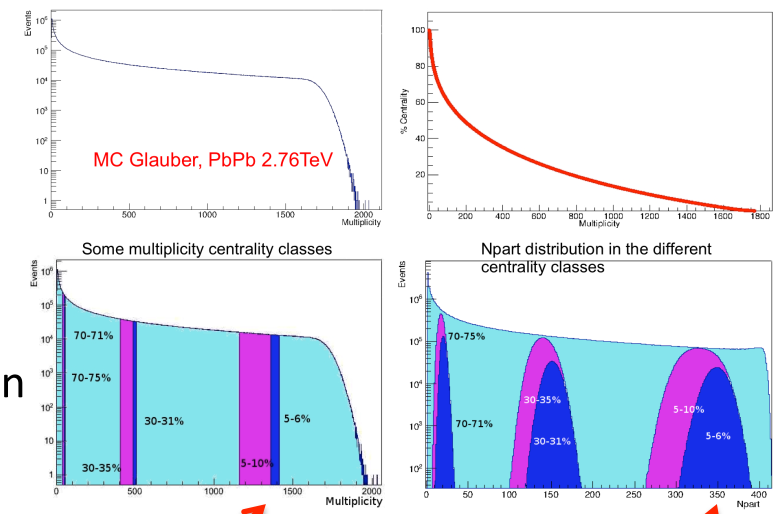
$$\chi_1^B = \frac{1}{VT^3}\langle N_B\rangle\,,$$

$$\chi_2^B = \frac{1}{VT^3}\langle(\delta N_B)^2\rangle\,,$$

$$\chi_3^B = \frac{1}{VT^3}\langle(\delta N_B)^3\rangle\,,$$

$$\chi_4^B = \frac{1}{VT^3}\left(\langle(\delta N_B)^4\rangle - 3\langle(\delta N_B)^2\rangle^2\right),$$

MC Glauber calculations Pb+Pb 2.76 TeV [1]



- For the class of selected events, the mean volume V and temperature T are supposed to be fixed during the CBWC procedure with r^{th} multiplicity bin .

- But narrow *class in multiplicity*, e.g. of **1%** width, does not mean narrow distribution neither in *the impact parameter b* nor in Npart[1]
  - **So, the volume V is not fixed in the event-by-event study with CBWC .**
  - **A narrow r^{th} multiplicity bin still contains trivial volume fluctuations of N_B.**
  - **Event N_{part} should be estimated with the highest accuracy .**

See also Talk by Grigory Feofilov 05/07/2025, 16:30 , Section 4

[1] T. A. Drozhzhova,V. N. Kovalenko,A. Yu. Seryakov,G. A. Feofilov, Physics of Atomic Nuclei, September 2016, Volume 79, Issue 5, pp 737–748

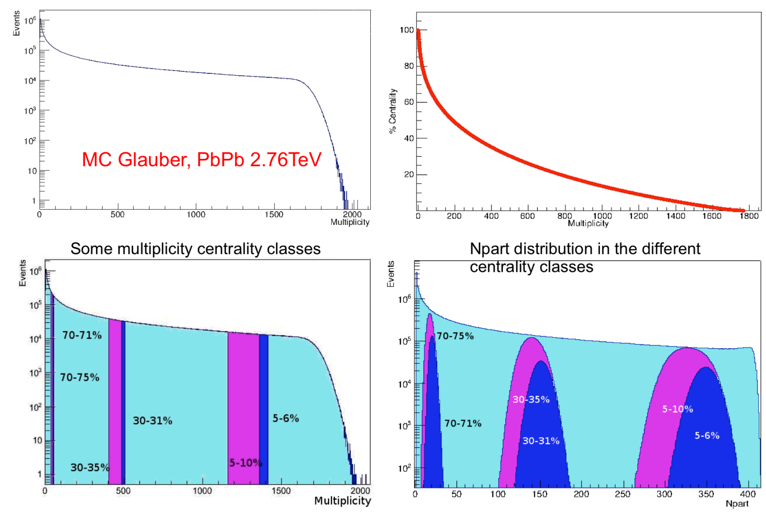
Problems of centrality class selection

- Narrow *class in multiplicity*, e.g. of **1%** width, does not mean narrow distribution neither in *the impact parameter b* nor in Npart[1]

- Event N_{part} should be estimated with the highest accuracy.

