

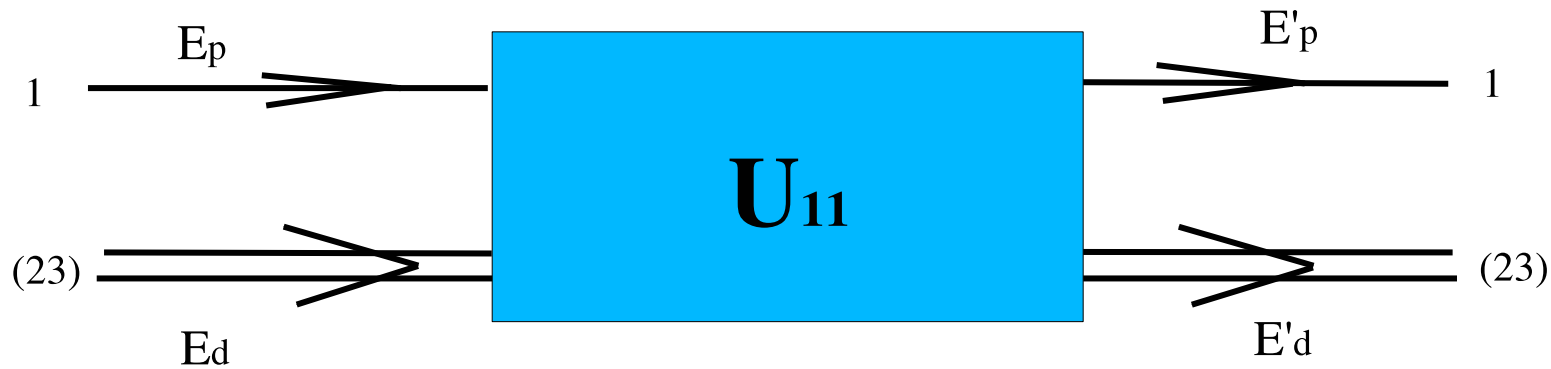


Nadezhda Ladygina
JINR, LHEP

Description of polarization observables in dp elastic scattering at a GeV energies

- The theoretical model is suggested for description of $dp \rightarrow dp$ reaction in the full angular range
- The model allows calculating both differential cross section and polarization observables in the deuteron energy range between 500 and 2000 MeV.
- The results are presented in comparison with the data.

$$dp \rightarrow dp$$



The matrix element of the transition operator U_{11} defines reaction amplitude

$$U_{dp \rightarrow dp} = \delta(E_d + E_p - E'_d - E'_p) \mathcal{J} = \langle 1(23) | [1 - P_{12} - P_{13}] U_{11} | 1(23) \rangle$$

Alt-Grassberger-Sandhas equations for rearrangement operators: Nucl.Phys. B2, 167 (1967)

E.Schmid, H.Ziegelmann The Quantum Mechanical Three-Body Problem

$$\begin{aligned}U_{11} &= t_{13}g_0U_{21} + t_{12}g_0U_{31} \\U_{21} &= g_0^{-1} + t_{23}g_0U_{11} + t_{12}g_0U_{31} \\U_{31} &= g_0^{-1} + t_{23}g_0U_{11} + t_{13}g_0U_{21}\end{aligned}$$

$t_1 = t(2, 3)$, etc., is the t -matrix of the two-particle interaction

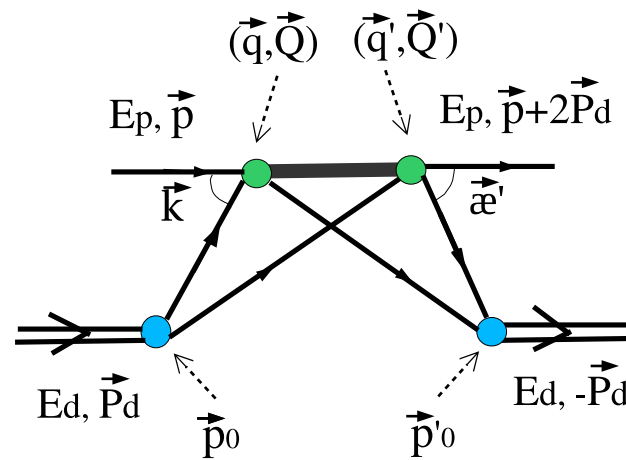
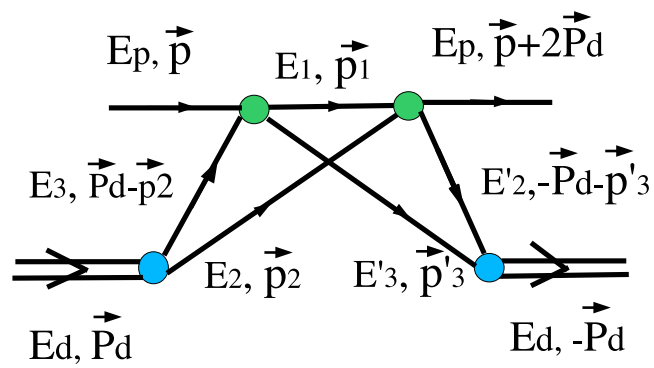
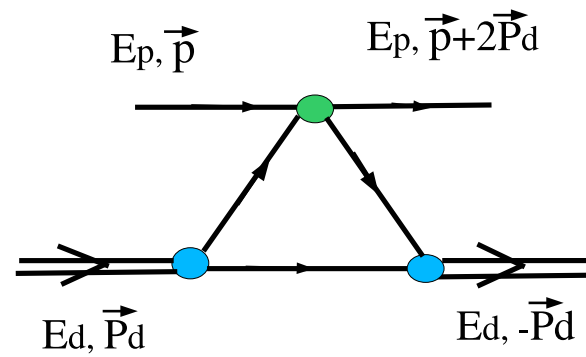
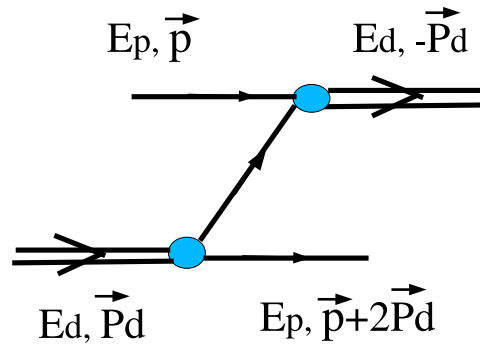
g_0 is the free three-particle propagator

Iterating AGS-equations up to second order terms over t one obtains

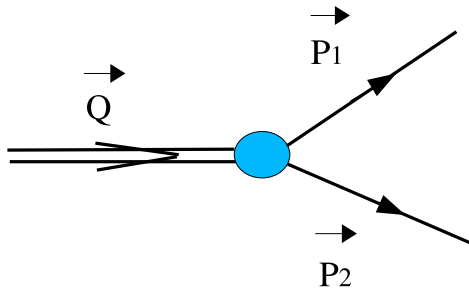
$$U_{11} = -2P_{12}g_0^{-1} + 2t_{12}^{sym} + 2t_{12}^{sym}g_0t_{13}^{sym}$$

$t_{ij}^{sym} = [1 - P_{ij}]t_{ij}$ - antisymmetrized t -matrix

Diagrams



Lorenz transformation



$$L(\vec{u})p_1 = (E^*, \vec{p})$$
$$L(\vec{u})p_2 = (E^*, -\vec{p})$$

with velocity

$$\vec{u} = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}$$

The c.m. energy of one of the nucleons E^* is related with Mandelstam variable s by

$$E^* = \sqrt{s}/2 \quad .$$

Let's introduce new variables $\vec{Q}$ and $\vec{k}$ which can be expressed through $\vec{p}_1$ and $\vec{p}_2$

$$\vec{Q} = \vec{p}_1 + \vec{p}_2$$
$$\vec{k} = \frac{(E_2 + E^*)\vec{p}_1 - (E_1 + E^*)\vec{p}_2}{E_1 + E_2 + 2E^*} \quad .$$

Deuteron wave function

The deuteron wave function in the rest has the standard form

$$\begin{aligned} & \langle m_p m_n, \vec{p} | \Omega_d | \vec{0}, \mathcal{M}_d \rangle = \\ & \frac{1}{\sqrt{4\pi}} \langle m_p m_n, \vec{p} | \left\{ u(p) + \frac{w(p)}{\sqrt{8}} [3(\vec{\sigma}_1 \hat{p})(\vec{\sigma}_2 \hat{p}) - (\vec{\sigma}_1 \vec{\sigma}_2)] \right\} | \vec{0}, \mathcal{M}_d \rangle \end{aligned}$$

$u(p)$ and $w(p)$ - S - and D - components of the deuteron.

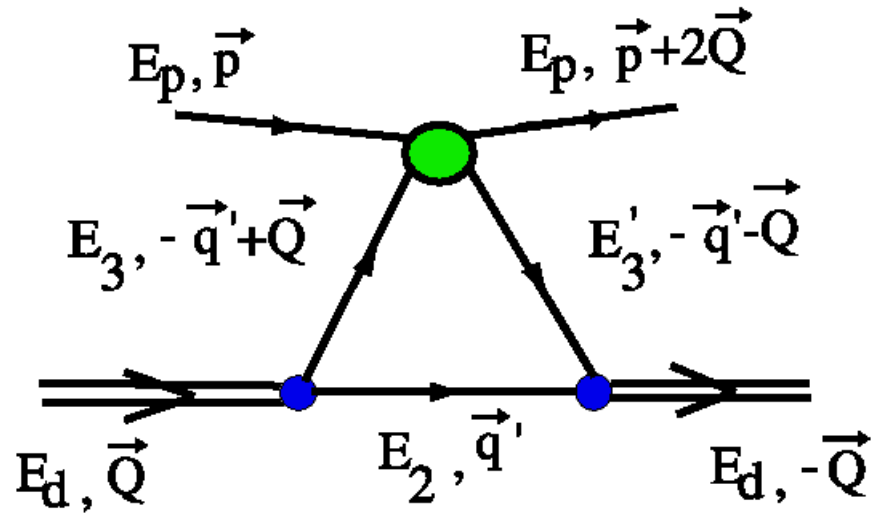
Then the deuteron wave function in the moving frame is

$$\langle \vec{p}_1 \vec{p}_2, m_1 m_2 | \Omega_d | \vec{Q}, \mathcal{M}_d \rangle \sim \langle \vec{p}, m'_1 m'_2 | W_{1/2}^\dagger(\vec{p}_1, \vec{u}) W_{1/2}^\dagger(\vec{p}_2, \vec{u}) \Omega_d | \vec{0}, \mathcal{M}_d \rangle$$

where $W_{1/2}$ is Wigner rotation operator

$$\begin{aligned} W_{1/2}(\vec{p}_i, \vec{u}) &= \exp \{ -i\omega_i (\vec{n}_i \vec{\sigma}_i) / 2 \} = \cos(\omega_i / 2) [1 - i(\vec{n}_i \vec{\sigma}_i) \operatorname{tg}(\omega_i / 2)] \\ \vec{n}_i &= \frac{\vec{p}_i \times \vec{u}}{|\vec{p}_i \times \vec{u}|} \end{aligned}$$

Single Scattering contribution



$$\mathcal{I}_{SS} = {}_{1(23)}\langle \vec{p}' m'; -\vec{Q} \mathcal{M}'_d | \Omega_d^\dagger(23) | [1 - P_{12}] | t_{NN} \Omega_d(23) | \vec{Q} \mathcal{M}_d; \vec{p} m \rangle_{1(23)}$$

Nucleon-Nucleon t -matrix

W.G.Love, M.A.Franey, Phys.Rev.C24, 1073 (1981)

