Calculation of target dependence of the isotope distributions in heavy-ion reactions at energies from 35 to 140 MeV per nucleon in the modified transport-statistical model

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Outline

- **Motivation.**
- **Description of heavy-ion collisions with modified transport (BNV)-statistical (SMM) approach . Comparison with experimental data for reactions with four different projectiles ^{18}O (35 MeV per nucleon) , ^{86}Kr (64 MeV per nucleon), ^{40}Ca and ^{48}Ca (140 MeV per nucleon) on two targets : ^{181}Ta and ^{9}Be**
- **Comparison with other model calculations (EPAX, Abrasion-Ablation and Fracs)**
- **Explanation of target ratio dependence on mass number *A* from the point of view of BNV-SMM calculations .**
- **Conclusion**

To model heavy-ion collision microscopically we use kinetic theory:

Transport theory: Boltzmann-Nordheim-Vlasov (BNV) approach

time evolution of the one-body phase space density: f(r,p;t)

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla} f - \vec{\nabla} U \,\vec{\nabla}_p f = I_{coll}\left[f, \sigma \right]$$

Physical input:

mean field potential U (equation of state) and in-medium elastic cross section

F. Bertsch, S. Das Gupta, Phys. Rep. **160** (1988) 189
V. Baran, M. Colonna, M. Di Toro, Phys. Rep., **410** (2005**)** 335

Density functional
U(ρ(r)) = Nuclear Mean Field + Symmetry terms + Coulomb

$$U(\rho) = A\left[\frac{\rho}{\rho_0}\right] + B\left[\frac{\rho}{\rho_0}\right]^d + C(-1)^k\left(\rho_n - \rho_p\right)/(\rho_n + \rho_p) + U_{\mathrm{coul}}$$

$$A = -356 MeV, B = 303 MeV, d = 7/6, k = 1(p), 2(n), C = 36 MeV$$

Solution of transport equation

Partial integro-differential equation for *f(r,p;t)* is solved by simulation with the test particle method:
N -finite element test particles (TP) per nucleon.
Each TP carries charge and isospin number.
A – number of nucleons in the system
g – the shape of the TP in space
ρ – the density

$$f(\vec{r},\vec{p},t)=\frac{1}{N}\sum_{i}^{NA} g(\vec{r}-\vec{r}_i(t))\,\bar{g}(\vec{p}-\vec{p}_i(t))$$

$$g=\mathrm{e}^{-(\vec{r}-\vec{r}_i(t))^2/L^2}\ \ldots;\ \bar{g}=\mathrm{e}^{-(\vec{p}-\vec{p}_i(t))^2/l^2}$$

$$\rho(r;t)=\int d\vec{p}\ f(\vec{r},\vec{p};t)$$

Equations of motion of TP

(Newton equation of motions:):

$$\frac{\partial\vec{p}_i(t)}{\partial t}=-\vec{\nabla}_r U(r_i,t)\qquad \frac{\partial\vec{r}_i(t)}{\partial t}=\frac{\vec{p}_i(t)}{m}$$

Velocity Verlet (or leapfrog) algorithm, accuracy $(dt)^2$:

$$\vec{p}_i(t+\frac{1}{2}\Delta t)=\vec{p}_i(t)-\frac{1}{2}\Delta t\vec{\nabla}_r U(r_{i(t)})$$

$$\vec{r}_i(t+\Delta t)=\vec{r}_i(t)+\Delta t\ \vec{p}_i(t+\frac{1}{2}\Delta t)/m$$

$$\vec{p}_i(t+\Delta t)=\vec{p}_i(t+\frac{1}{2}\Delta t)-\frac{1}{2}\Delta t\vec{\nabla}_r U(r_{i(t+\Delta t)})$$

Collision term

$$I_{coll}[f_1,\sigma] =$$

$$\frac{g}{h}\int d^3p_2\, d^3p_3\, d^3p_4\ \sigma(12,34)\,\delta(\vec{p}_1+\vec{p}_2-\vec{p}_3+\vec{p}_4)\,\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3+\varepsilon_4)\left[\bar{f}_1\bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3 \bar{f}_4\right]$$

Pauli blocking factors for final state $(1-f(r,v_i;t)) \equiv (1-f_i) := \bar{f}_i$

g degeneracy

Collision term: treatment by stochastic simulation

1.Select in each time step dt TPs with distance $d \leq \sqrt{\sigma/\pi}$

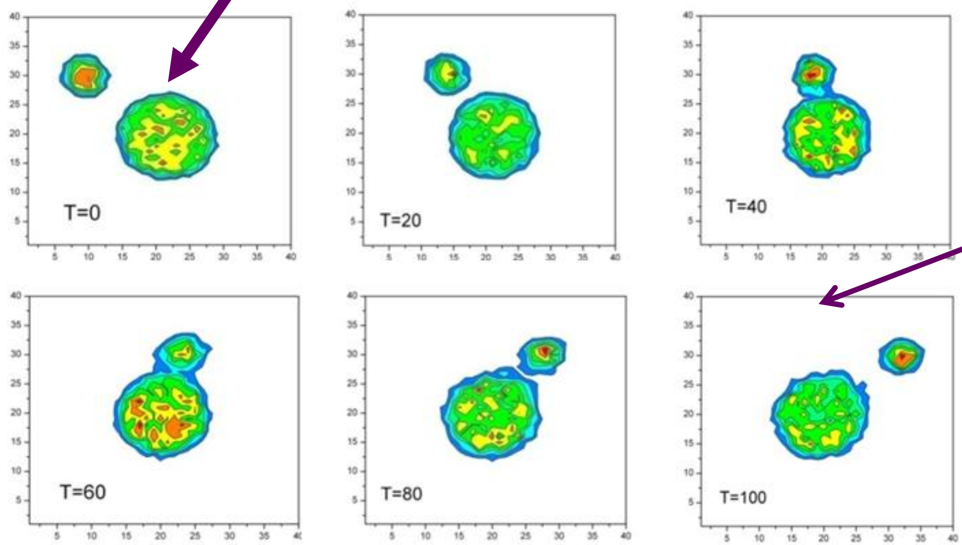
2.Collide with probability $P = \sigma_{el}/\sigma_{max}$ with random scattering angle

