

Joint Institute for Nuclear Research (JINR)  
The Bogoliubov Laboratory of Theoretical Physics (BLTP)



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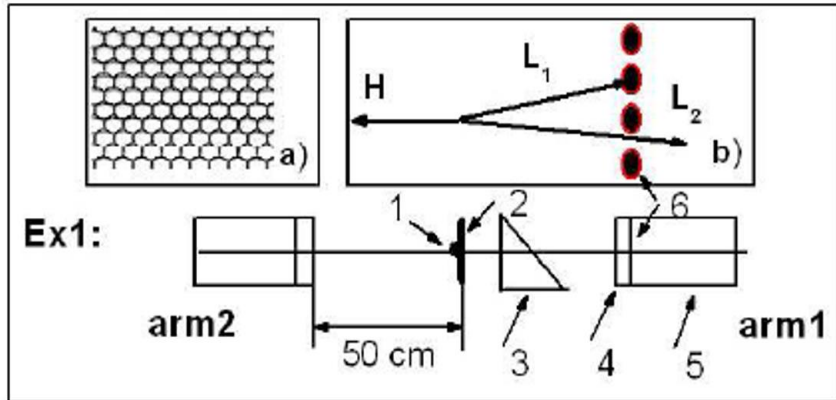
## Decay of the spontaneous fission isomers in the Coulomb field of third nucleus

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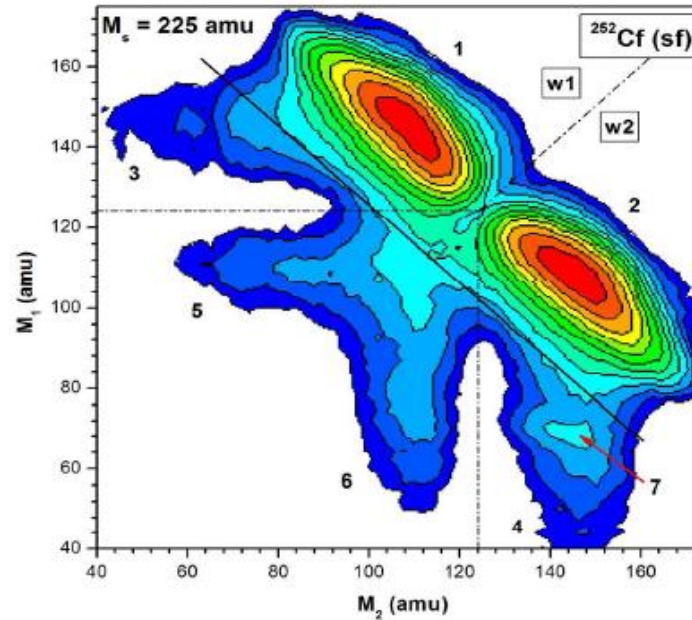
with permanent consulting support in details  
of the interpreted experiments of Prof. Pyatkov Yu. and Dr. Kamanin D. (FLNR, JINR)

Year 2025, Saint Petersburg

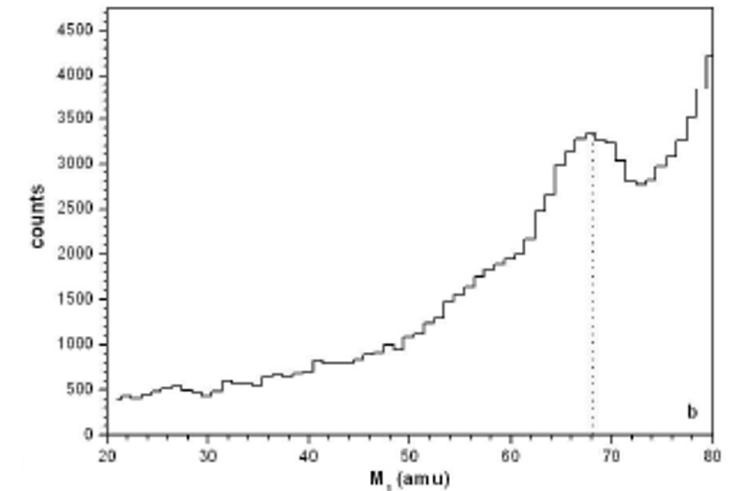
# Introduction



Lay out of the FOBOS setup used for studies of  $^{252}\text{Cf}(\text{sf})$



Contour map of the mass-mass distribution of the collinear fragments of  $^{252}\text{Cf}$ , detected in coincidence in the two opposite arms of the FOBOS spectrometer.