N.B.Ladygina,nucl-th/0805.3021

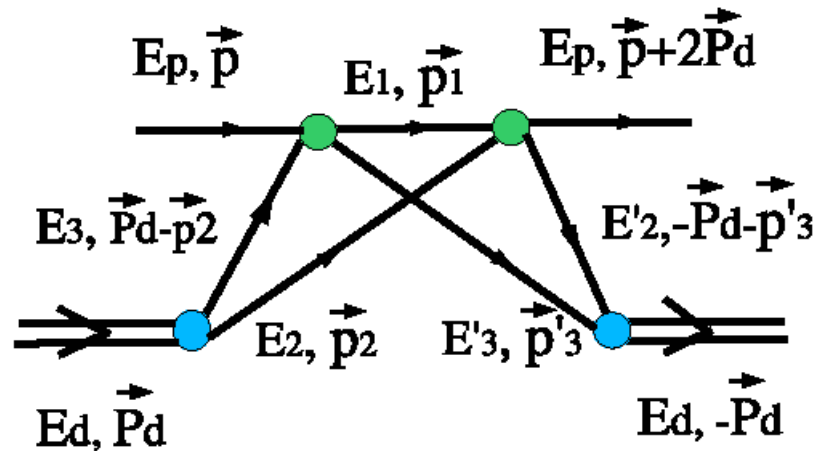
$$\begin{aligned} \langle \kappa' m'_1 m'_2 | t | \kappa m_1 m_2 \rangle &= \langle \vec{\kappa}' m'_1 m'_2 | A + B(\vec{\sigma}_1 \hat{N}^*)(\vec{\sigma}_2 \hat{N}^*) + C(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{N}^* + \\ &\quad D(\vec{\sigma}_1 \hat{q}^*)(\vec{\sigma}_2 \hat{q}^*) + F(\vec{\sigma}_1 \hat{Q}^*)(\vec{\sigma}_2 \hat{Q}^*) | \vec{\kappa} m_1 m_2 \rangle \end{aligned}$$

where the orthonormal basis is combinations of the nucleons relative momenta in the initial $\vec{\kappa}$ and final $\vec{\kappa}'$ states

$$\hat{q}^* = \frac{\vec{\kappa} - \vec{\kappa}'}{|\vec{\kappa} - \vec{\kappa}'|}, \quad \hat{Q}^* = \frac{\vec{\kappa} + \vec{\kappa}'}{|\vec{\kappa} + \vec{\kappa}'|}, \quad \hat{N}^* = \frac{\vec{\kappa} \times \vec{\kappa}'}{|\vec{\kappa} \times \vec{\kappa}'|}$$

The amplitudes A, B, C, D, F are the **functions of the center-of-mass energy and scattering angle**. The radial parts of these amplitudes are taken as a sum of Yukawa terms.

Double scattering contribution

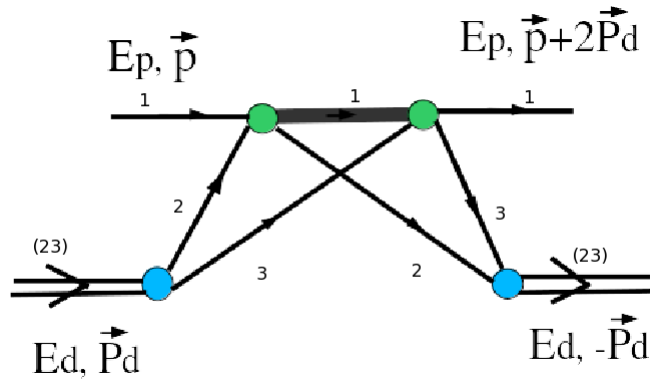


$$\mathcal{J}_{DS} = \begin{aligned} &< 1(23) | [1 - P_{12}] | t_{NN} | N(1)N(2) > | N(3) > g_0 \\ &< N(2) | < N(1)N(3) | t_{NN} [1 - P_{13}] | (23)1 > \end{aligned}$$

g_0 is a free three-particle propagator:

$$\begin{aligned} g_0 &= \frac{1}{E_d + E_p - E_1 - E_2 - E'_3 + i\varepsilon} = \\ &= \mathcal{P} \frac{1}{E_d + E_p - E_1 - E_2 - E'_3} - i\pi\delta(E_d + E_p - E_1 - E_2 - E'_3) \end{aligned}$$

Δ -contribution



$$\mu^2 = E_{\Delta}^2 - \vec{p}_{\Delta}^2$$

Δ -contribution is defined by two $N\Delta$ matrices

$$\mathcal{J}_{\Delta} = \begin{aligned} &< 1(23) | [1 - P_{12}] | t_{N\Delta} | \Delta(1) N(2) > | N(3) > g_0 \\ &< N(2) | < \Delta(1) N(3) | t_{\Delta N} [1 - P_{13}] | (23) 1 > \end{aligned}$$

g_0 —a free three-particle propagator:

$$g_0 = \frac{1}{E_d + E_p - E'_2 - E_3 - E(m_{\Delta}) + i\Gamma(E_{\Delta})/2}$$

the distribution function of Δ -energy:

$$\rho(\mu) = \frac{1}{2\pi} \frac{\Gamma(\mu)}{(E_{\Delta}(\mu) - E_{\Delta}(m_{\Delta}))^2 + \Gamma^2(\mu)/4},$$

and wave functions of the initial and final deuterons.

Δ -isobar definition

The potential for the $NN \rightarrow N\Delta$ transition is based on the π - and ρ - exchanges:

$$t_{N\Delta}^{(\pi)} = -\frac{f_\pi f_\pi^*}{m_\pi^2} F_\pi^2(t) \frac{q^2}{m_\pi^2 - t} (\vec{\sigma} \cdot \hat{q})(\vec{S} \cdot \hat{q})(\vec{\tau} \cdot \vec{T})$$
$$t_{N\Delta}^{(\rho)} = -\frac{f_\rho f_\rho^*}{m_\rho^2} F_\rho^2(t) \frac{q^2}{m_\rho^2 - t} \{(\vec{\sigma} \vec{S}) - (\vec{\sigma} \cdot \hat{q})(\vec{S} \cdot \hat{q})\}(\vec{\tau} \cdot \vec{T})$$

with coupling constants:

$$\begin{aligned} f_\pi &= 1.008 & f_\pi^* &= 2.156 \\ f_\rho &= 7.8 & f_\rho^* &= 1.85 f_\rho \end{aligned}$$

The hadronic form factor has a pole form:

$$F_x(t) = (\Lambda_x^2 - m_x^2)/(\Lambda_x^2 - t)^n, \quad \begin{aligned} n &= 1 && \text{for } \pi - \text{meson} \\ n &= 2 && \text{for } \rho - \text{meson} \end{aligned}$$

The reaction amplitude is defined through 12 terms:

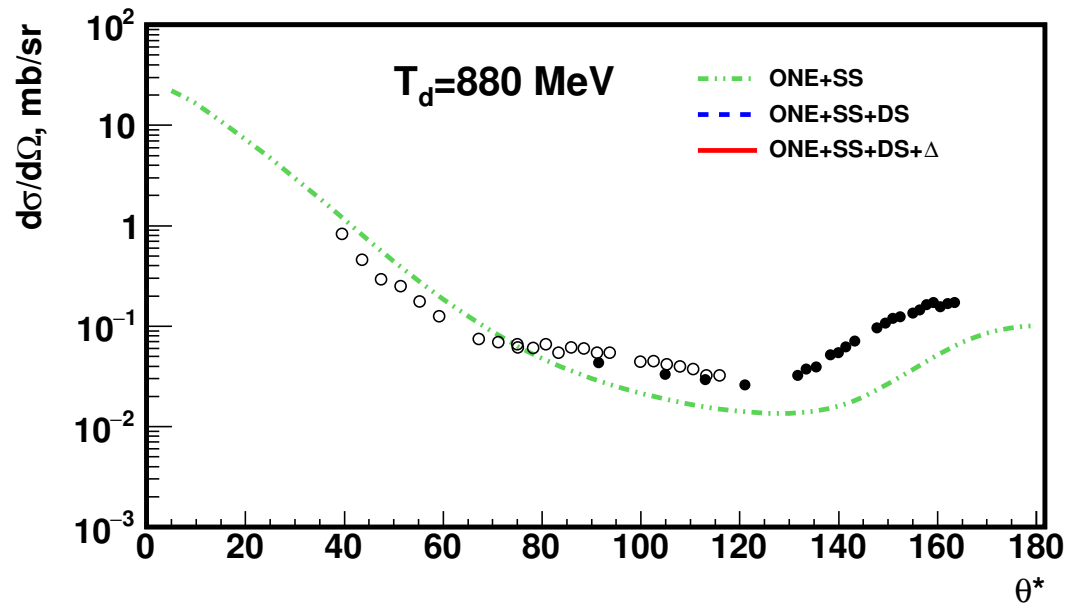
$$\begin{aligned} \mathcal{J}_{dp \rightarrow dp} = & \langle 1M'_d | < \frac{1}{2} m' | F_1 + F_2(\vec{S}\vec{y}) + F_3 Q_{xx} + F_4 Q_{yy} + \\ & F_5(\vec{\sigma}\vec{x})(\vec{S}\vec{x}) + F_6(\vec{\sigma}\vec{x}) Q_{xx} + F_7(\vec{\sigma}\vec{y}) + F_8(\vec{\sigma}\vec{y})(\vec{S}\vec{y}) + F_9(\vec{\sigma}\vec{y}) Q_{xx} + \\ & F_{10}(\vec{\sigma}\vec{y}) Q_{yy} + F_{11}(\vec{\sigma}\vec{z})(\vec{S}\vec{z}) + F_{12}(\vec{\sigma}\vec{z}) Q_{yz} | \frac{1}{2} m > | 1M_d > \end{aligned}$$

σ_i, S_i are the spin operators for $s = 1/2$ (Pauli matrices) and $S = 1$
 Q_{ij} is the quadrupole tenzor:

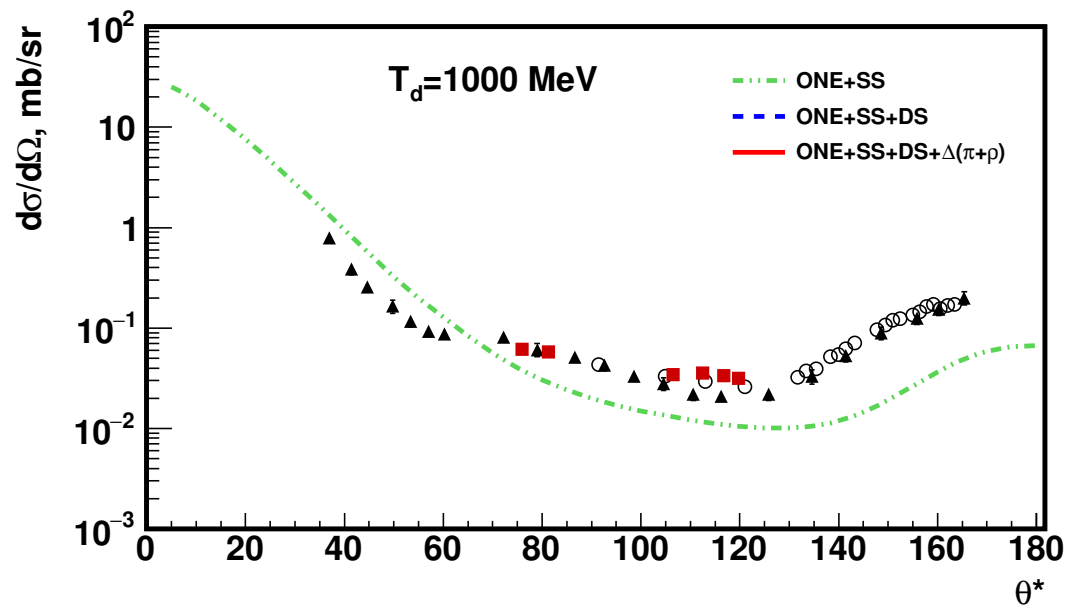
$$Q_{ij} = \frac{1}{2}(S_i S_j + S_j S_i) - \frac{2}{3}\delta_{ij}$$

$$\boxed{C_{i,j,k,l} = \frac{\text{Tr}(\mathcal{J} \sigma_i S_j \mathcal{J}^\dagger \sigma_k S_l)}{\text{Tr}(\mathcal{J} \mathcal{J}^\dagger)}} \quad (1)$$