3.Check Pauli blocking of final state in phase space

The computational code was developed on the basis of the code developed in LNS-INFN Catania in collaboration with M. Di Toro and H.H. Wolter

STEP 1: stochastic modeling of the system of two approaching ions.
Minimization of energy in Woods-Saxon potential, taking into account Coulomb and Symmetry energy. The initial values of coordinates ***R*** and momenta ***P*** in the center of mass system are added

STEP 2: Evolution in time in the mean field until freeze-out time: only Coulomb potential, nuclear forces between fragments are negligible



Density contour plots in the reaction
^{18}O(35 *A* MeV) + ^{181}Ta at *b* = 9 fm
(t=0, 20, 40, 60, 80, 100 fm / *c* (10 fm/*c*=3.3*10-23 c))

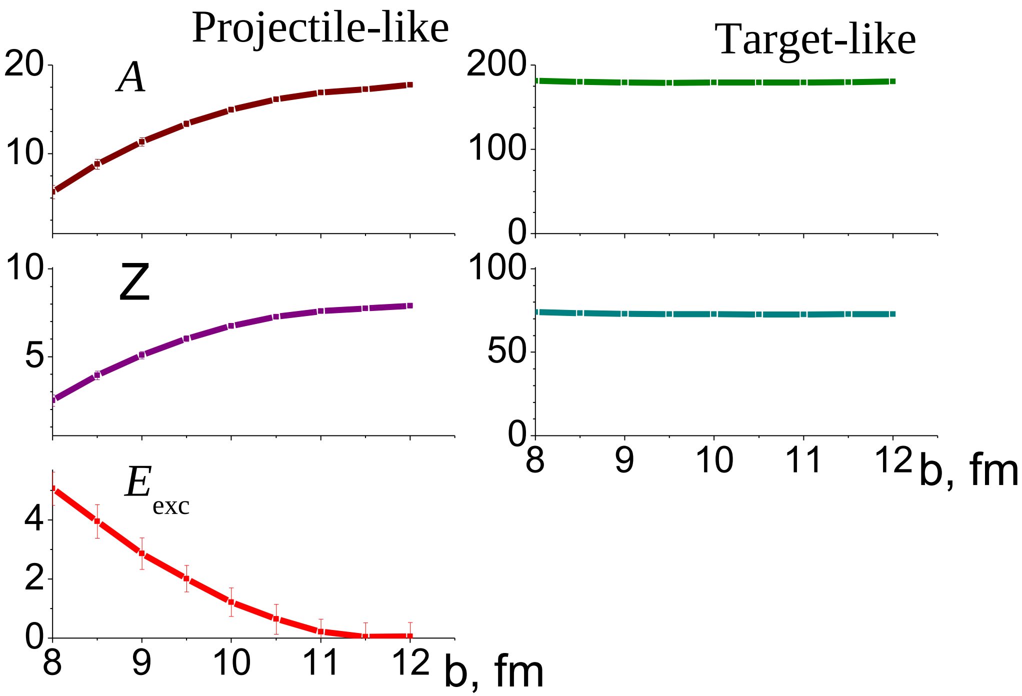
Identify final fragments by coalescence method
Here:
Cut-off criterion in density

$$(\varrho(r;t_{\text{freeze-out}})<0.02)$$

Fragments characteristics:
mass number *A*, charge *Z*, intrinsic energy E_{int}, momentum ***P***, coordinates ***R***

Calculations with 200 TP for nucleon

^{18}O(35 *A* Mev)+^{181}Ta



The results smoothly depends on the value of impact parameter *b*

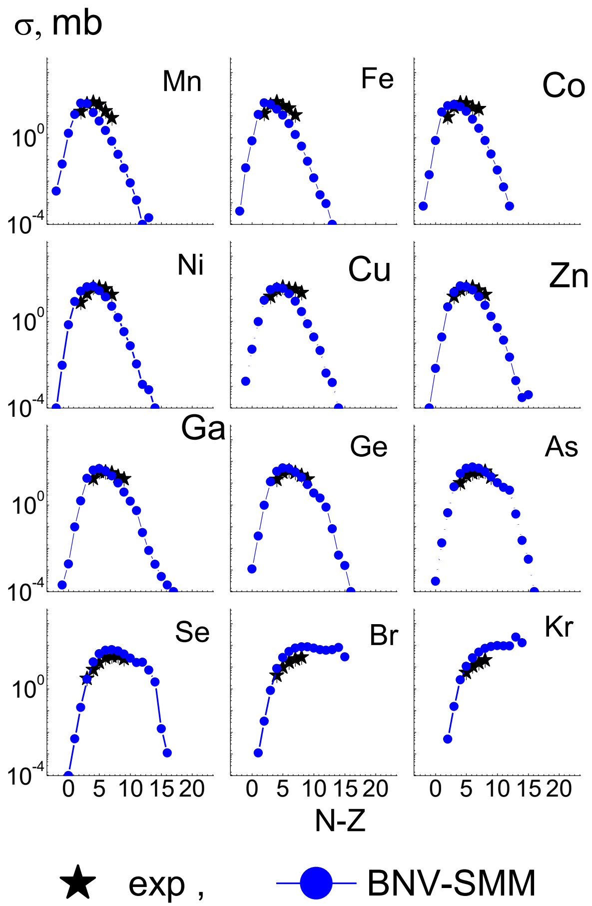
Heavier ion stays practically the unchanged!

