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HEAVY AND LIGHT MESONS IN THE FRAME OF THE QUARK MODEL WITH SEPARABLE INTERACTION KERNEL

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INTRODUCTION

- A complete understanding of the full range of hadronic spectra from the lightest to the heaviest remains challenge.

- Direct calculations of hadron properties from the first principles of QCD still encounter some technical and conceptual difficulties.

- The use of different approaches is determined by the flavour structure of mesons:
  - the behaviour of light flavours is determined by chiral symmetry and requires fulfillment of low-energy theorems in QCD;
  - the strong dynamics of heavy quarks is more simple, since they behave like classical particles

THE QUARK MODEL WITH SEPARABLE INTERACTION

We start from the Bethe-Salpeter equation in the ladder approximation:

$$\Gamma_H(q,P) = -\frac{4}{3}\int \frac{d^4p}{(2\pi)^4} D(q-p)\gamma_\alpha S_1(p_1)\Gamma_H(p,P)S_2(p_2)\gamma_\alpha$$

Considering the interaction kernel in separable form

$$D(q-p) = D_0\varphi(q^2)\varphi(p^2)$$

with 4-dimentional form-factor function in Gaussian form $\varphi(q^2) = e^{-q^2/\Lambda_H^2}$

and vertex function rewritten as $\Gamma_H(p,P) = N_H\,\varphi(p^2)\,\gamma_H$

the dressed quark propagator is considered in the Euclidean space $S_i(p_i) = \dfrac{1}{i(p_i\cdot\gamma)+m_i}$

The meson-quark coupling constants N_H are determined by the condition:

$$1 = N_c\frac{P_\mu}{2P^2}\frac{\partial}{\partial P_\mu}\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}\left\{\Gamma_H(p,P)S_1(p_1)\Gamma_H(q,P)S_2(p_2)\right\}$$

SYSTEM OF EQUATIONS: PSEUDOSCALAR MESONS

The meson-quark coupling constants N_H are determined by the condition:

$$1 = -i\frac{N_c N_{ps}^2}{2P^2} P^\mu \int \frac{dp}{(2\pi)^4} \varphi^2(p^2) \cdot$$
$$\cdot \{b_1 \mathrm{tr}\left[i\gamma_5 S_1(p_1)\gamma_\mu S_1(p_1) i\gamma_5 S_2(p_2)\right] + b_2 \mathrm{tr}\left[S_1(p_1) i\gamma_5 S_2(p_2)\gamma_\mu S_2(p_2) i\gamma_5\right]\}$$

For pseudoscalar mesons weak leptonic decay constant can be obtained from the matrix element of the axial current

$$f_P P^\mu := \langle 0|\bar{\mathcal{Q}}(T^P)^T \gamma_\mu \gamma_5 \mathcal{Q}|P(p)\rangle$$

$$P^\mu f_P = N_c N_P \int \frac{dp}{(2\pi)^4} \varphi(p^2) \mathrm{tr}\{(\mathrm{i}\gamma_5)\mathrm{S}_1(\gamma_\mu\gamma_5)\mathrm{S}_2\}.$$

SYSTEM OF EQUATIONS: VECTOR MESONS

The meson-quark coupling constants N_H are determined by the condition:

$$\begin{aligned} 1 &= -i\frac{N_c N_{\rm v}^2}{6P^2} P^\mu \int \frac{dp}{(2\pi)^4} \varphi^2(p^2) \epsilon^{\rho\sigma} \cdot \\ &\cdot \{ b_1 {\rm tr}\left[\gamma_\rho S_1(p_1)\gamma_\mu S_1(p_1)\gamma_\sigma S_2(p_2)\right] + b_2 {\rm tr}\left[S_1(p_1)\gamma_\rho S_2(p_2)\gamma_\mu S_2(p_2) i\gamma_\sigma\right]\} \end{aligned}$$