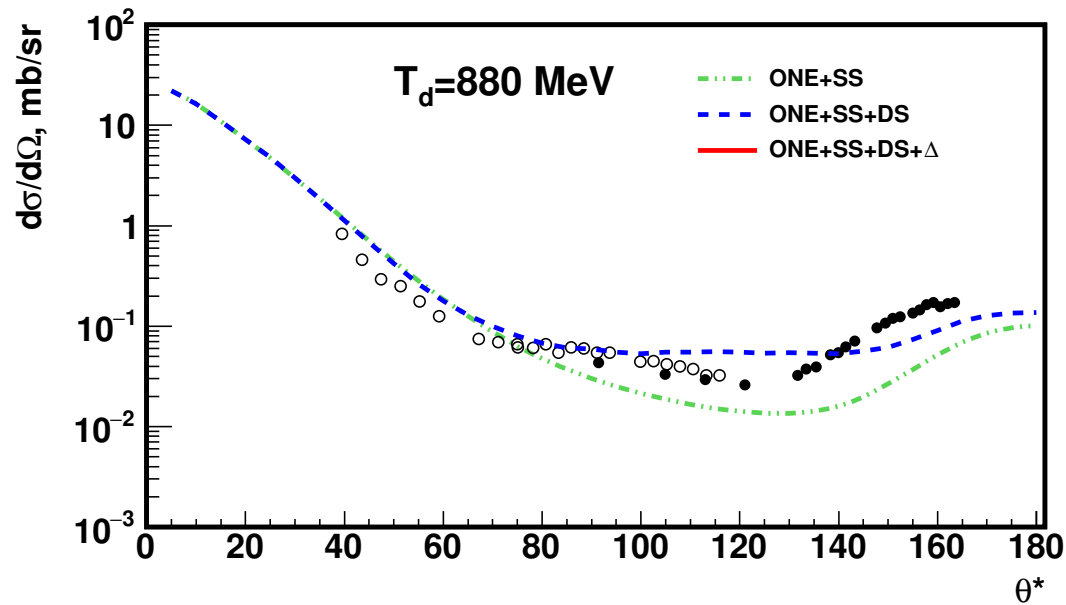
$$\sigma \sim \text{Tr}(\mathcal{J} \mathcal{J}^\dagger), \quad A_y = \frac{\text{Tr}(\mathcal{J} S_y \mathcal{J}^\dagger)}{\text{Tr}(\mathcal{J} \mathcal{J}^\dagger)}, \quad A_{yy} = \frac{\text{Tr}(\mathcal{J} Q_{yy} \mathcal{J}^\dagger)}{\text{Tr}(\mathcal{J} \mathcal{J}^\dagger)}$$



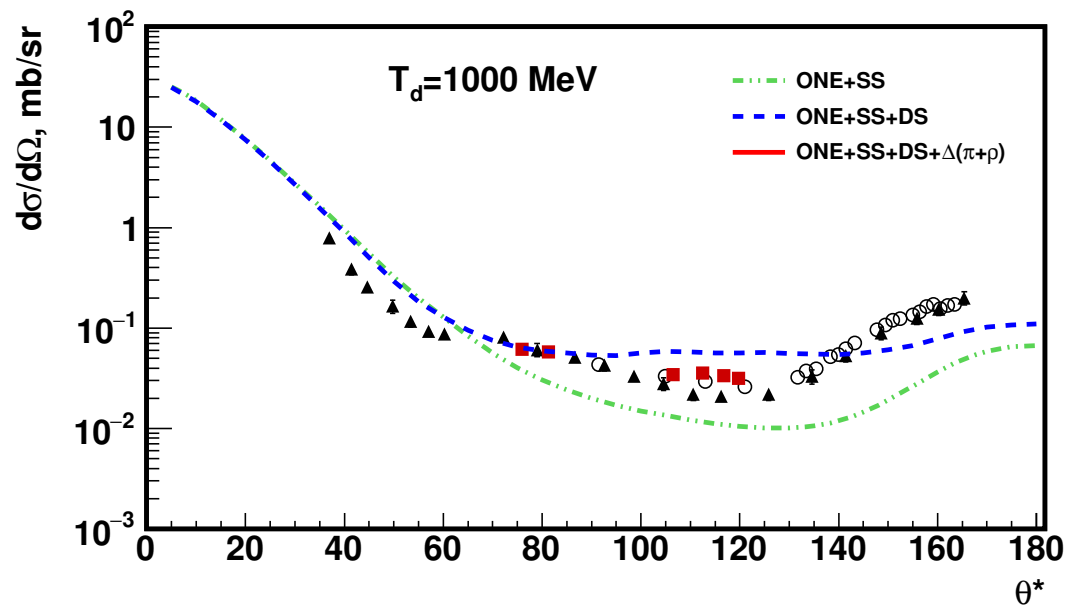
● - N.E.Booth et al.,
 Phys.Rev.D4, p.1261
 (1971), $T_d = 850 \text{ MeV}$
 ○ - J.C.Alder et al.,
 Phys.Rev.C6, p.2010
 (1972), $T_d = 940 \text{ MeV}$



▲ - J.S. Vincent et al.,
 Phys. Rev. Lett.
 24, 236 (1970), $T_d = 1160 \text{ MeV}$
 ■ - A.A. Terekhin, et
 al., Eur. Phys. J. A55,
 129(2019)

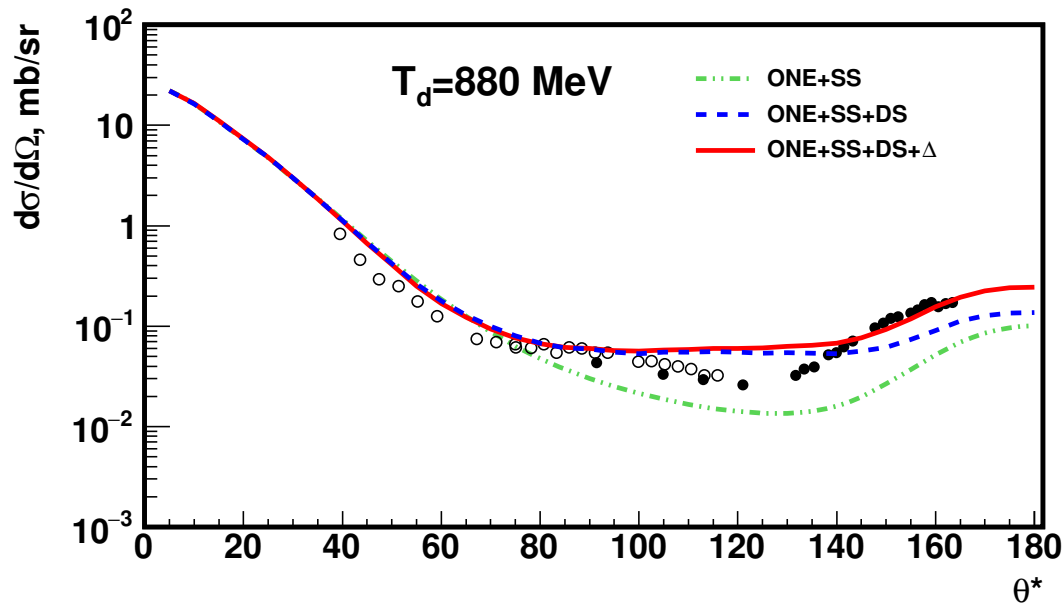


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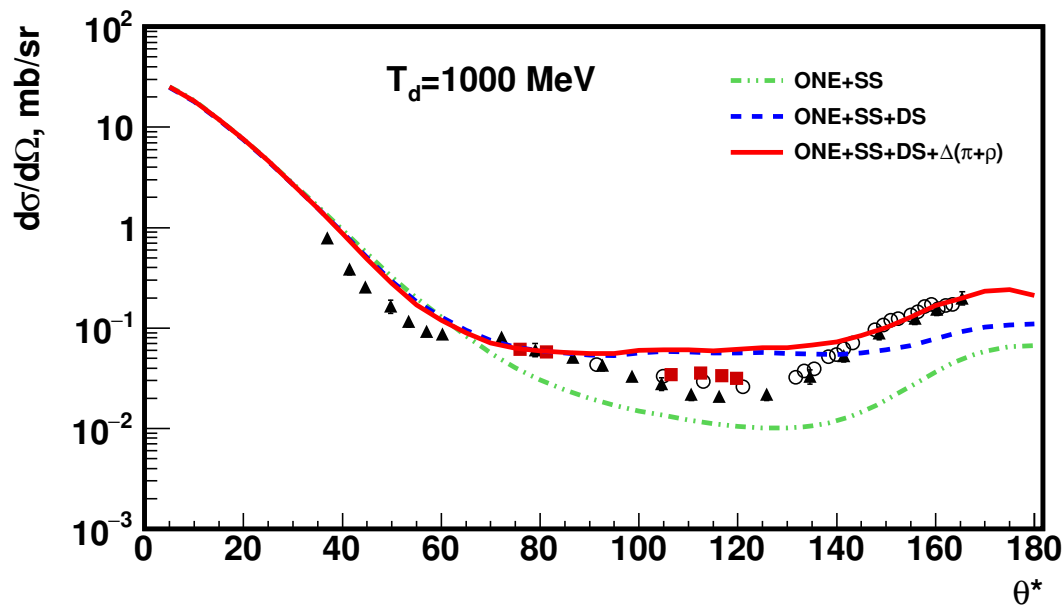


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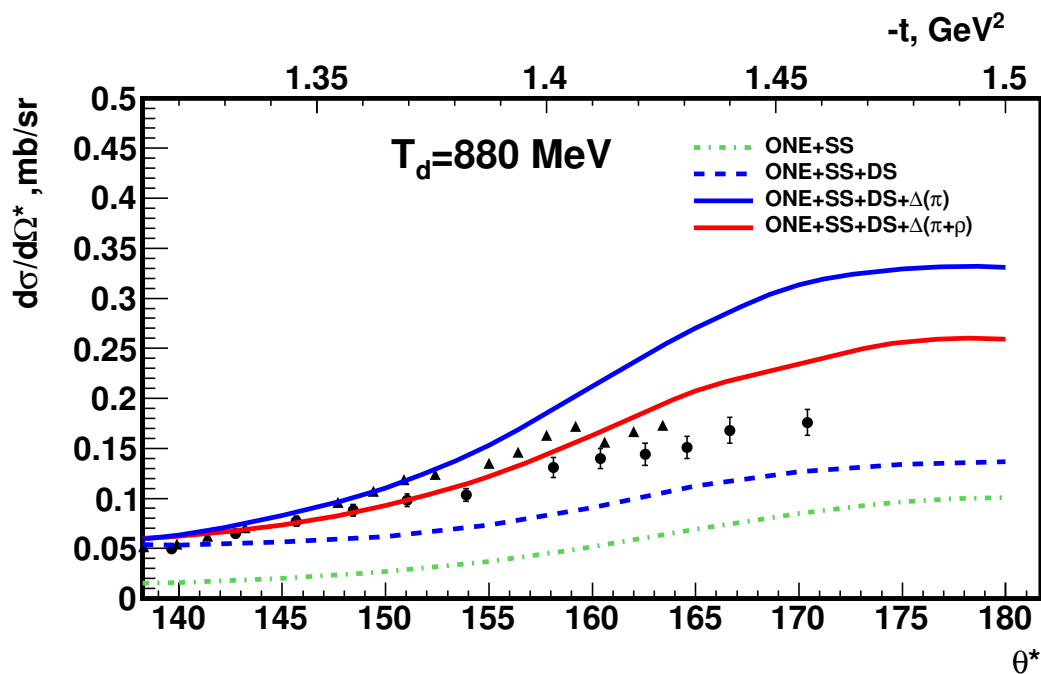