Transport calculations are fulfilled with the rather large step in *b* (δb=0.5fm) and then the parameters *A*, *Z*, ***P***, E_{exc} in the intermediate steps in *b* (δb=0.03fm) are calculated with the use of the interpolation method.

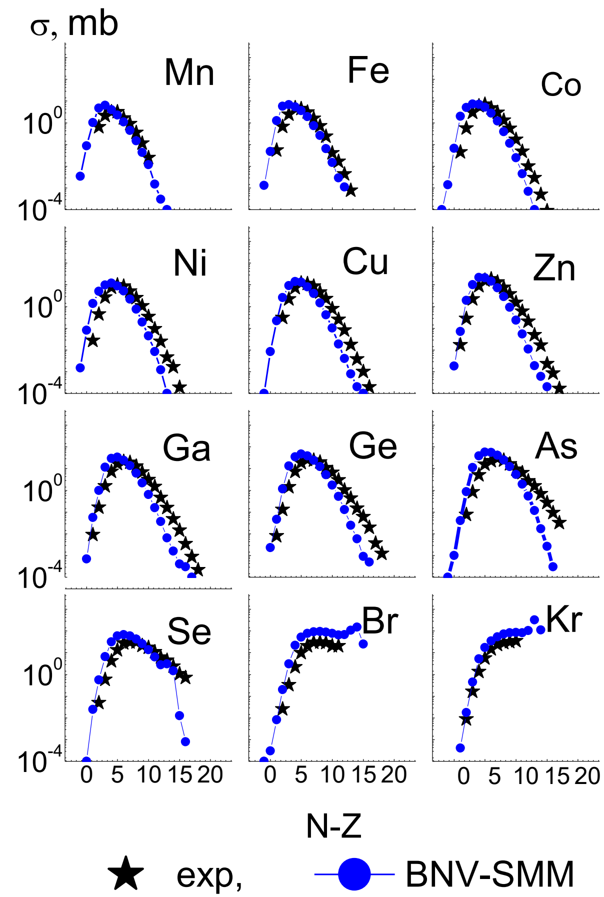
To de-excite the primary fragments the statistical code is used → **SMM** code, P. Bondorf, et al.,Phys. Rep. 257, 133 (1995)

STEP 3: Isotope distributions for two reactions, Experiment and BNV-SMM calculations:

^{86}Kr(64 *A*Mev)+^{181}Ta



^{86}Kr(64 *A*Mev)+^{9}Be



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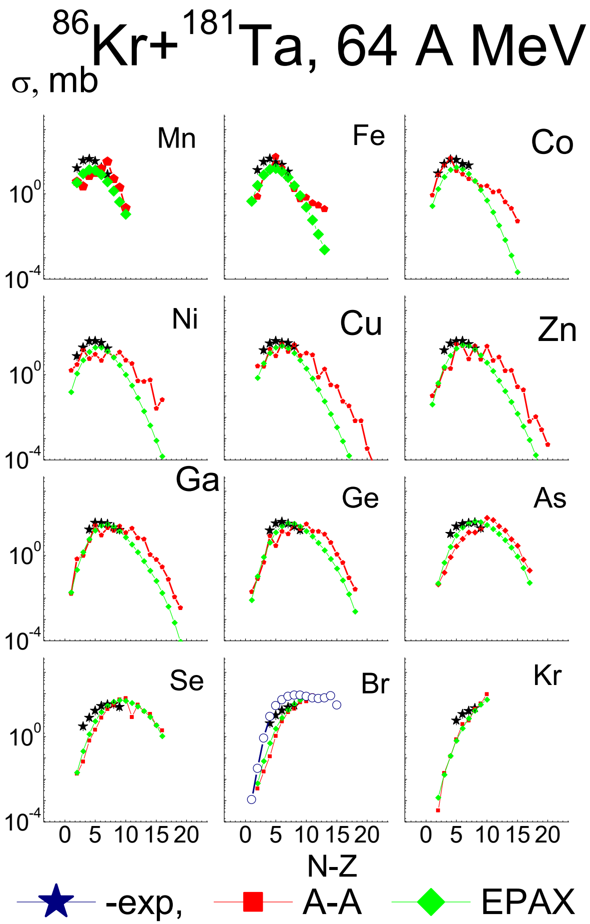
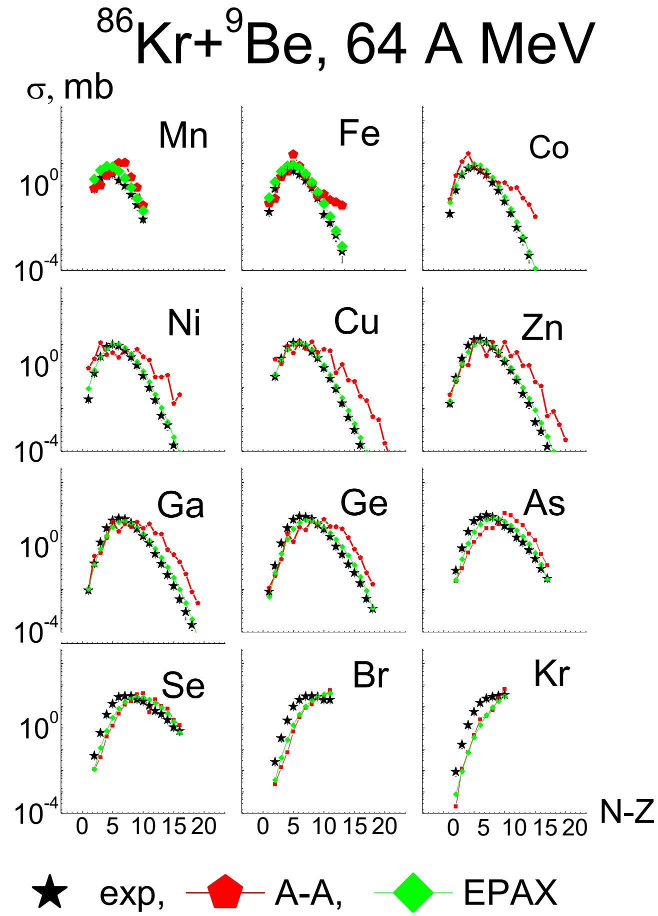
Approaches commonly used to predict isotope distributions produced in nuclear collisions at Fermi energies:

- **EPAX (**an Empirical PArametrization of fragmentation CROSS sections) *K. Summerer and B. Blank, Phys. Rev. C. 61, 034607 (2000).*
- **Abrasion-Ablation model (A-A),** *Bowman J.D., Swiatecki W.J., Tsang C.F. // LBL Report. 1973. LBL-2908.*
- **FRACS** (Mei B. Improved empirical parameterization for projectile fragmentation cross sections // Phys. Rev. C. 2017. V. 95. P. 034608.

Cold residues for reactions, experiment, EPAX and Abrasion-Ablation models





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Target ratio of forward emitted fragments

A.G. Artukh et al. / Nuclear Physics A 701 (2002) 96c–99c 99c

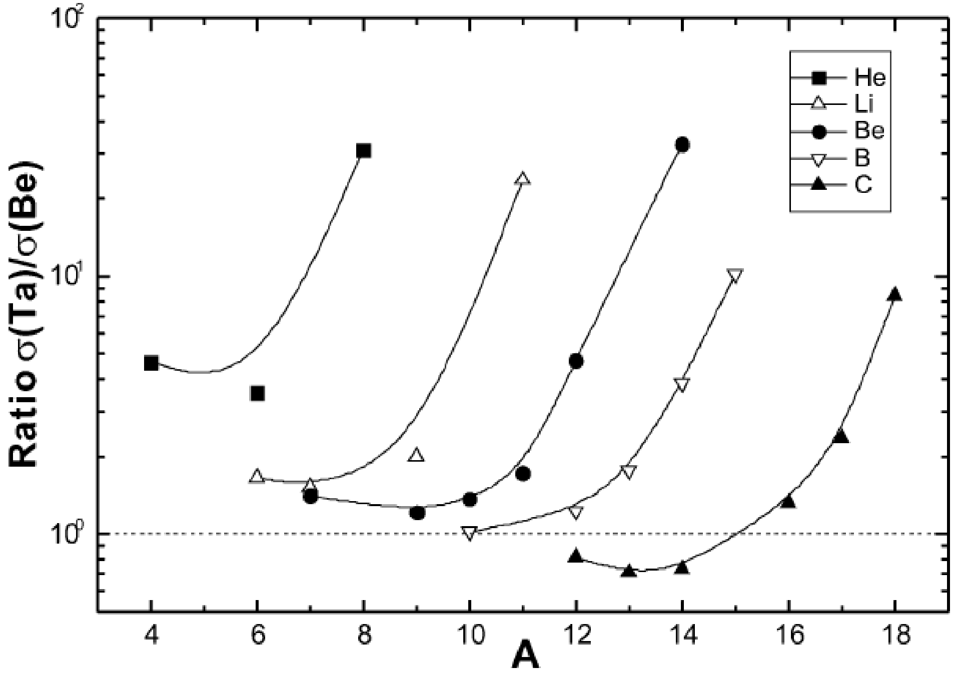

Fig. 3. A comparison of cross sections in production of neutron-rich isotopes for elements He, Li, Be, B and C induced in the reactions of ^{18}O (35 *A* MeV) with a heavy target ^{181}Ta and a light target ^{9}Be.

Target Ratio σ(Kr+Ta) / σ(Kr+Be), 64 *A* MeV

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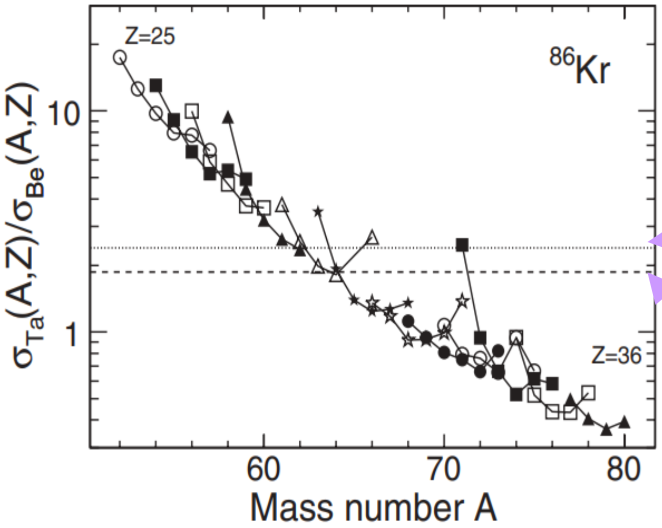


$$\frac{\sigma_{\rm Ta}(A,Z)}{\sigma_{\rm Be}(A,Z)} = \frac{\left(A_{\rm Kr}^{1/3} + A_{\rm Ta}^{1/3}\right)^2}{\left(A_{\rm Kr}^{1/3} + A_{\rm Be}^{1/3}\right)^2} = 2.4,$$