The vector leptonic decay can be written in the same way

$$f_{\rm v} M_{\rm v} \epsilon^{\mu\nu} := \langle 0|\bar{\mathcal{Q}}(T^P)^T \gamma^\mu \mathcal{Q}|V_\nu(p)\rangle$$

$$f_{\rm v} M_{\rm v} \epsilon^{\mu\nu} = N_c N_{\rm v} \int \frac{dp}{(2\pi)^4} \varphi(p^2) \epsilon^{\mu\rho} {\rm tr}\{\gamma_\rho {\rm S}_1({\rm p}_1)\gamma_\nu {\rm S}_2({\rm p}_2)\}.$$

INTEGRATION TECHNIQUES

The one-loop two-point integral

$$I(P^2) = \int \frac{dp}{\pi^2} F(p^2) \frac{1}{[p_1^2 + m_1^2]\,[p_2^2 + m_2^2]},$$

after applying the Feynman parametrization all integrals can be presented as

$$I(P^2) \;=\; \int_0^1 d\alpha \int_0^\infty dt \frac{t}{(1+t)^2}\,[F(z_0)]$$

with $\quad z_0 \;=\; tD + \dfrac{t}{1+t} R^2 \qquad D = \sum_i \alpha_i (q_i^2 + m_i^2) - R^2 \qquad R = \sum_i \alpha_i q_i$

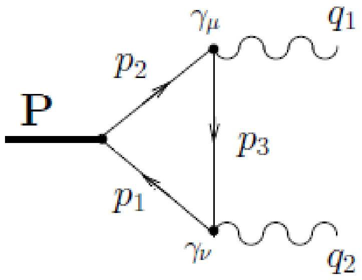
Finally all matrix elements can be presented presented as combination of simple integrals of type

$$I(a_0 \ldots a_n, m, n, F) = \int_0^1 \{d\alpha_i\} \prod_i \alpha_i^{a_i} \int_0^\infty dt \frac{t^m}{(1+t)^n}[F(z_0)].$$

LIGHT MESONS

- Definition of parameters of the model:
  - Pion photo-decay
  - Rho-meson: hadronic and radiative decays
- Pion transition form factor

THE $\pi^0 \to \gamma\gamma$ DECAY



$$T^{\mu\nu}(q_1,q_2) = N_c N_p \int \frac{dp}{(2\pi)^4}\varphi(p^2)\mathrm{tr}\{i\gamma_5 S_i(p_2)\gamma_\mu S_i(p_3)\gamma_\nu S_i(p_1)\}.$$

The second amplitude $T^{(2)}$can be written by the replacement $q_1 \leftrightarrow q_2$

Employing the pseudoscalar vertex function $\Gamma_H = i\gamma_5 g_{Hqq}$

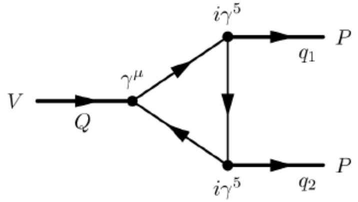
$$\begin{aligned} T(q_1,q_2) &= i\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu\epsilon_2^\nu q_1^\alpha q_2^\beta\left(N_c Q_q^2\right)\frac{mN_\pi}{4\pi^2}I(Q,q_1,q_2) = \\ &= i\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu\epsilon_2^\nu q_1^\alpha q_2^\beta\left(N_c Q_q^2\right)G_{\pi\gamma\gamma}(Q,q_1,q_2), \end{aligned}$$

$Q_q = (e_u^2 - e_d^2)$ with $e_u = 2/3e$ and $e_d = -1/3e$

$$g_{\pi\gamma\gamma} = G_{\pi\gamma\gamma}(M_\pi^2,0,0) \simeq \frac{mN_\pi}{4\pi^2\Lambda_\pi^2}I(M_\pi^2,0,0)$$

$$\boxed{\Gamma(\pi^0\to\gamma\gamma) = \frac{M_\pi^3}{64\pi}(4\pi\alpha)^2 g_{\pi\gamma\gamma}^2}$$

$\boldsymbol{\rho}$ −MESON



$$T^\mu(q_1, q_2) = N_c N_\rho N_\pi \int \frac{dp}{(2\pi)^4} \varphi(p^2)$$
$$\mathrm{tr}\{\gamma_\mu S_2(p_2) i\gamma_5 S_3(p_3) i\gamma_5 S_1(p_1)\}$$