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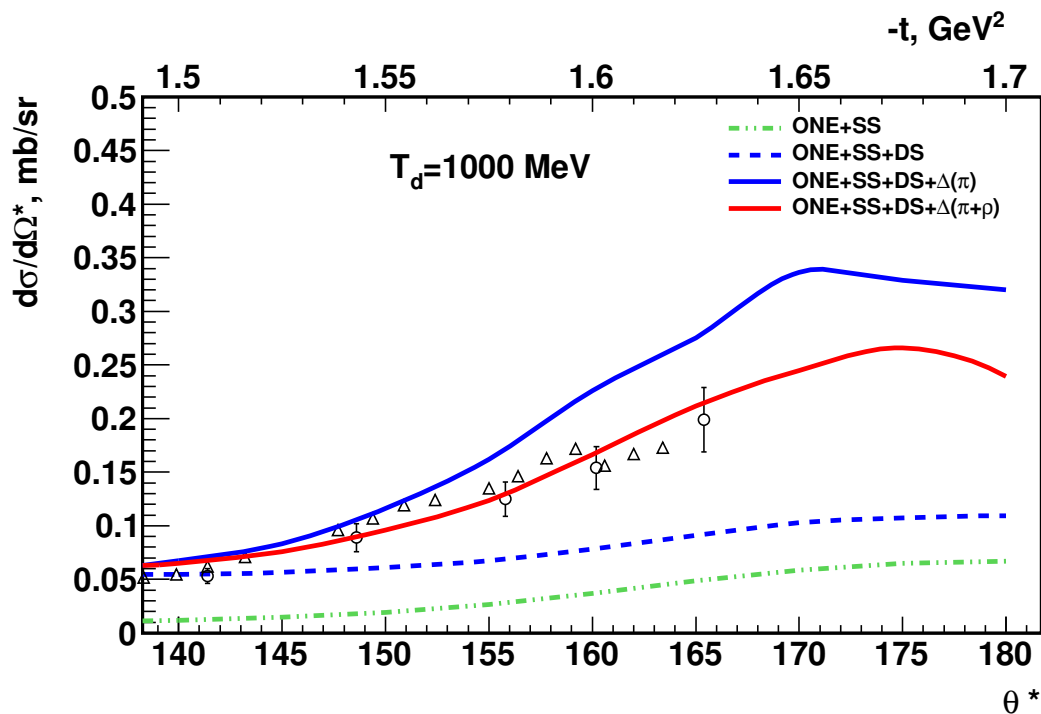


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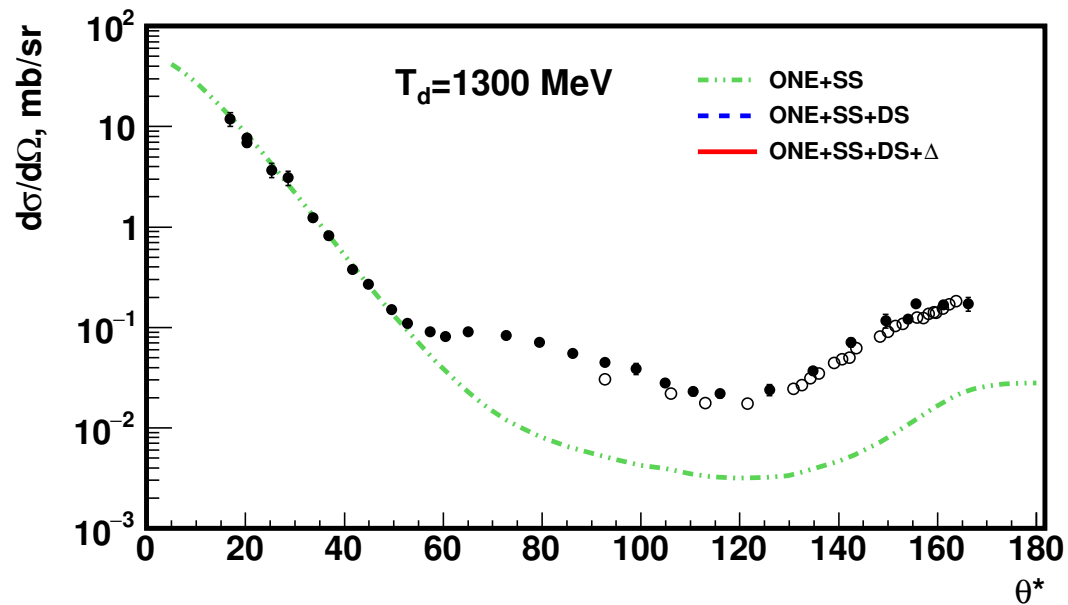
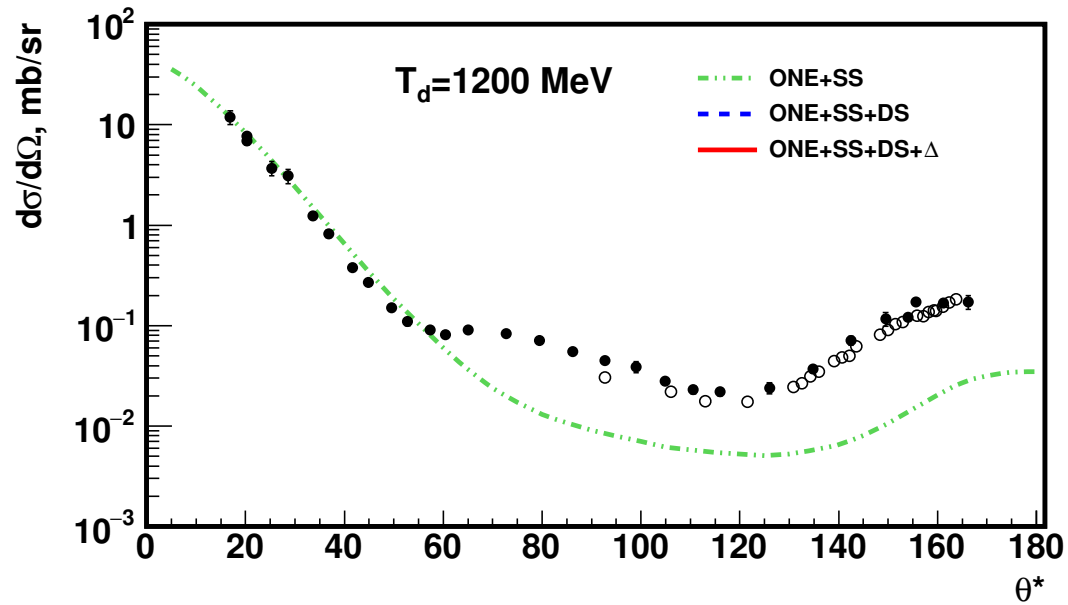
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- $\bullet$ - N.E. Booth et al., Phys.Rev.D4, p.1261 (1971), $T_d = 850\text{MeV}$
- $\blacktriangle$ - J.C. Alder et al., Phys.Rev.C6, p.2010 (1972), $T_d = 940\text{MeV}$

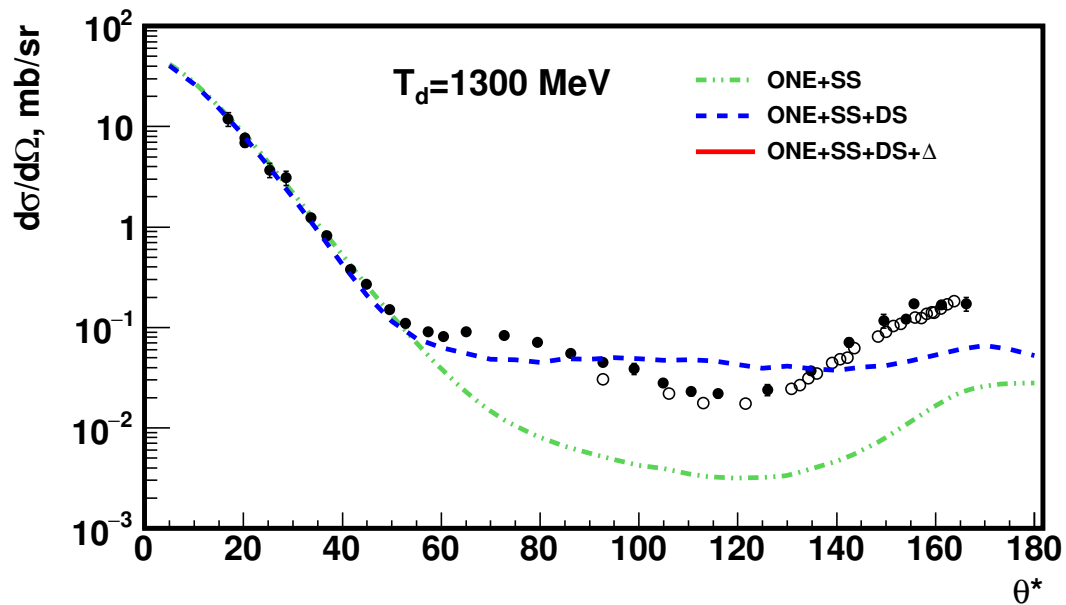
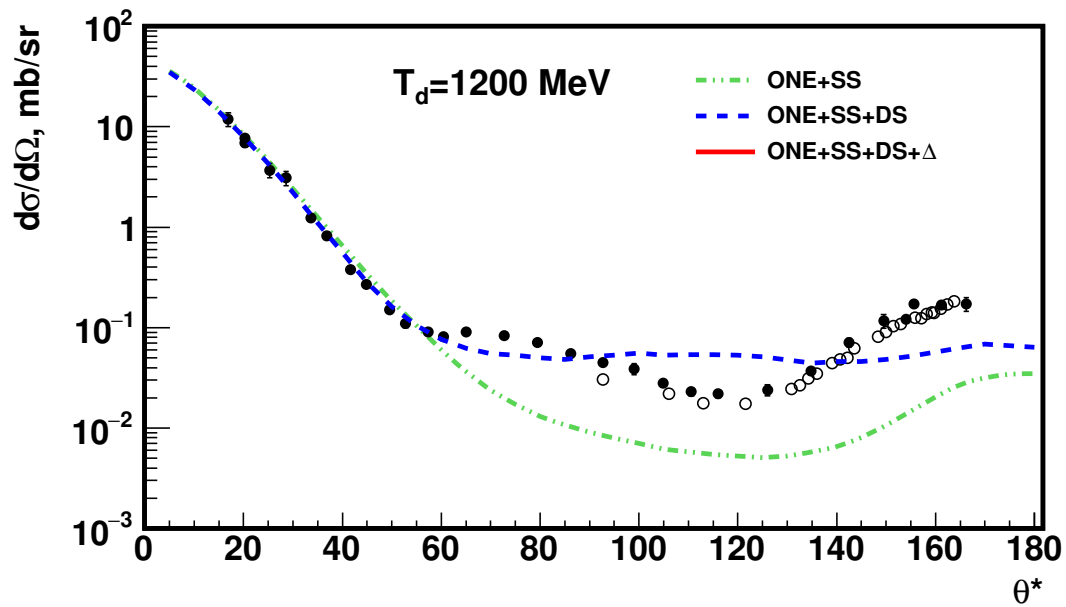


- $\circ$ - J.S. Vincent, Phys.Rev.Lett. 24, p.236 (1970), $T_d = 1169\text{MeV}$
- $\triangle$ - J.C. Alder et al., Phys.Rev.C6, p.2010 (1972), $T_d = 940\text{MeV}$