FIG. 10. Ratios of the fragmentation cross sections on Ta and Be targets, $\sigma_{\rm Ta}(A,Z)/\sigma_{\rm Be}(A,Z)$, for fragments with $25 \leqslant Z \leqslant 36$ for the ^{86}Kr beam. Only ratios with relative errors smaller than 25% are shown. Open and solid symbols represent odd and even elements starting with $Z = 25$. The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Eq. (4), respectively.

$$\frac{\sigma_{\rm Ta}(A,Z)}{\sigma_{\rm Be}(A,Z)} = \frac{\left(A_{\rm Kr}^{1/3} + A_{\rm Ta}^{1/3} - 2.38\right)}{\left(A_{\rm Kr}^{1/3} + A_{\rm Be}^{1/3} - 2.38\right)} = 1.9.$$

RARE ISOTOPE PRODUCTION
By

Michal Mocko

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics and Astronomy

2006

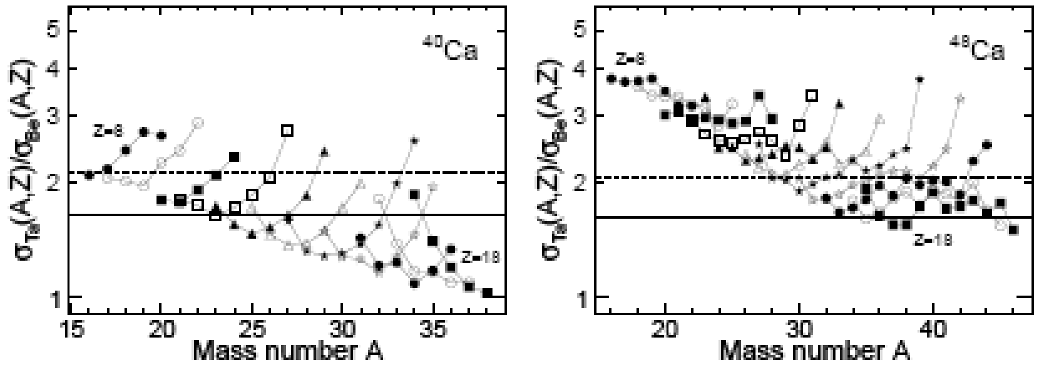


Figure 4.23: Target ratios of the fragmentation cross sections $\sigma_{Ta}(A,Z)/\sigma_{Be}(A,Z)$, of fragments $8 \leq Z \leq 18$ for two projectiles ^{40}Ca (left panel) and ^{48}Ca (right panel). The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Equation (4.22), respectively.

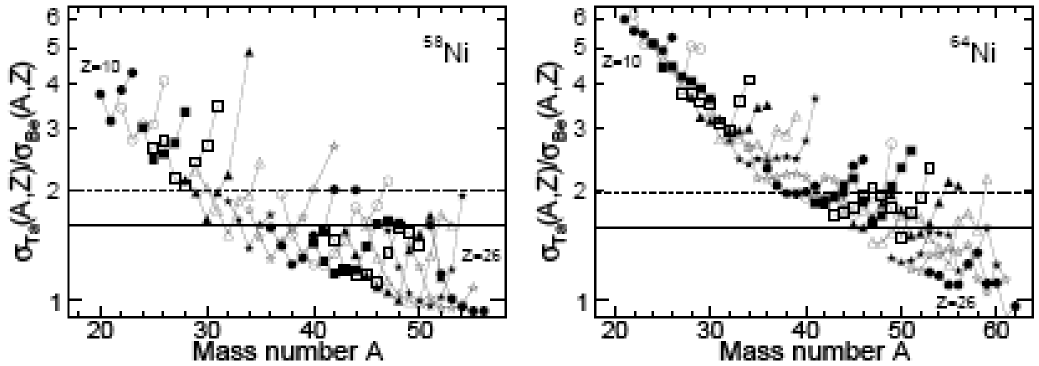
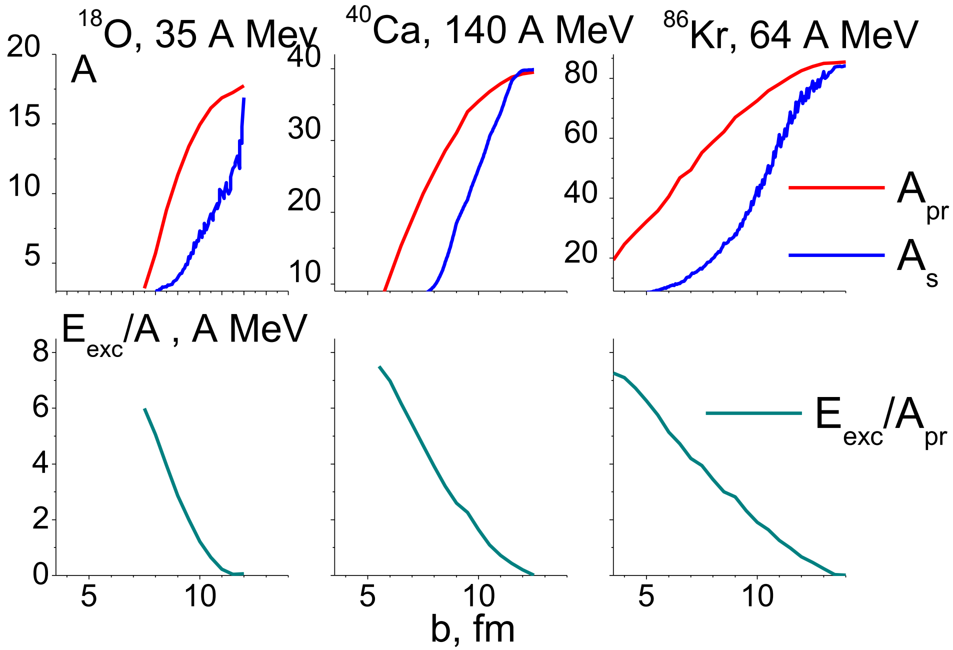