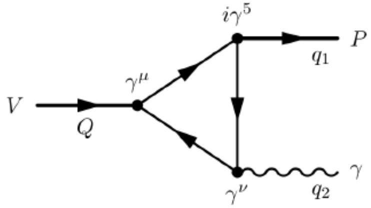
with form factor $\varphi(p^2) = \varphi_\rho(p^2)\varphi_\pi^2(p^2)$

The matrix element splits into two terms:

$$T^\mu(p_1, p_2) = (p_1 - p_2)^\mu f^+(t) + (p_1 + p_2)^\mu f^-(t)$$

with $f^-(t = M_\rho^2) = 0,\ \frac{1}{2} f^+(t = M_\rho^2) = g_{\rho\pi\pi}$

$$\boxed{\Gamma_{\rho\pi\pi} = \frac{1}{6\pi}\frac{k^3}{M_\rho^2} g_{\rho\pi\pi}^2}$$



$$T^{\mu\nu}(q_1, q_2) = i\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu(Q)\epsilon_2^{*\nu}(q_2)\cdot$$
$$\cdot Q^\alpha q_2^\beta (N_c Q_q e)\frac{m N_p N_v}{4\pi^2} I(Q, q_1, q_2)$$

$$\boxed{\Gamma_{VP\gamma} = \frac{1}{3}\alpha k^3 g_{VP\gamma}^2}$$

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PARAMETERS OF THE MODEL

Parameters of the model: masses of quarks (m_q, m_c, m_b), parameters Λ_H

Physical data used for parameter fitting: $m_\pi = 0.139, m_\rho = 0.77\ GeV$

	π	Exp.	ρ	Exp.
f_H, GeV	0.131	0.131	0.2	0.2
$\Gamma_{\pi\gamma\gamma}$, eV	7.76	7.82 [1]	-	-
$\Gamma_{\rho\pi\pi}$, GeV	-	-	0.151	0.150 [2]
$\Gamma_{\rho\pi\gamma}$, keV	-	-	71.8	84.2 [3]

[1] PRD **33**, (1986) 3199–3202
[2] Phys. Rev. Lett. **106**, (2011) 162303
[3] PoS LATTICE2018 (2018) 065, [2]

Basic model parameters and constants for light mesons:

$m_{u(d)}$, GeV	Λ_π, GeV	N_π	Λ_ρ, GeV	N_ρ
0.223	1.14	3.724	0.75	3.612

THE $\gamma^* \rightarrow \pi^0\gamma$ TRANSITION FORM FACTOR

$$T^{\mu\nu}(q_1,q_2) = N_c N_p \int \frac{dp}{(2\pi)^4}\varphi(p^2)\mathrm{tr}\{i\gamma_5 S_i(p_2)\gamma_\mu S_i(p_3)\gamma_\nu S_i(p_1)\}.$$

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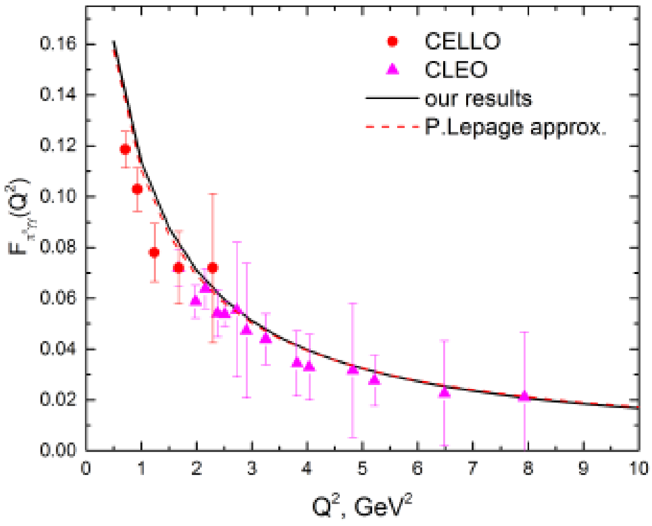
$Q_q = (e_u^2 - e_d^2)$ with $e_u = 2/3e$ and $e_d = -1/3e$ for the pion