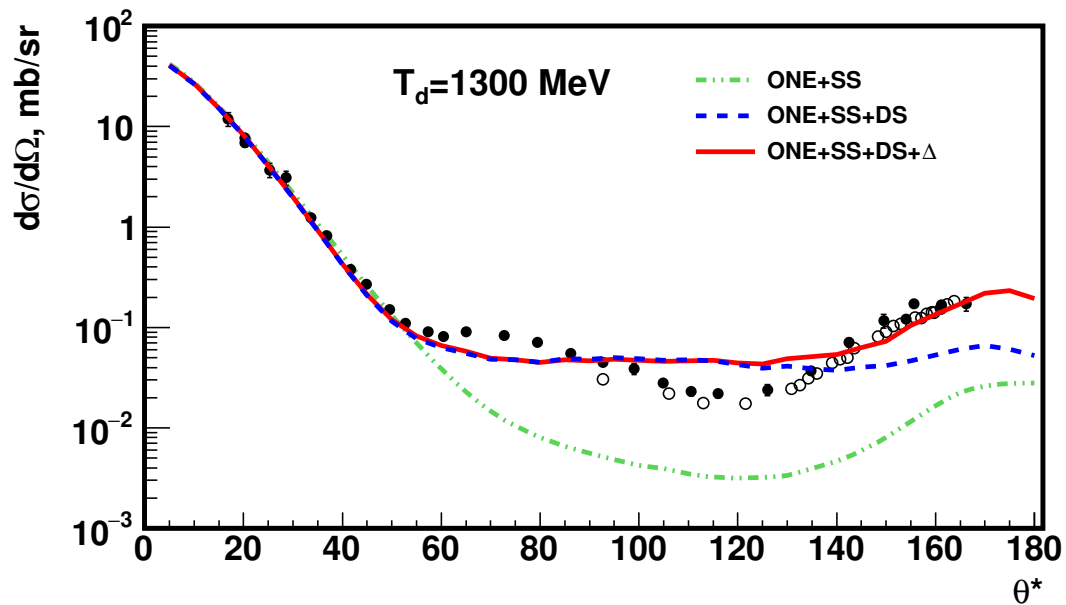
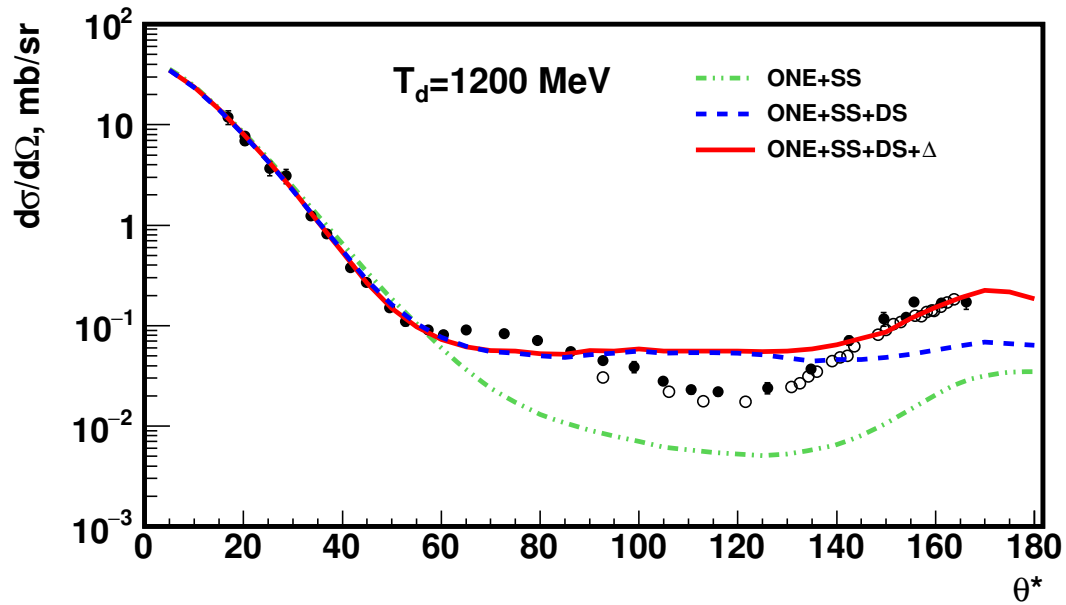
● - E.T.Boschitz et al., Phys.Rev.C6, p.457 (1972)

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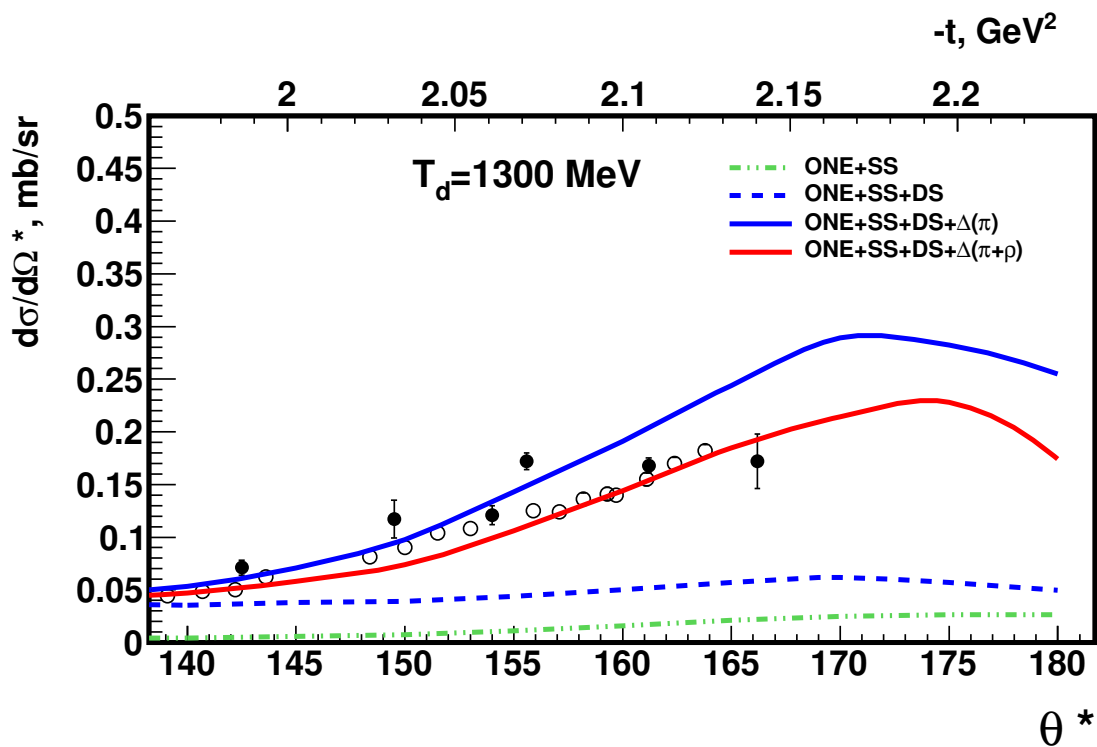
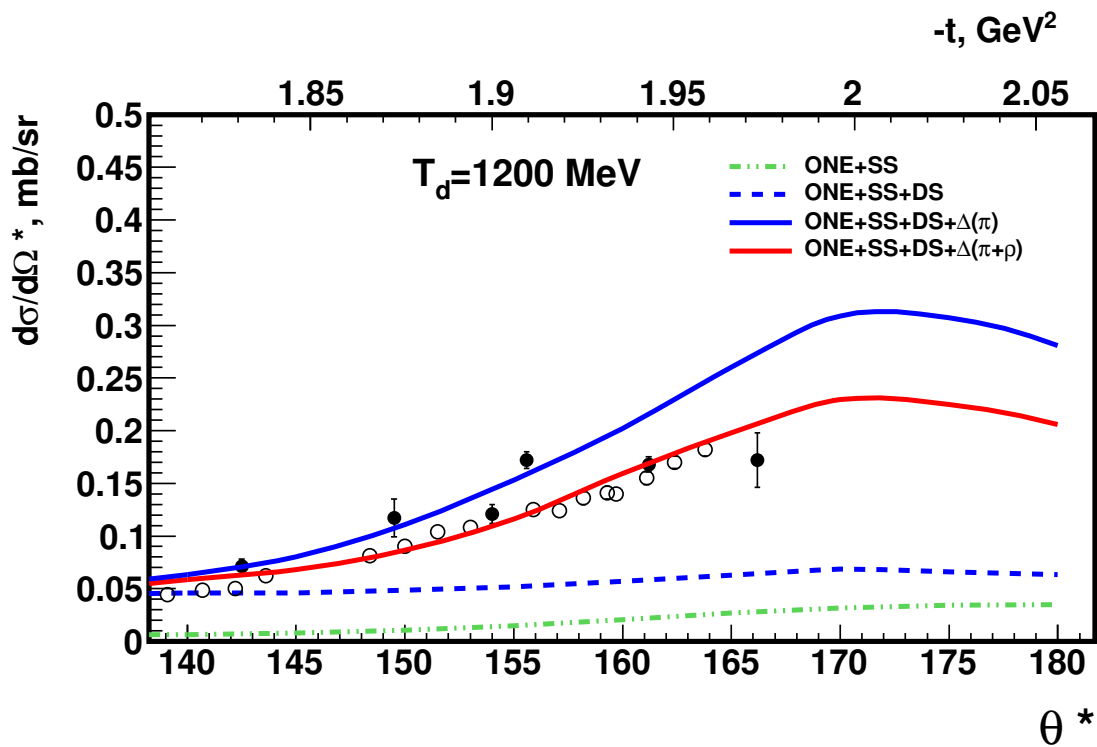
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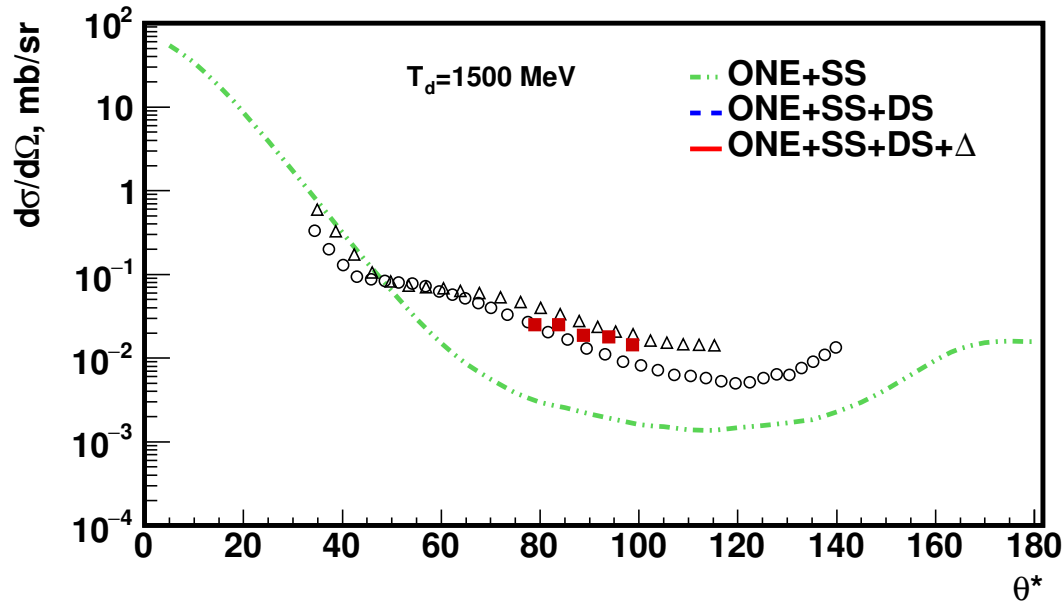
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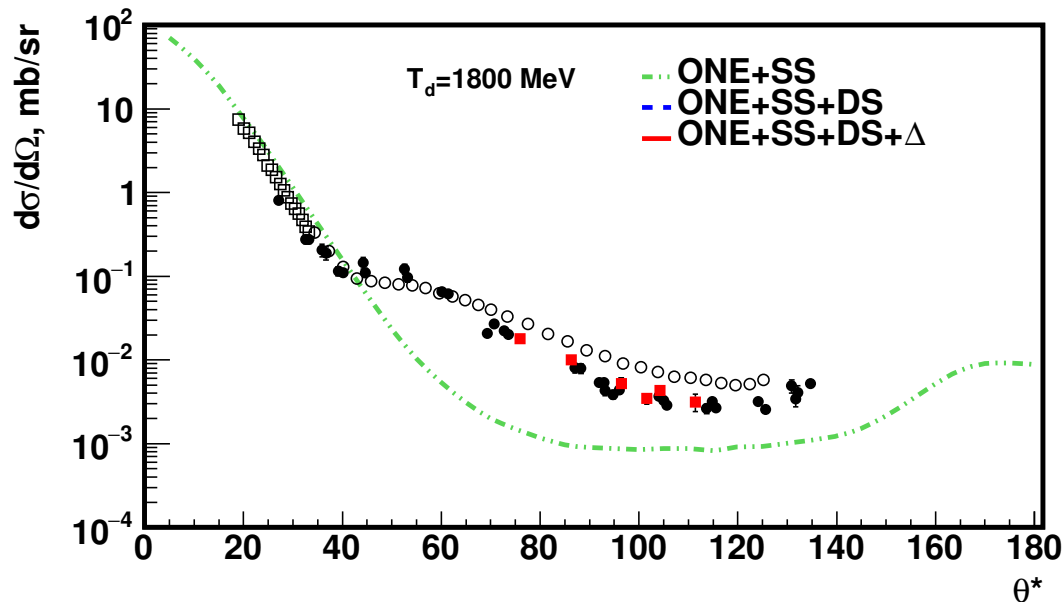
○ - J.C.Alder et al., Phys.Rev.C6, p.2010 (1972)