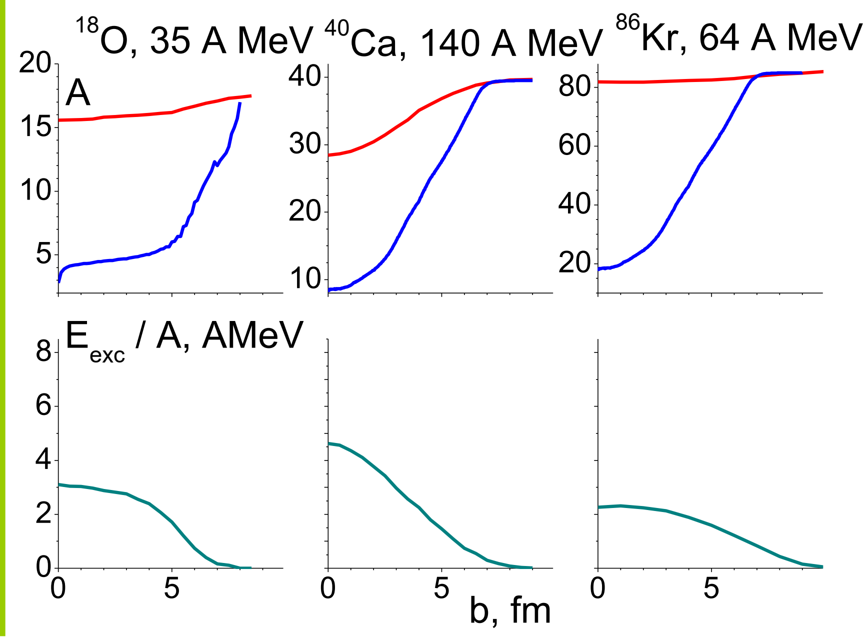
Figure 4.24: Target ratios of the fragmentation cross sections $\sigma_{Ta}(A,Z)/\sigma_{Be}(A,Z)$ of fragments $10 \leq Z \leq 26$ for two projectiles ^{58}Ni (left panel) and ^{64}Ni (right panel). The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Equation (4.22), respectively.

Impact parameter dependence of the mass *A* of primary and secondary fragments and the excitation energy E_{exc} of the primary fragments

^{181}Ta target



^{9}Be target



In the process of interaction of two different ions, the lighter one is more affected by the collision with the larger particle loss than in the heavier one.