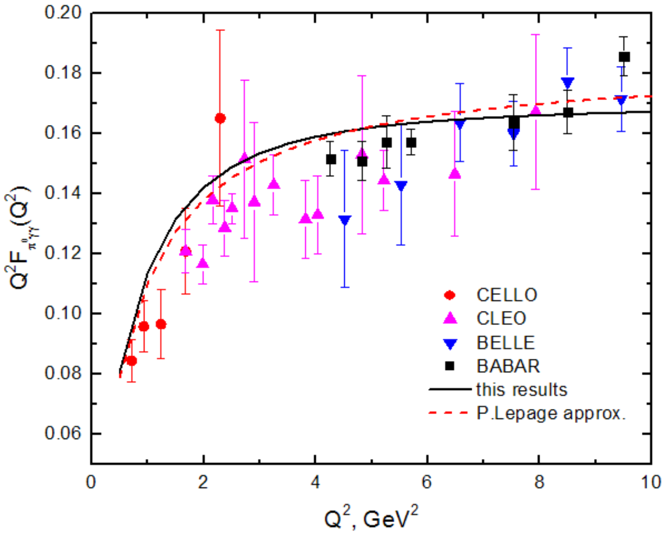
$$F_{\pi\gamma}(Q^2) = e^2 G_{\pi\gamma\gamma}(M_\pi^2, Q^2, 0)$$

THE $\gamma^* \to \pi^0\gamma$ TRANSITION FORM FACTOR

Sold line corresponds to the S. Brodsky and P. Lepage prredictopn within non-perturbative QCD [PRD 24, (1981) 1808-1817]

$$F_{\pi\gamma}(Q^2) = \frac{2f_\pi}{8\pi^2 f_\pi^2 + Q^2},$$

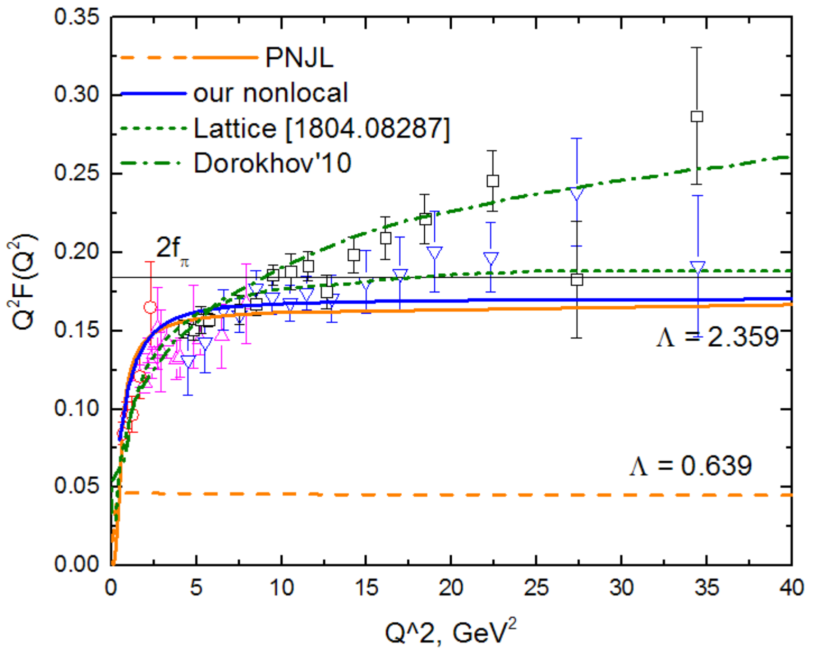




Experimental data: CLEO PRD 57 (1998) 33-54, CELLO Z. Phys. 49, (1991) 401–409, Belle PRD86 (2012) 092007, BaBar PRD 80 (2009) 052002