○ - E. Gülmez et al.,
 Phys.Rev.C42, p.2067,
 1991, $T_d = 1.585 \text{ GeV}$

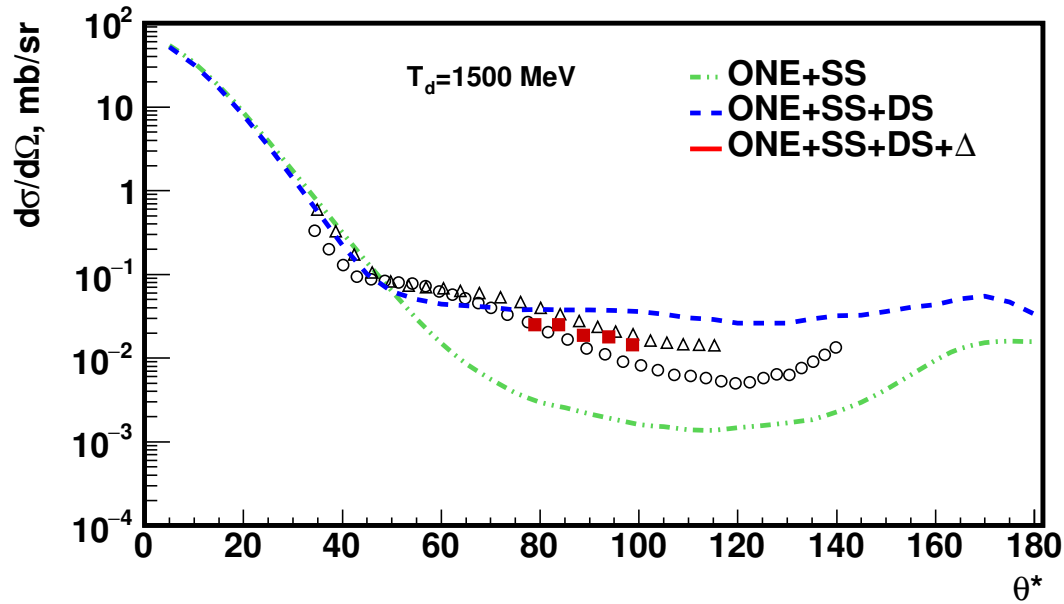
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 et al., Eur. Phys. J.
 A55, 129(2019)



□ - C. Fritzsche et al.,
 Phys.Lett.B 784, 277
 (2018), $T_d = 1.8 \text{ GeV}$

● - Bennet et al.,
 Phys. Rev. Lett.
 19, p.387 (1967),
 $T_d = 2 \text{ GeV}$



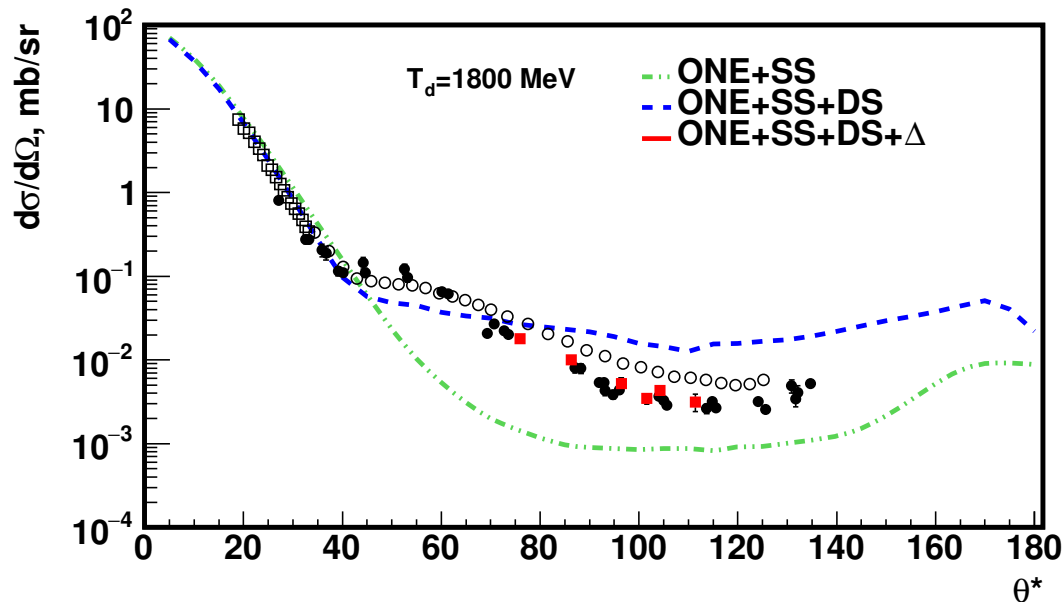
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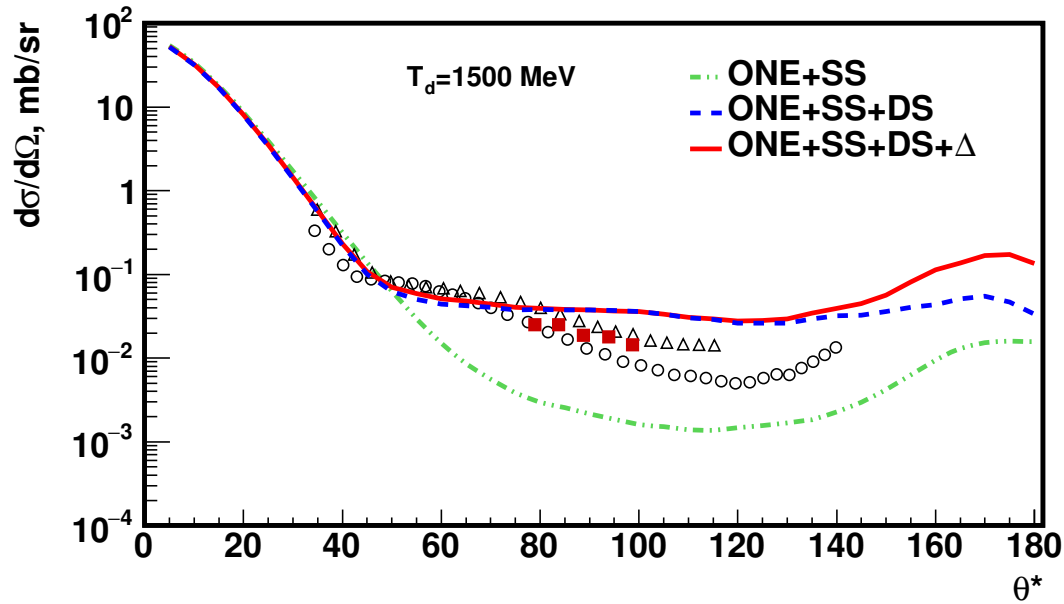
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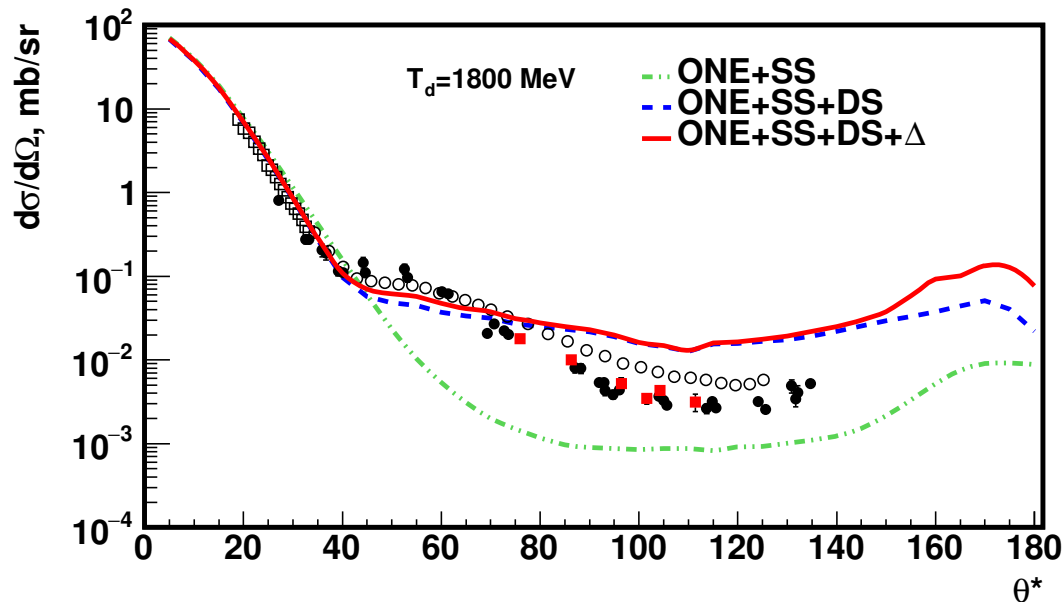
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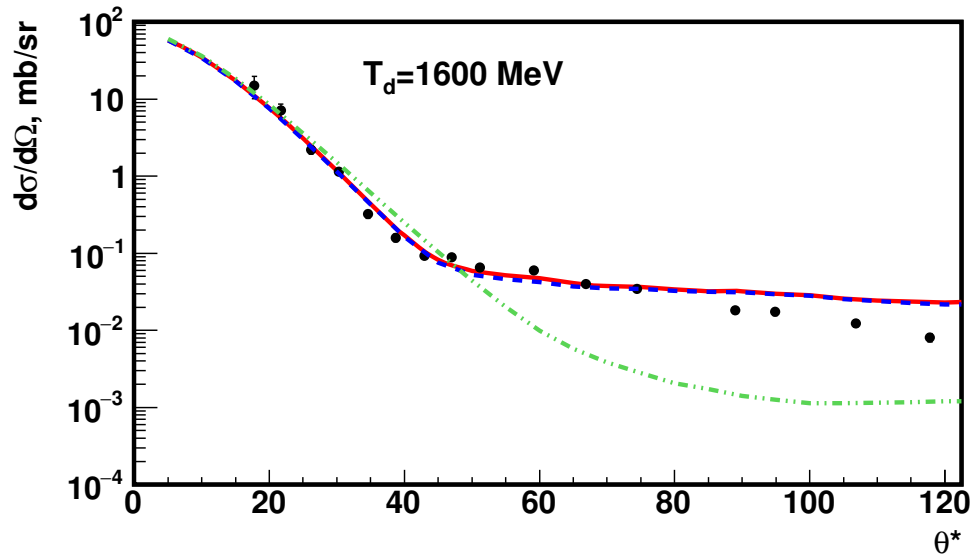
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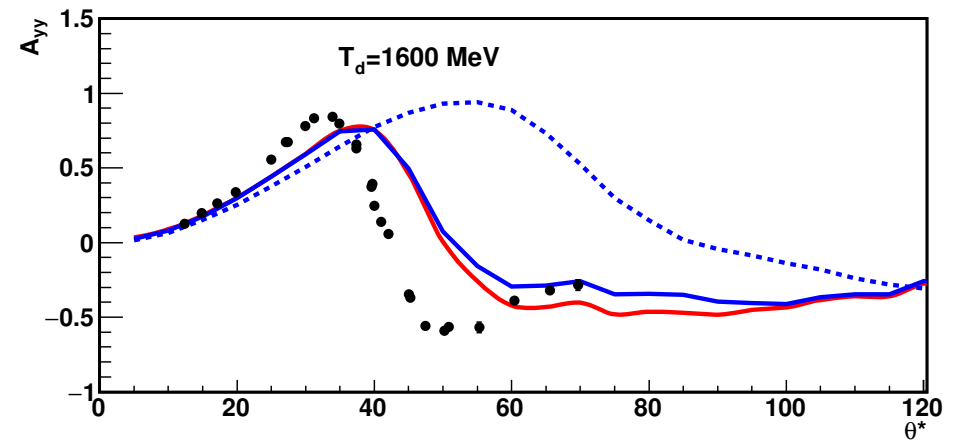
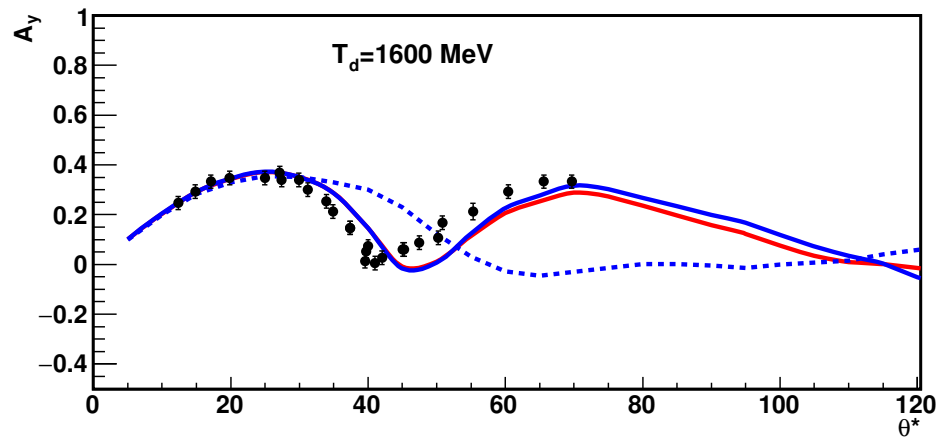