- **Work is in progress for the event impact parameter and relevant centrality determination in nucleus-nucleus collisions in experiments at the NICA**

MC Glauber calculations Pb+Pb 2.76 TeV [1]



Talk by Кирилл Галактионов (СПбГУ)
01/07/2025, 18:00 , Section 4

Talk by Dim Idrisov (INR RAS)
01/07/2025, 18:40 , Section 4

Talk by Svetlana Simak (SPbSU)
03/07/2025, 11:23 , Section 4

[1] T. A. Drozhzhova, V. N. Kovalenko, A. Yu. Seryakov, G. A. Feofilov, Physics of Atomic Nuclei, September 2016, Volume 79, Issue 5, pp 737–748

Proposal: New Bin Width Correction procedure (CBWC –V)

Ratios of cumulants $C_2/C_1=\sigma^2/M$, $C_3/C_2.=S\sigma$, and $C_4/C_2=\kappa\sigma^2$ were used to reduce the volume dependence.

However, the average values of σ, *S* and *k* **are calculated assuming the fixed value of volume V in all events!.**

- **We propose to use the** reduced cumulants, similar to **[1], but on the event-by-event basis, following the new CBWC-V procedure with V^r defined in each r^{th} multiplicity bin via $<N^r_{part}>$:**

$M = <N> = C_1$,

$\delta N = N - <N>$

$\sigma^2 = <(\delta N)^2> = C_2$,

$S = <(\delta N)^3>/\sigma^3 = = C_3/C_2^{3/2}$,

$k = <(\delta N)^4>/\sigma^4 - 3 = = C_4/C_2^2$

$c1 = M/V^r = <N^r/V^r>$,

$\delta N^r = N^r/V^r - <N^r/V^r>$

$c2 = \sigma_r^2 = <(\delta N^r)^2>$,

$S_r = <(\delta N^r)^3>/\sigma_r^3$,

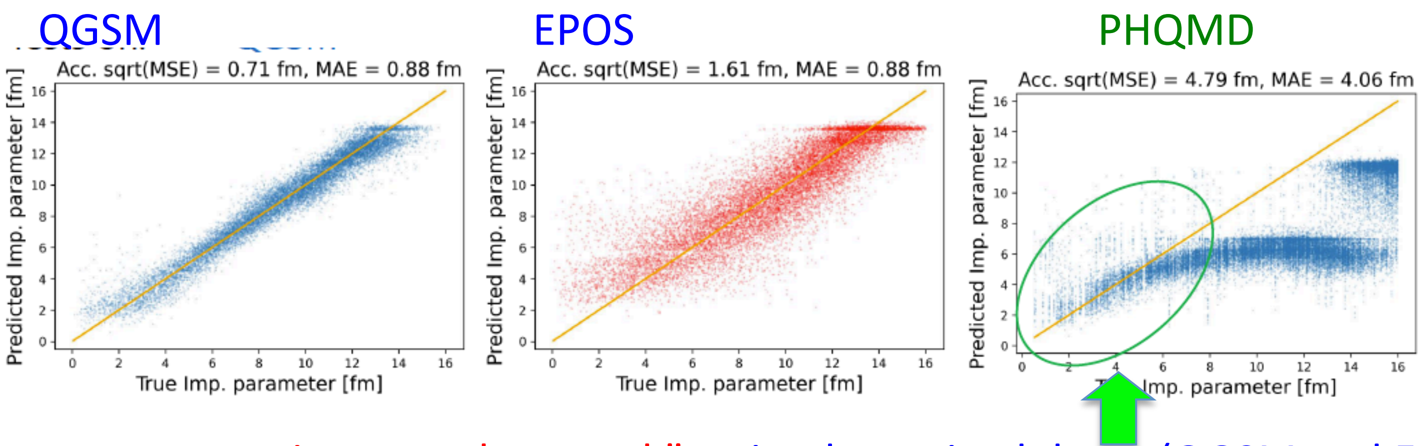
$k_r = <(\delta N^r)^4>/\sigma_r^4 - 3$

- **We assume that for any r^{th} multiplicity bin the relevant mean volume V^r is proportional to the number of nucleons-participants: $V^r = V_0 N^r_{part}$**
- Here a volume factor $V_0 = 2.83\ fm^3$ (see in [1]).
- **Thus we obtain the reduced deviation**

$\delta N^r = N^r/V^r - <N^r/V^r>$ for the relevant distribution of conserved quantity N^r

[1] V. Skokov, B. Friman and K. Redlich, "Volume fluctuations and higher order cumulants of the net baryon number". arXiv:1205.4756v2

"Deep reconstruction neural network" trained on mixed data by K. Galaktionov [1]



- "Deep reconstruction neural network" trained on mixed data (QGSM and EPOS) was shown [1] -- to be capable to select some class of central events (CCE) **in PHQMD**
- The events from this class could be used in further event-by-event analysis.
- Event-by-event number of spectator nucleons (N_s) from the energy measured in a calorimeter, could bring us the estimates of the values of N^r_{part} for each event and the relevant errors for N^r_{part} value – by using Bayes' formula as described in [2]
- **Volume V^r is estimated in each event : $V^r = V_0 N^r_{part}/2$**

COMMENT GF: Many thanks to Evgeniy Anronov who pointed at the lost ½ !