CONCLUSION - I

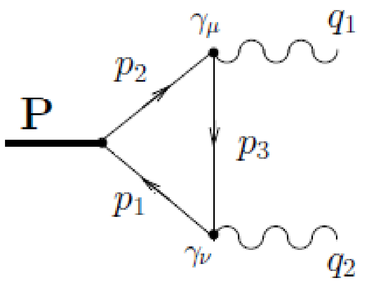
- perturbative QCD/Lattice QCD at high Q does not reproduce the behavior presented by BaBar data.
- there exist couple of models (A.E. Dorokhov, arXiv:0905.4577; A.V. Radyushkin, arXiv:0906.0323) which show logarithmic behavior of TFF at high momenta
- non-perturbative QCD well works at low Q



HEAVY MESONS

- $\eta_c(\eta_b) \to \gamma\gamma$ decays and transition form factors
- ${}^{J}/_{\psi}, \Upsilon, D^*$ radiative decays

THE $\gamma^* \to H\gamma$ FORM FACTOR



$$T^{\mu\nu}(q_1,q_2) = N_c N_p \int \frac{dp}{(2\pi)^4}\varphi(p^2)\mathrm{tr}\{i\gamma_5 S_i(p_2)\gamma_\mu S_i(p_3)\gamma_\nu S_i(p_1)\}.$$

The second amplitude $T^{(2)}$can be written by the replacement $q_1 \leftrightarrow q_2$

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$Q_c = 2/3e$, $Q_b = -1/3$e for heavy pseudoscalars

$$g_{H\gamma\gamma} = G_{H\gamma\gamma}(M_H^2,0,0) \simeq \frac{mN_H}{4\pi^2\Lambda_H^2} I(M_H^2,0,0)$$

$$\boxed{\Gamma(H\to\gamma\gamma) = \frac{M_H^3}{64\pi}(4\pi\alpha)^2 g_{H\gamma\gamma}^2}$$

$$\boxed{F_{H\gamma}(Q^2) = e^2 G_{H\gamma\gamma}(M_H^2,Q^2,0)}$$

TRANSITION FORM FACTORS η_c, η_b

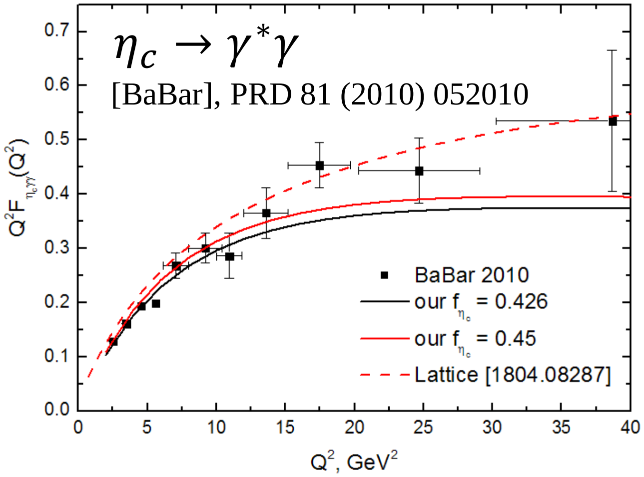
Model parameters and constants for heavy mesons

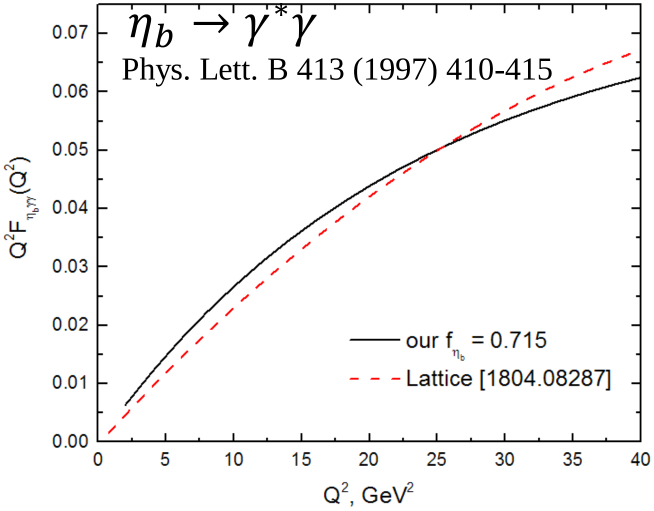
$m_c = 1.6$, $m_b = 4.77$

	M_H, GeV	Λ_H, GeV	N_H	f_H (our)	f_H (refs)
η_c	2.985	2.775	3.546	0.426	0.42[18]
η_b	9.39	2.81	8.568	0.715	0.705[18]