$$T_d = 1600 \text{ MeV}$$



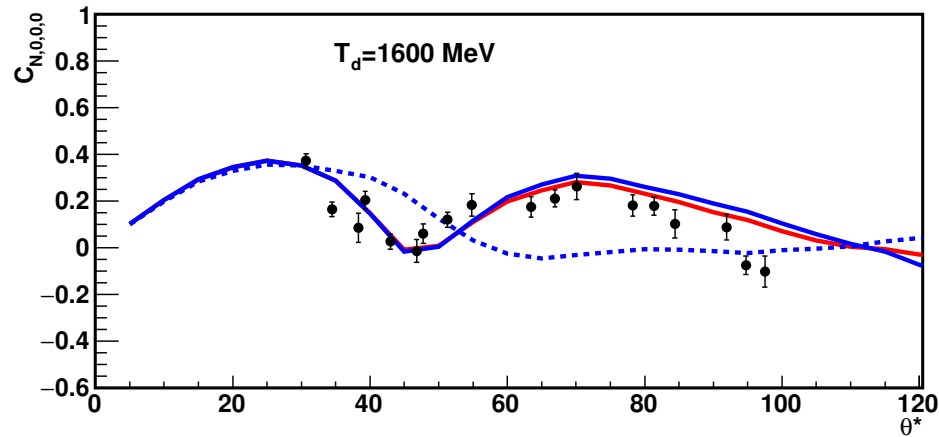
V.Ghazikhanian et al.,
Phys.Rev.C43,p.1532(1991)

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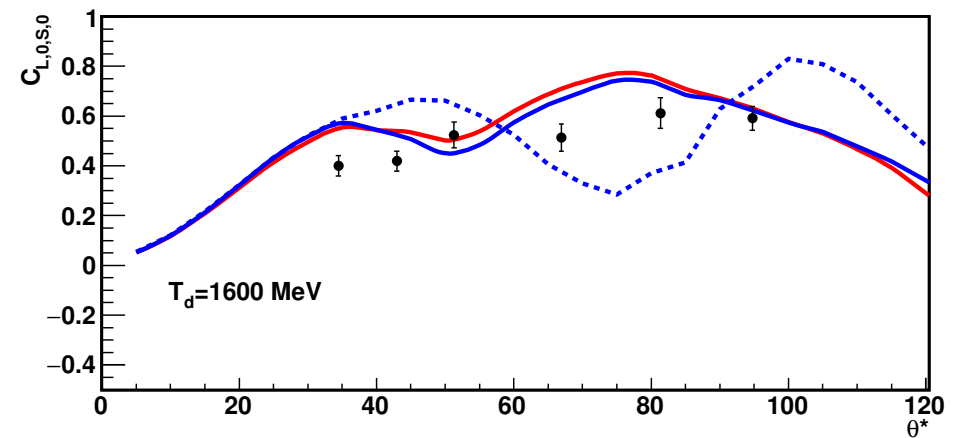
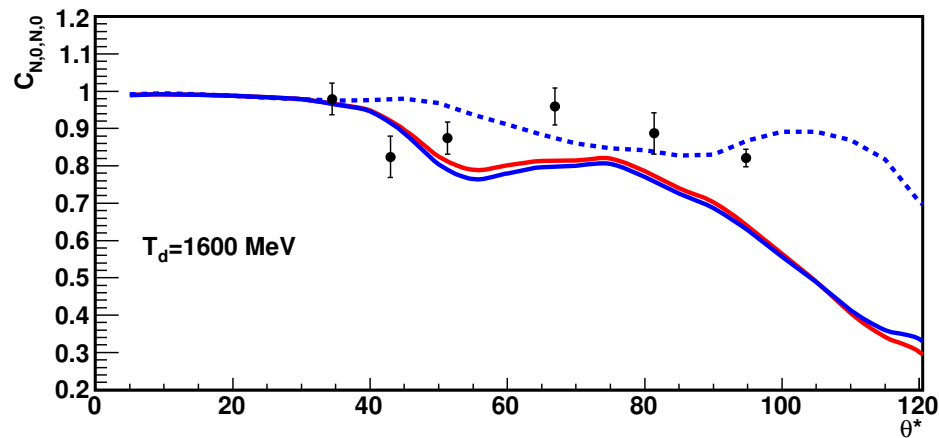


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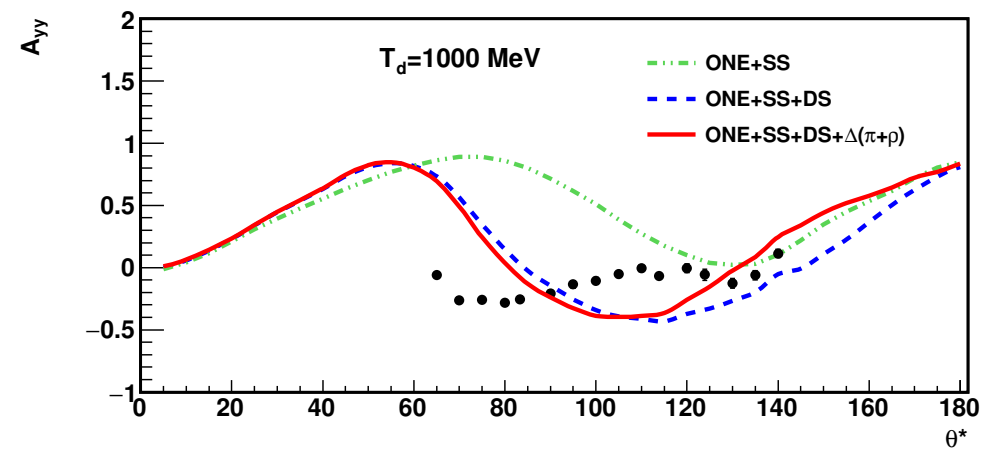
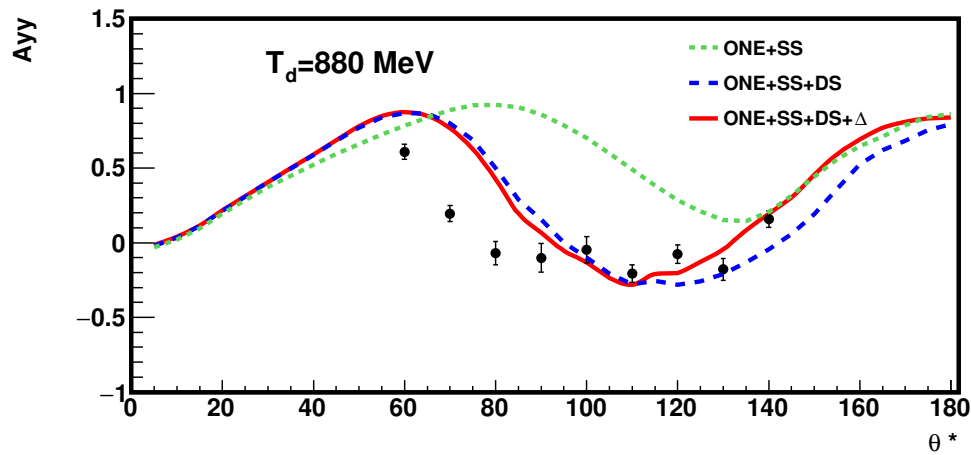
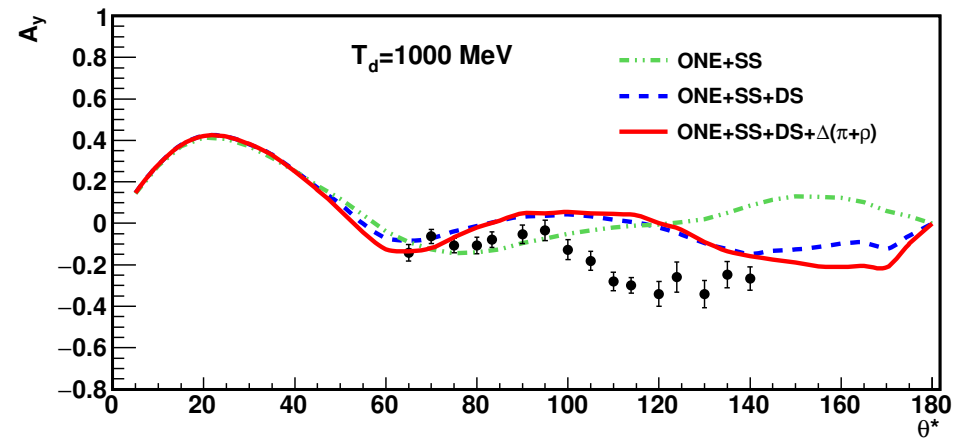
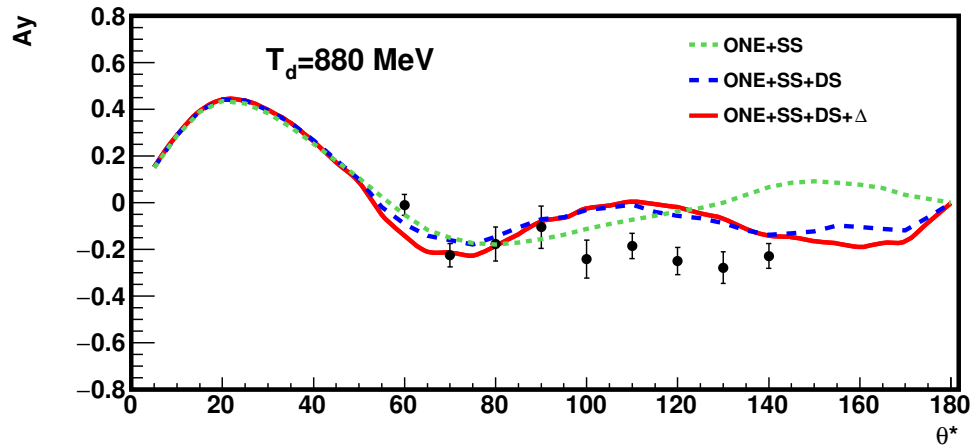


