Application of the holographic equations of state for modeling experiments on heavy ion collisions

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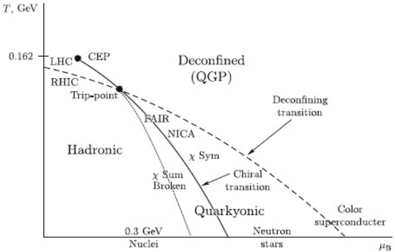
Open problems of the QCD phase diagram research

The aims of the work

1. To propose **a method for setting up holographic equations of state (EoS)** for simulations of real experiments with heavy ions for detailed study of the phase diagram at $\mu_B > 0$.
2. To implement holographic equations of state **into a software package** for solving equations of relativistic hydrodynamics
3. To demonstrate **the results of a multi-stage simulation of a heavy ion collision experiment using a holographic EoS**



Pic.: I. Ya. Aref'eva, Theoret. and Math. Phys., 217, 3 (2023)

Lattice QCD results:

- The **great** results for $\mu_b \approx 0$.
- Area $\mu_b > 0$ is unavailable because of **the sign problem**.

AdS/CFT duality

Gerard 't Hooft - the Holographic principle (1993):

Information about matter in space is a "flat hologram"at the boundary of this space (no more than one degree of freedom per Planck area).

Maldacena - AdS/CFT - duality (1998)J. Maldacena // Adv.Theor.Math.Phys.2:231-252,1998] :

Under certain boundary conditions, there is a duality of QCD and Einstein's theory in the low-energy limit

Thermodynamic characteristics of quark-gluon plasma:

T, ε, P, μ_b
EoS: $P = P(\varepsilon)$

Parameters of the AdS_5 space deformation:

$T(z_h)$, $S(z_h)$, where z_h is the characteristic of the gravitational horizon, T, S are the temperature and entropy of the corresponding black brane

Holographic EoS of QGP within the framework of the model with a double dilaton field (I.Ya. Arefieva)

An ansatz proposed is[I. Aref'eva et al. // JHEP 06, 090 (2021)] :

$$ds^2 = \frac{L^2}{z^2} b(z) \left[-g(z)dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)} \right],$$

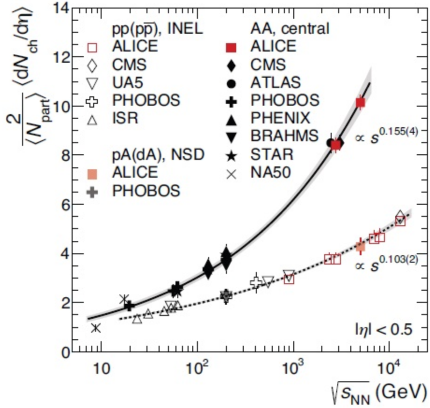
A deforming factor is $b(z) = e^{2A(z)}$, L is the radius of AdS, g(z) is a thermodynamic blackening function

$A(z) = -a ln(bz^2 + 1)$ - model of "light quarks"[O. Andreev, V. Zakharov // Phys.Rev.D74, 025023 (2006)]

Advantages of the approach:

- The possibility of studying the properties of nuclear matter at sufficiently large μ_b
- An anisotropy parameter introduced within the framework of this model allows us to holographically restore the experimentally confirmed dependence of multiplicity (with $\nu = 4.5$)
- One of the model parameters is selected in accordance with the Regge spectra of ρ mesons (the case of light quarks)

The anisotropic model



The experimental dependence of the multiplicity density on energy

$$M \propto s_{NN}^{0.15},$$

[K. Aamodt, et al. // Phys. Rev. Lett. 105 252301 (2010)]

The result within isotropic holographic models

$$M \propto s_{NN}^{\frac{1}{3}}, \nu = 1$$

[S. Gubser et al. // Phys. Rev. D 78 066014 (2008)]

The result of Arefieva's group (the anisotropic case)

$$M \propto s^{\frac{1}{6}} \approx s_{NN}^{0.16}, \nu = 4.45$$

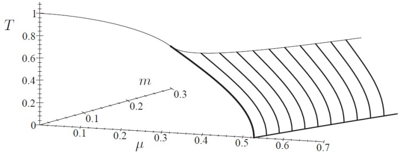
[I. Aref'eva, A. Golubtsova // JHEP 04 011 (2015)]

A holographic equation for the physical masses of quarks

The main idea

The parameters of the light quark model are fitted in accordance with lattice QCD calculations for the masses of quarks closest to the physical ones [M. Halasz et al. // Phys. Rev. D 58, 096007 (1998)].