(→ Then $\langle N^r/V^r \rangle$ could be calculated for the CCE selected and,

finally : $\delta N^r = N^r/V^r - \langle N^r/V^r \rangle$ for each event

[1] K. Galaktionov -- talk at NUCLEUS-2025 conference

[2] F.F. Valiev , V.V. Vechernin, G.A. Feofilov, **Estimation of the Accuracy of Determining the Number of Spectator Nucleons from the Energy Measured in a Calorimeter in A + A Collisions",** Bull.Russ.Acad.Sci.Phys. 88 (2024) 8, 1312-1318

Relative Accuracy of Determining the Number of Spectator Nucleons[1]

Example[1]

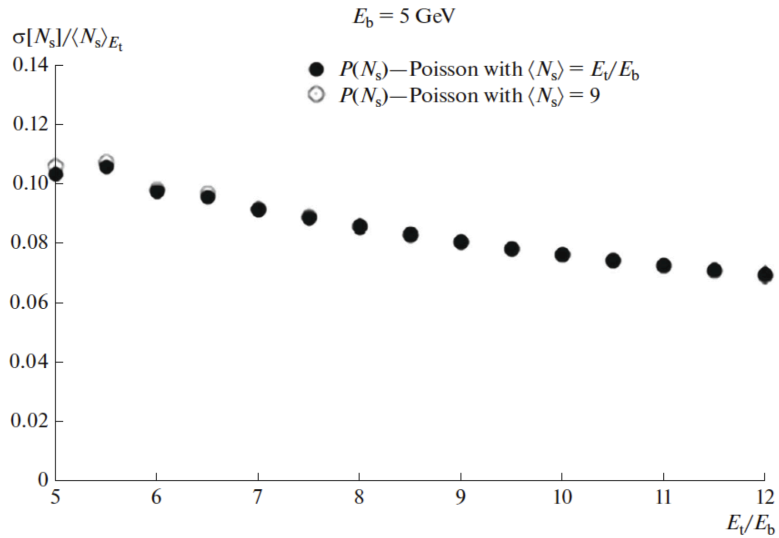


Fig. 5. Relative error in determining N_s at this E_t in the case where $P(N_s)$ is a Poisson distribution with mathematical expecta-tions $N_s = E_t/E_b$ (filled points) and $N_s = 9$ (open points) at the measured energy $E_t/E_b < 12$.

.

[1] F.F. Valiev , V.V. Vechernin, G.A. Feofilov, **Estimation of the Accuracy of Determining the Number of Spectator Nucleons from the Energy Measured in a Calorimeter in A + A Collisions",** Bull.Russ.Acad.Sci.Phys. 88 (2024) 8, 1312-1318

Relative Accuracy of Determining the Number of Spectator Nucleons[1]

Example[1]

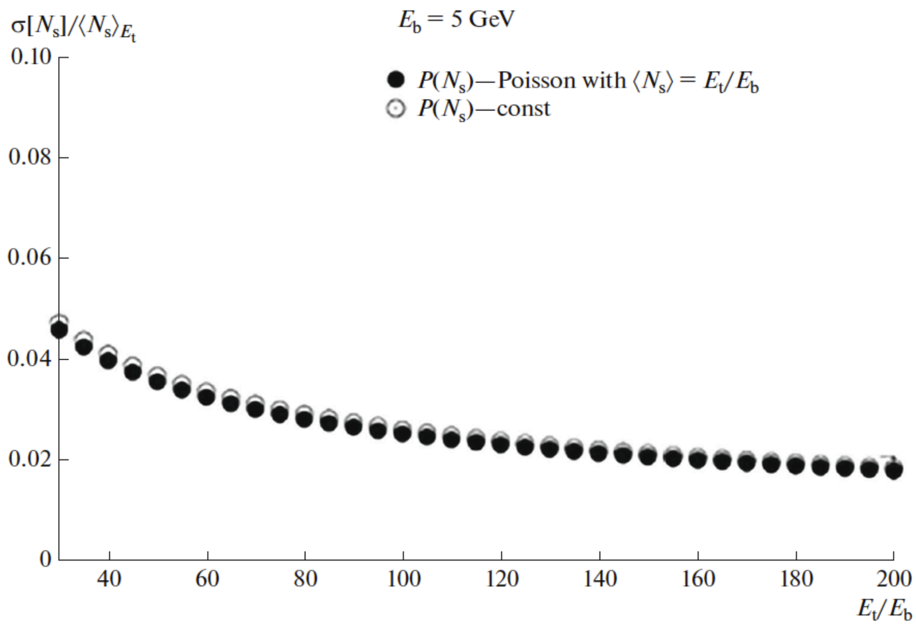


Fig. 6. Relative error in determining N_s at this E_t in the case where $P(N_s)$ is a Poisson distribution with mathematical expectation $N_s = E_t/E_b$ (filled points) and a uniform distribution (open points) at the measured energy $E_t/E_b > 30$.