	$\Gamma_{H\gamma\gamma}$ keV (our)	$\Gamma_{H\gamma\gamma}$ keV (refs)	PDG
η_c	5.03	4.88[1] - 6.788 [2]	5.1
η_b	0.18	0.17 - 0.69 [3]	-

[1]1804.08287 [hep-ph],
[2] 2305.06231 [hep-lat]]
[3][hep-ph/0609268]





$J/\psi \to \eta_c\gamma, \quad \Upsilon \to \eta_b\gamma, \quad D^* \to D\gamma$

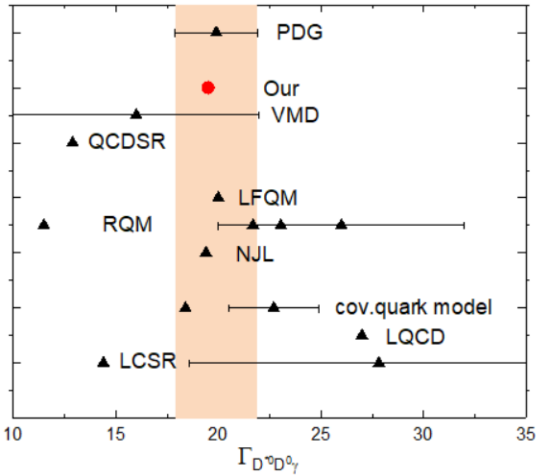
Model parameters and constants for heavy mesons

	M_H, GeV	Λ_H, GeV	N_H	f_H (our)	f_H (refs)
J/ψ	3.075	2.03	3.238	0.435	0.399
Υ	9.405	4.22	3.489	0.705	
D^*	2.031	1.29	2.916	0.24	0.223
D	1.889	1.42	4.325	0.213	0.203

	our	refs.
$J/\psi \to \eta_c\gamma$	2.28 keV	1.83-2.49 keV (Lattice QCD)
$\Upsilon \to \eta_b\gamma$	25.3 eV	5.8 [1], 45[2]

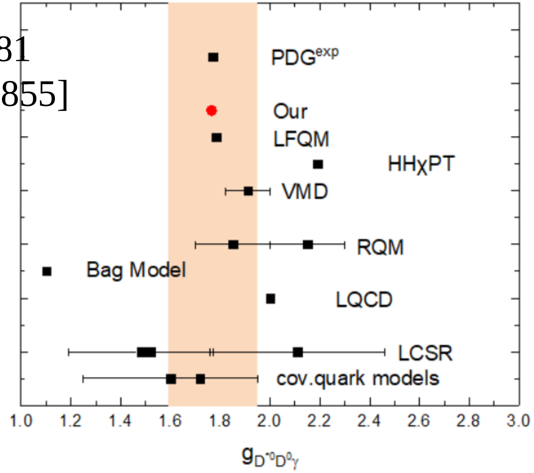
CLEO experiment: $\Gamma_{J/\psi\eta_c\gamma}$= 1,83 keV [arXiv:0805.0252]

KEDR experiment: ($\Gamma_{J/\psi\eta_c\gamma}$= 2,17 keV) [arXiv:1002.2071]



[1]arXiv:0210381

[2 arXiv:1208.2855]



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CONCLUSION-II

- An approach for describing properties of light and heavy mesons is developed in the framework of the effective quark model with nonlocal interaction. A set of model parameters is obtained for light mesons and heavy mesons. The meson-quark couplings are obtained in explicit form.

- All the parameters are fixed via observables such as the meson mass and decay constants.

- The model is applied to calculate the physical processes: decay of pseudoscalar mesons in two photons, the radiative decays of vector mesons. All the results are consistent with the experimental data or the results of other models.

THANK YOU FOR ATTENTION