Calculation of the target ratio

$R_J(A_s) = \sigma_J(A_s)_{Ta} / \sigma_J(A_s)_{Be}$,

for projectiles ^{40}Ca and ^{18}O

^{40}Ca,
140 AMev

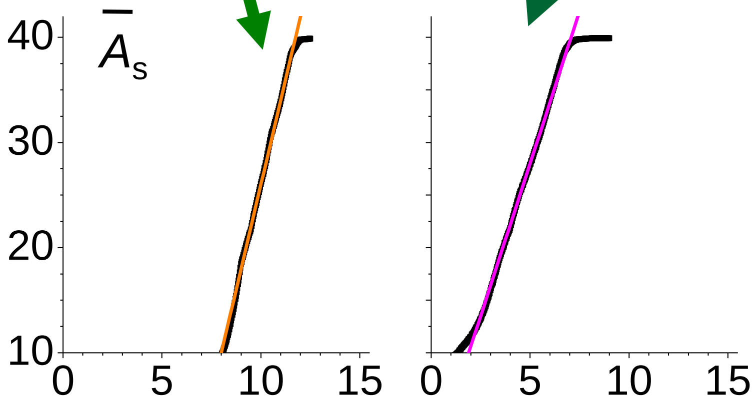
^{181}Ta

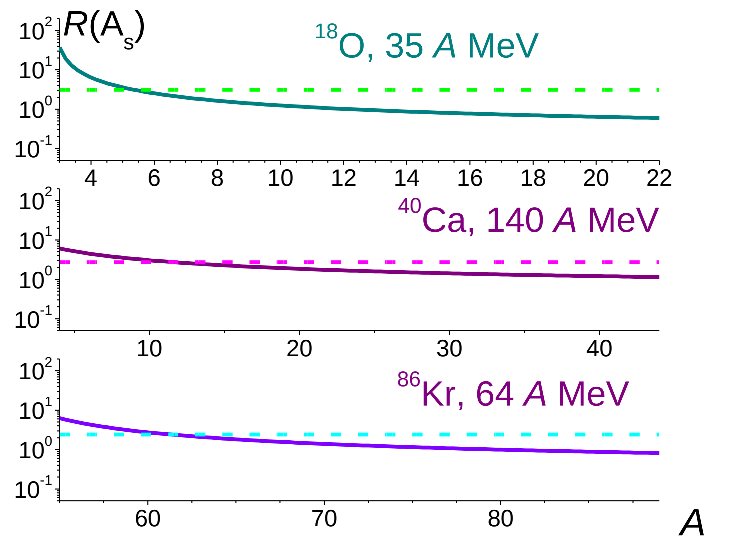
^{9}Be

$A_s = 5.8 * b - 1$

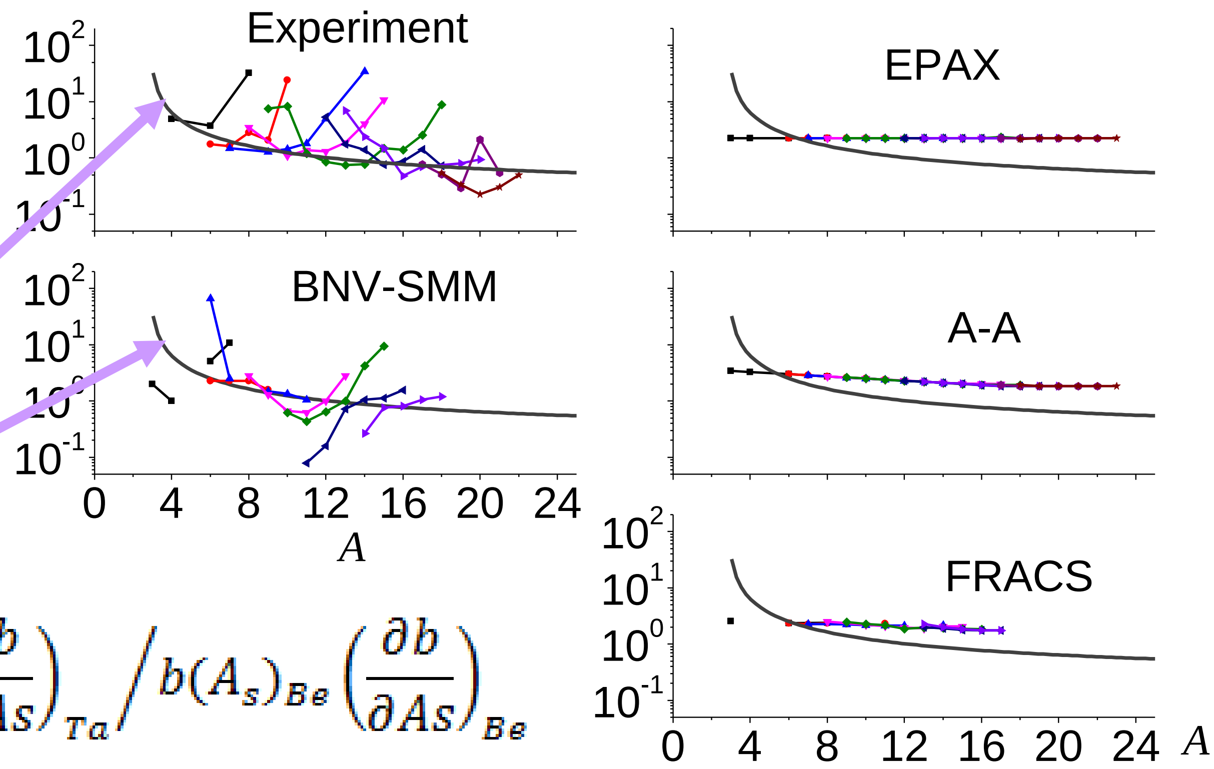
$A_s = 8 * b - 54$

$$R(A_s) = b(A_s)_{Ta} \left(\frac{\partial b}{\partial As}\right)_{Ta} \Big/ b(A_s)_{Be} \left(\frac{\partial b}{\partial As}\right)_{Be}$$





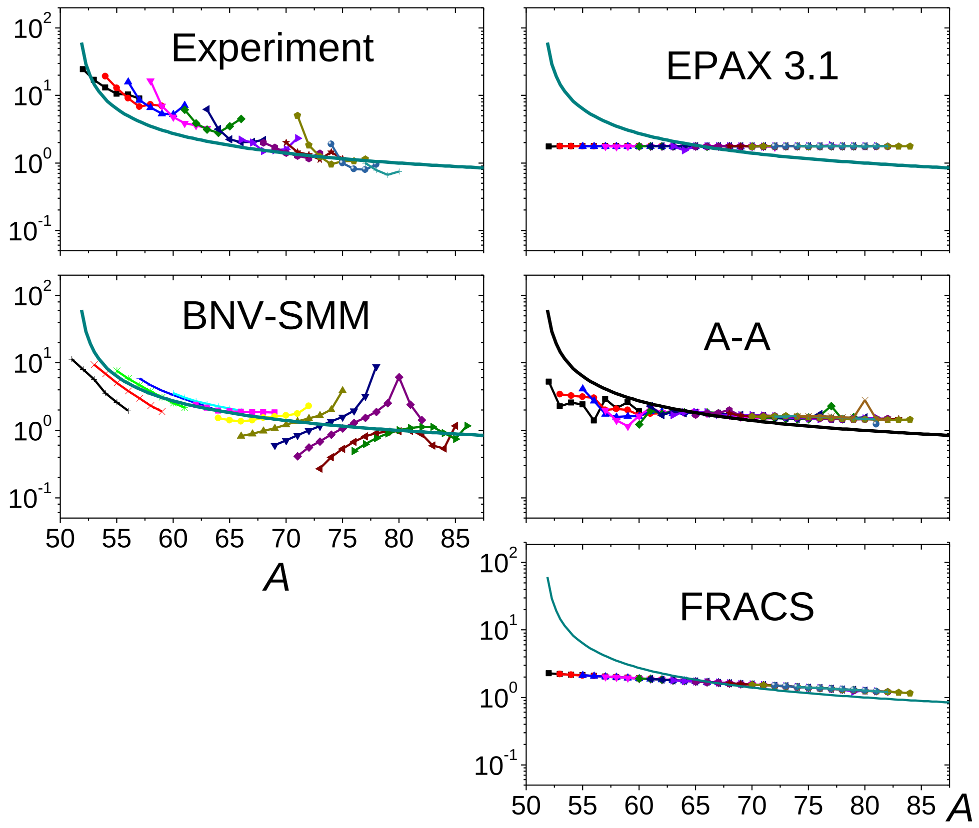
Target Ratio $\sigma(^{18}O+^{181}Ta)/\sigma(^{18}O+^{9}Be)$, 35 *A* MeV, Experiment and model calculations