It can be assumed that there is no significant qualitative difference of the QCD phase structure of QCD in the chiral limit and in the case of the physical masses of quarks



The method of free parameters calibration [J. Grefa et al. // Phys. Rev. D 104, 034002 (2021)]:

Dimensionless thermodynamic quantities are multiplied by the scale factor Ł (power of L= power of a quantity in GeV). Other parameters:

- ν is an anisotropy parameter .
  $\nu = 1$ for the isotropic case,
  $\nu = 4.5$ for the anisotropic case.
- Parameter L is a characteristic energy scale.
- Parameter G is a dimensionless gravitational constant
- The dimensionless parameter a and the parameter b appear in the dilaton field interaction function selected as $A(z) = -a ln(1 + bz^2)$

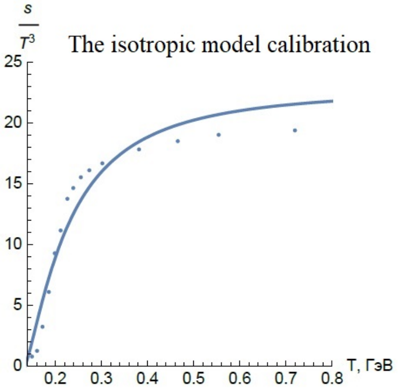
A holographic equation for the physical masses of quarks

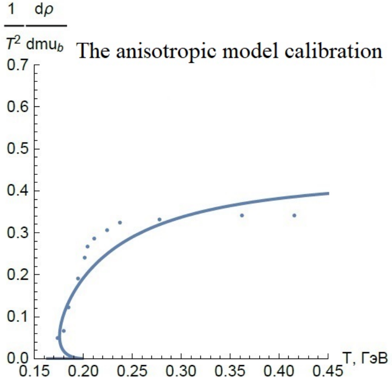
The least squares fit for isotropic case:

$L = 1.08$ GeV, $G = 0.34$
$a = 3.71$, $b = 0.0129$ GeV^2.

The least squares fit for anisotropic case:

$L = 1.01$ GeV, $G = 0.81$ $a = 3.949$,
$b = 0.03506$ GeV^2.





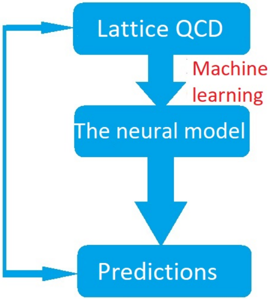
The model with an alternative deforming factor

$$A(z) = d\ln(az^2+1) + d\ln(bz^4+1);\ a, b \geq 0, d \leq 0$$

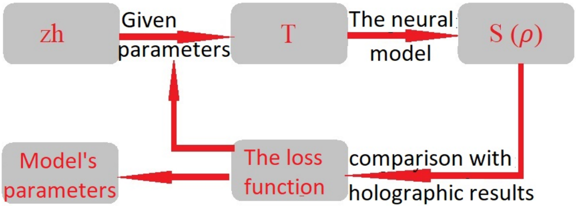
from [X. Chen, M. Huang // J. High Energ. Phys. 2025, 123 (2025)]

Machine learning application to the model calibration:

Task 1:
A model for testing



Task 2:
The optimization of parameters



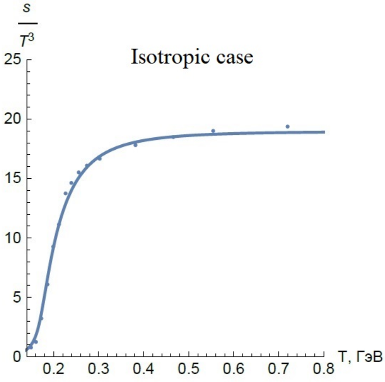
A holographic equation for the physical masses of quarks with the alternative factor

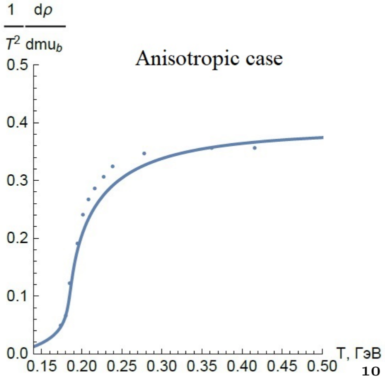
The least squares fit for isotropic case:

$L = 1.02$ GeV, $G = 0.41$ $a = 0.013$ GeV^2, $b = 0.037$ GeV^4, $d = -0.267$

The least squares fit for anisotropic case:

$L = 0.7$ GeV, $G = 0.43$ $a = 0.177$ GeV^2, $b = 0.202$ GeV^4, $d = -0.249$





Implementation of the holographic EoS in the packages of relativistic hydrodynamics

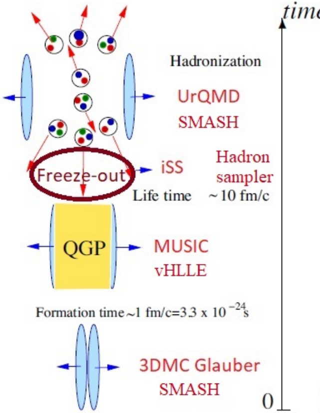
A program code was written based on the built-in MUSIC (https://github.com/MUSIC-fluid/MUSIC.git) and vHLLE (https://github.com/yukarpenko/vhlle.git) methods for reading tables constructed on lattice QCD results:

1. A two-dimensional table of thermodynamic quantities (T, p, μ_b) is being constructed based on the holographic formulae.
2. One uses the initial profile of energy and baryon densities to interpolate the values of thermodynamical quantities at each point of the grid initialized.
3. Evolution stops when the energy density reaches a certain critical value - freeze-out energy (preset)
4. Provides a special parameter of the MUSIC input file for quick table switching from the isotropic model to the anisotropic case.