--- ONE+SS
— ONE+SS+DS
— ONE+SS+DS+ Δ



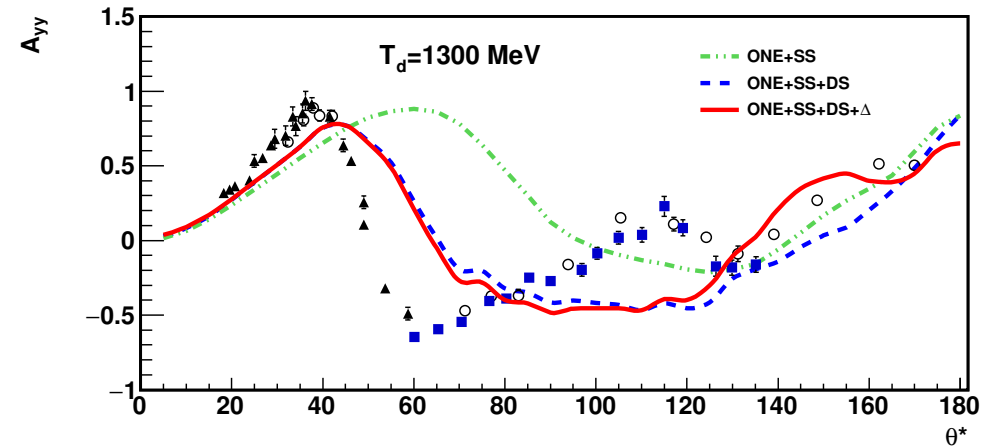
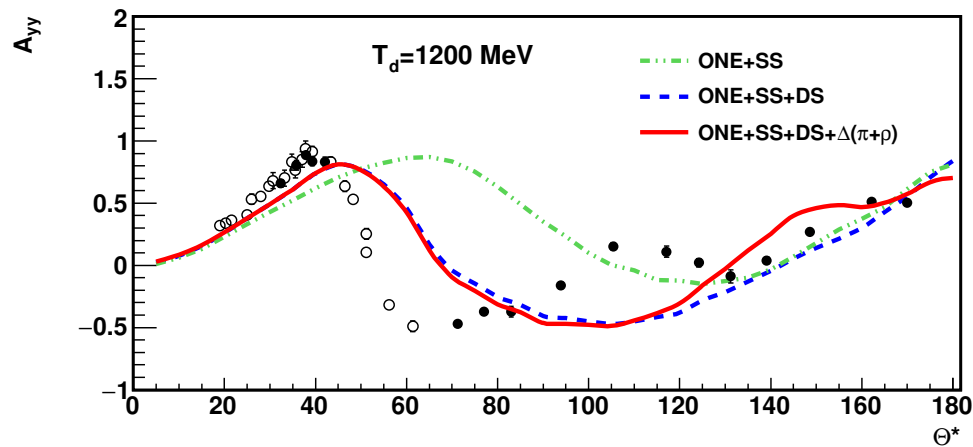
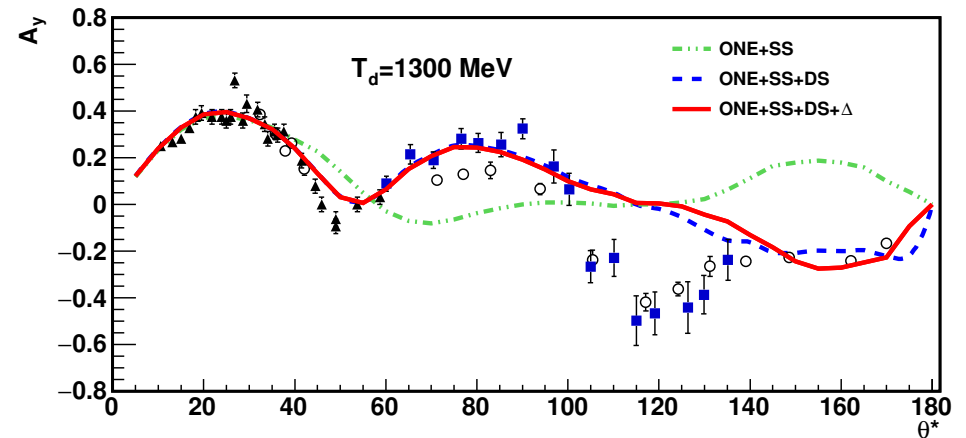
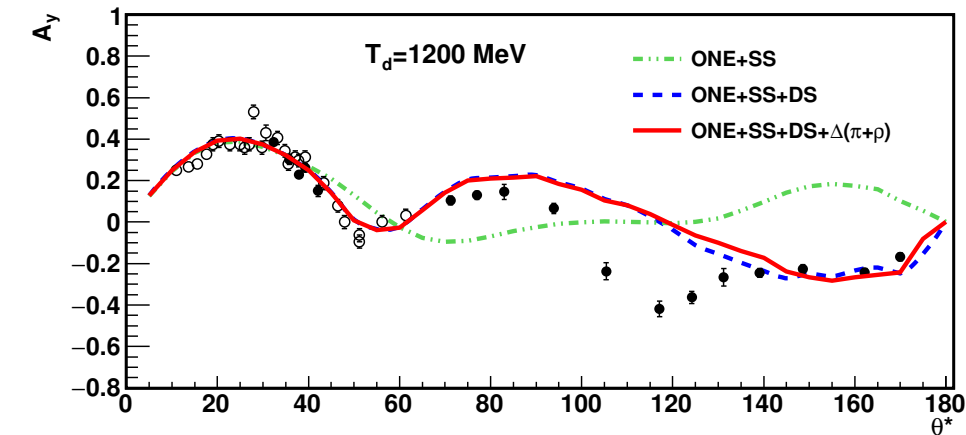
Conclusion

- dp elastic scattering has been considered taking into account four contributions: one-nucleon-exchange, single-scattering, double-scattering, and Δ -excitation in an intermediate state.
- A good description of the differential cross section has been obtained at the scattering angle $\theta^* \leq 100^\circ$ and $\theta^* \geq 130^\circ$ in the energy range between 880 and 1800 MeV.
- The description of the growth of the differential cross section at $\theta^* \geq 140^\circ$ became possible due to the inclusion of the Δ -isobar term in the consideration.
- A quite good description of the polarisation data at the deuteron energies $T_d=1600$ MeV on the deuteron vector A_y and tensor A_{yy} analysing powers, and on the proton analyzing power $C_{N,0,0,0}$ and proton polarization transfer $C_{N,0,0,0}$ $C_{N,0,0,N,0}$ $C_{L,0,S,0}$ has been obtained at the scattering angle $\theta^* \leq 100^\circ$
- Nevertheless, there are problems with the description of the differential cross section at the scattering angles $90^\circ < \theta^* \leq 140^\circ$



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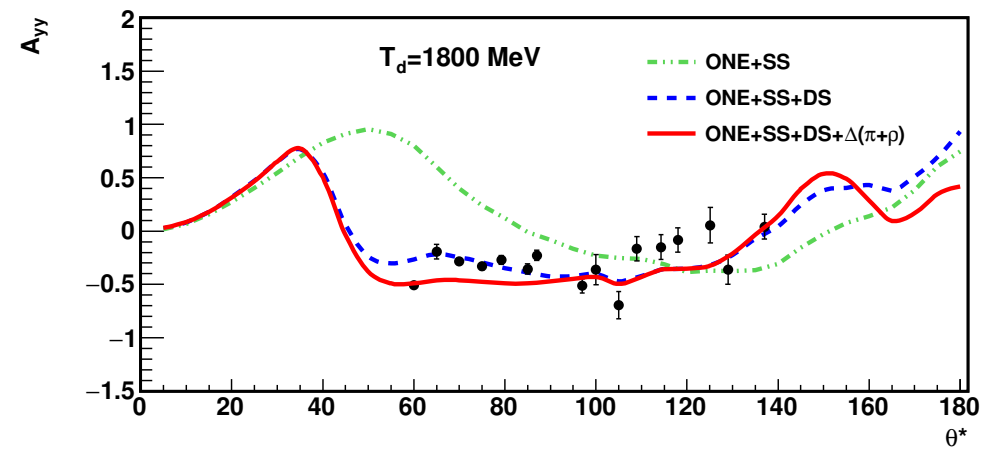
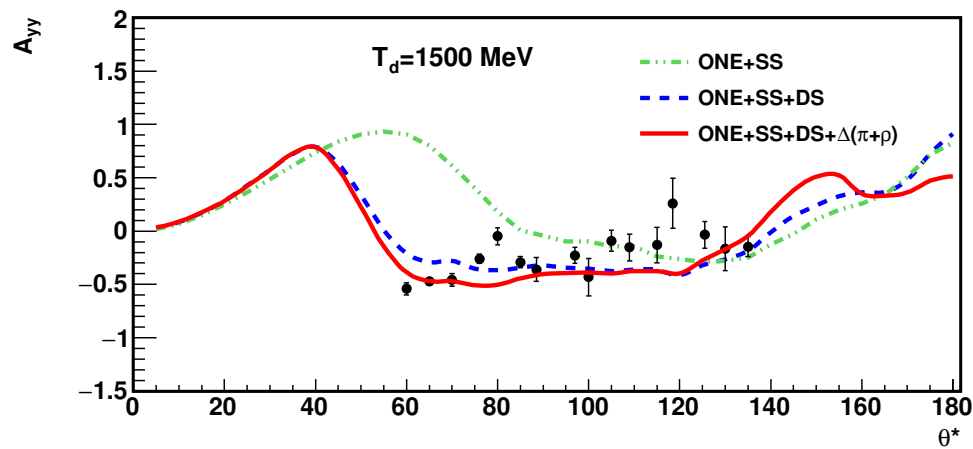
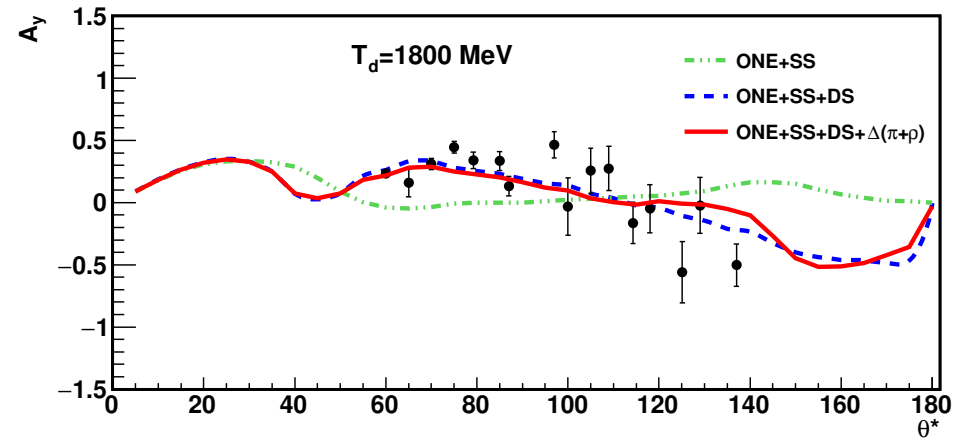
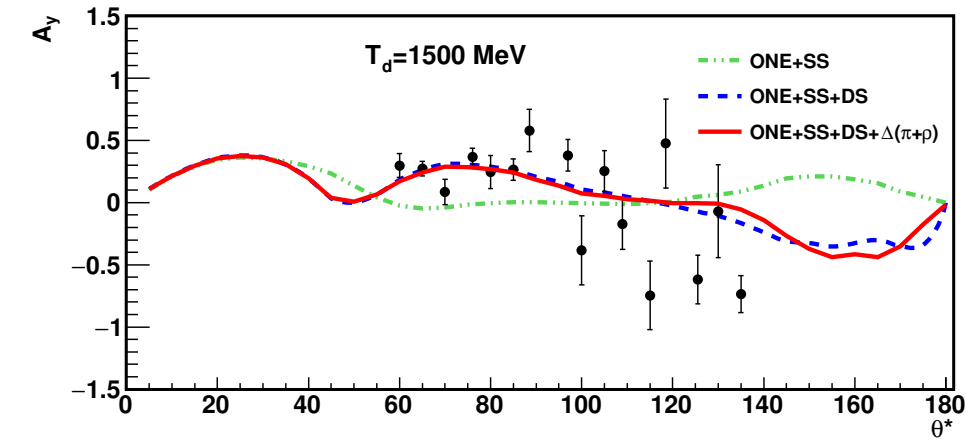
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