[1] F.F. Valiev , V.V. Vechernin, G.A. Feofilov, **Estimation of the Accuracy of Determining the Number of Spectator Nucleons from the Energy Measured in a Calorimeter in A + A Collisions”,** Bull.Russ.Acad.Sci.Phys. 88 (2024) 8, 1312-1318

Conclusions

- Class of central events (CCE) is selected using ML approach[1], and then it allows to proceed
- Values of event N^r_{part} should be estimated event-by-event with the highest accuracy.
- $V^r = V_0 N^r_{part}/2$ are defined event-by-event
- $\langle N^r/V^r \rangle$ could be calculated for the class of events selected
- Reduced deviations : $\delta N^r = N^r/V^r - \langle N^r/V^r \rangle$, are calculated for each event,

and, finally, we may obtain:

$$\sigma^2 = \langle (\delta N^r)^2 \rangle$$
$$S = \langle (\delta N^r)^3 \rangle / \sigma^3$$
$$k = \langle (\delta N^r)^4 \rangle / \sigma^4 - 3$$

.

[1] K. Galaktionov -- talk at NUCLEUS-2025 conference

[2] F.F. Valiev , V.V. Vechernin, G.A. Feofilov, Estimation of the Accuracy of Determining the Number of Spectator Nucleons from the Energy Measured in a Calorimeter in A + A Collisions", Bull.Russ.Acad.Sci.Phys. 88 (2024) 8, 1312-1318

Thank you for your attention!

Cumulants
and correlation length ξ of hot dense matter

- **Correlation length ξ of hot dense matter[1]**

  --- the cubic central moment of multiplicity $\langle(\delta N)^3\rangle \sim \xi^{4.5}$

  --- the quartic cumulant $\langle(\delta N)^4\rangle \sim \xi^{7}$

- **Correlation length ξ will diverge (reach the maximum value) at the critical point[2]**

[1] M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).
{2} M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys.Rev. D 60, 114028 (1999) [arXiv:hep-ph/9903292].

Hot topic! Proton and net-proton High-Order Cumulants and Susceptibilities

DEFINITIONS of central moments and cumulants [1]	
the proton multiplicity in a given event	N
the average over all events	<….>
Mean:	$M = \langle N \rangle = C_1$,
Deviation,	$\delta N = N - \langle N \rangle$
Variance	$\sigma^2 = \langle (\delta N)^2 \rangle = C_2$,
skewness	$S = \langle (\delta N)^3 \rangle / \sigma^3 = C_3/C_2^{3/2}$,
kurtosis	$k = \langle (\delta N)^4 \rangle / \sigma^4 - 3 = C_4/C_2^2$

$$\chi_1^B = \frac{1}{VT^3} \langle N_B \rangle ,$$

$$\chi_2^B = \frac{1}{VT^3} \langle (\delta N_B)^2 \rangle ,$$

$$\chi_3^B = \frac{1}{VT^3} \langle (\delta N_B)^3 \rangle ,$$

$$\chi_4^B = \frac{1}{VT^3} \left(\langle (\delta N_B)^4 \rangle - 3 \langle (\delta N_B)^2 \rangle^2 \right) ,$$

- $\delta N_B = N_B - \langle N_B \rangle$ can be measured event by event
- $\langle \cdot\cdot\cdot \rangle$ -- the ensemble average

Ratious of cumulants and susceptibility ratios:
$\sigma^2/M = \chi_2/\chi_1$, $S\sigma = \chi_3/\chi_2$,
$\kappa\sigma^2 = \chi_4/\chi_2$

- Ratios of the cumulants are used to reduce volume dependence:

$C_2/C_1 = \sigma^2/M$, $C_3/C_2 = S\sigma$, and $C_4/C_2 = \kappa\sigma^2$.

[1] -Xiaofeng Luo (STAR Collab.) , Probing the QCD Critical Point with Higher Moments of Net-proton Multiplicity Distributions, arXiv: 1106.2926v1,J. Phys.: Conf. Ser. 316, 012003 (2011),
DOI:https://doi.org/10.1088/1742-6596/316/1/012003; Skokov, 1205.4756v2.pdf arXiv:1205.4756:;
[2] S. Ejiri, F. Karsch, and K. Redlich, **Hadronic fluctuations at the QCD phase transition**,Phys. Lett. B 633, 275 (2006).