$$R(A_s) = b(A_s)_{Ta}\left(\frac{\partial b}{\partial As}\right)_{Ta} \Big/ b(A_s)_{Be}\left(\frac{\partial b}{\partial As}\right)_{Be}$$

Experiment: Combas set-up, FLNR, JINR
Artukh, A.G, et al. Nucl. Phys.A701, p96. (2001).

Target ratio $\sigma(^{86}Kr+^{181}Ta) / \sigma(^{86}Kr+^{9}Be)$, 64 A MeV, Experiment and model calculations



Experiment:
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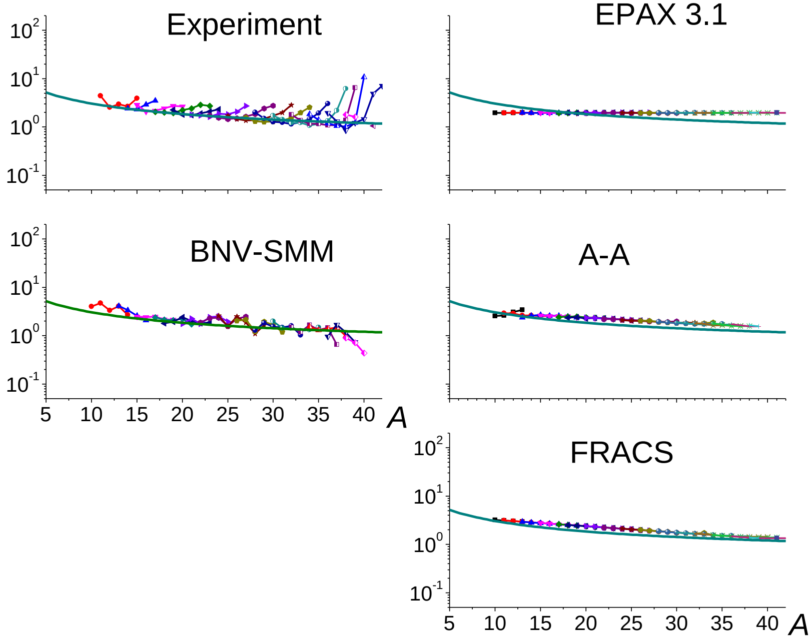
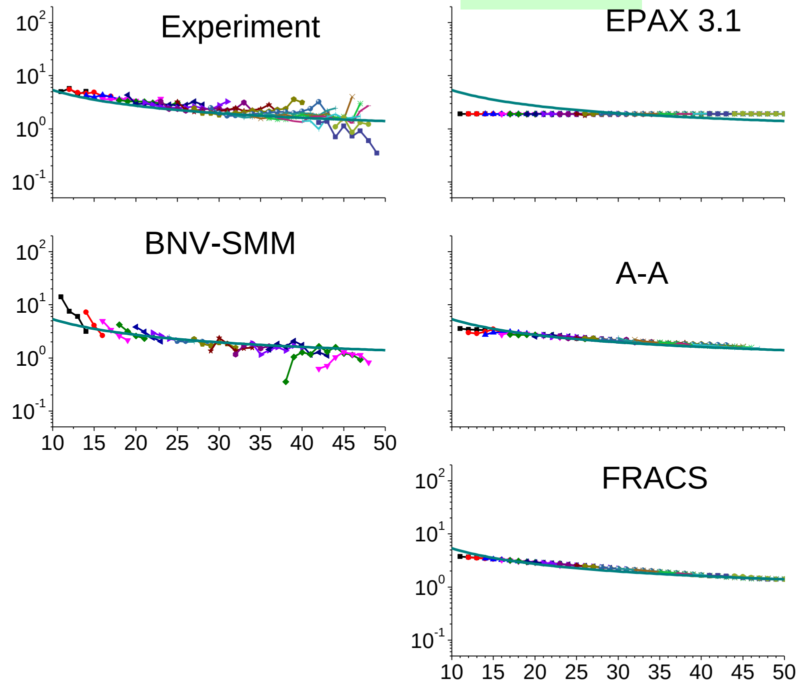
Target ratio σ($^{40/48}$Ca+^{181}Ta) /σ($^{40/48}$Ca+^{9}Be), 140 *A*MeV, Experiment and model calculations

^{40}Ca



^{48}Ca



Experiment: Rare isotope production, Dissertation , M. Mocko, 2006

Conclusions

We found that our calculations describe fragments close to N-Z = 0 rather well, but for neutron rich isotopes our calculations give smaller values than the experiment

The target ratios point out on the importance of taking into account two important characteristics of the reaction:

1) **target mass (light or heavy)**
2) **impact parameter value**

The increase of the yields of the neutron-rich isotopes in the reactions on heavy targets in comparison with the light ones may be connected with the presence of the pick-up reactions


Thank you
for
attention