Evolution of the QGP within the iEBE-MUSIC(https://github.com/chunshen1987/iEBE-MUSIC.git) and vHLLE-SMASH(https://github.com/smash-transport/smash-vhlle-hybrid.git) packages

The structure of the QGP evolution:

1. An initial profile of energy and baryon density is calculated using the 3D Monte Carlo Glauber (or SMASH) model.
2. The MUSIC (vHLLE) package (modified with a holographic equation of state) accepts this profile as an input and performs the QGP evolution.
3. The iSS (SMASH hadronic sampler) package performs Monte Carlo sampling to obtain a finite set of particles.
4. Transport model UrQMD (SMASH-afterburner) allows one to get the final spectrum of hadrons



The comparison of K^+ and π^- spectra by transverse mass with the results NA49[NA49 collab. // Phys.Rev.C66:054902,2002]

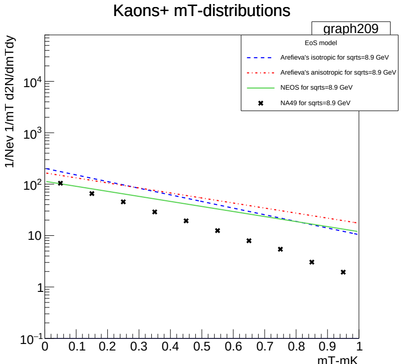
Pb-Pb collisions, $|y| < 2.6$

$\sqrt{s}$=8.9 GeV, b<2.5 fm

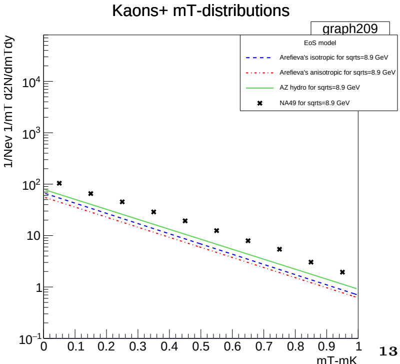
The standart deforming factor is used:

$$A(z) = -a ln(bz^2 + 1)$$

K^+ with MUSIC



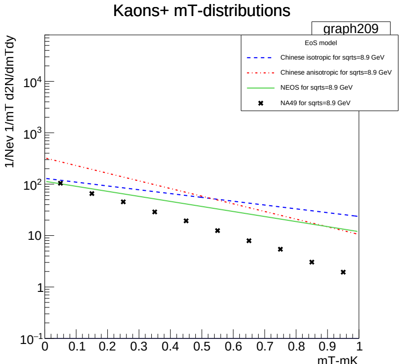
K^+ with vHLLE



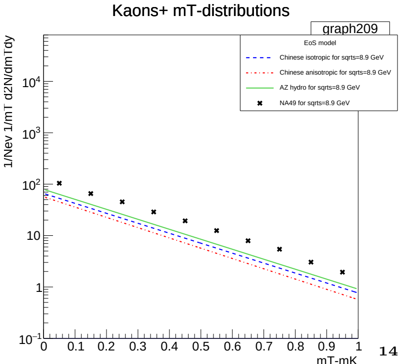
The comparison of K^+ and π^- spectra by transverse mass with the results NA49 with the alternative deforming factor

Pb-Pb collisions, $|y| < 2.6$
$\sqrt{s}$=8.9 GeV, b<2.5 fm
The alternative deforming factor is used:
$$A(z) = dln(az^2 + 1) + dln(bz^4 + 1)$$

K^+ with MUSIC



K^+ with vHLLE



Conclusion and potential for further development

Results

1. A method is proposed for tuning holographic models using lattice QCD results for the **physical quark masses**.

2.**Holographic equations of state are implemented into the software packages of relativistic hydrodynamics**.

3. The results of calculations in the iEBE-MUSIC and vHLLE-SMASH packages for **multi-stage simulation** of the QGP dynamics are considered.

4. A **model dependence of the results on the applied equations of state** is shown.

Future plans and prospects

1. For the better consistency of packages' componets, it is planned to use **the matching of holograhic EoS with the HRG equation**

2. It is possible to **account for the secondary magnetic field** generated by the outgoing charged particles

3. Using the approaches described, it is planned to study **fluctuations and long-range correlations as tools for studying the phase diagram**

Thank you for your attention!

Backup Slides

Tensorflow input

Task 1:

- The neural network trained is four fully connected layers on scheme 64-128-64-1.
- Activation functions scheme is relu-relu-relu-linear.
- Each layer utilizes a sigmoid activation function.
- Adam algorithm is used as an optimizer with MSE loss function.
- Learning rate=0.001

Task 2:

- A gradient descent algorithm to find the optimal parameter values.
- The loss function is the difference between the predictions of the holographic model and the values obtained from the neural network model
- The least squares method is employed to minimize the loss function.
- The "Adam" optimizer is used for training with constraints on paranmeters from slide № 9.
- In every training epoch, the loss function is calculated first and then the gradients should be computed. The parameter values are updated afterwise.