Projection of the Ni bump on the M1 axis.

Spontaneous fission of heavy nuclei like  $^{252}\text{Cf}$  is a cornerstone of experimental nuclear physics, yet its ternary decay—particularly collinear cluster tripartition (CCT)—remains enigmatic. The series of experiments of the FOBOS group in FLNR JINR pioneered the detection of unknown ternary decays using a specialized coincidence setup.

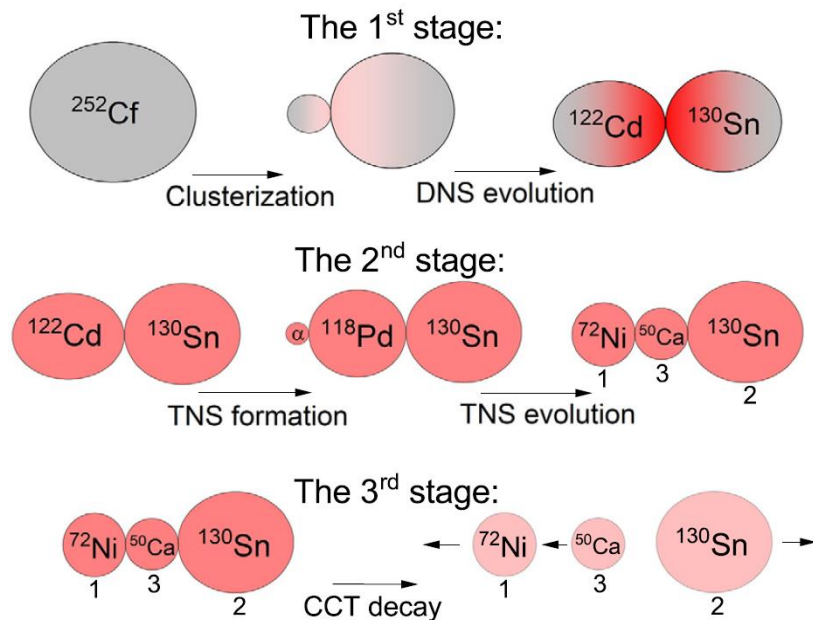
1. New indications of collinear tripartition in  $^{252}\text{Cf}(\text{sf})$  studied at the modified FOBOS setup / Yu V Pyatkov, DV Kamanin, AA Alexandrov et al. // Physics of Atomic Nuclei. — 2003. — Vol. 66. — Pp. 1631–1635.

2. Pyatkov, Yu V. Tech. Rep.: / Yu V Pyatkov, DV Kamanin, WH Trzaska: Flerov Laboratory of Nuclear Reactions, 2005.

3. Examination of evidence for collinear cluster tri-partition / Yu V Pyatkov, DV Kamanin, AA Alexandrov et al. // Physical Review C. — 2017. — Vol. 96, no. 6. — P. 064606.

# Model

In our model, spontaneous ternary fission of heavy nuclei takes place in three consecutive stages: first, the formation of asymmetric DNS and its evolution in mass (and charge) asymmetry coordinates to-wards symmetric DNS configurations, then the transformation of one of the nearly symmetric DNS nuclei into another DNS, thus forming the TNS, and finally the decay of TNS into three fragments.



Mechanism of formation of collinear cluster tri-partition (CCT) fragments.

The potential energy  $U_{DNS}$  of a dinuclear system is calculated as:

$$U_{DNS} = V_{int,DNS} + B_1 + B_2 - B_{CN}$$

where  $B_1$ ,  $B_2$  and  $B_{CN}$  are mass excesses of two nuclei forming the DNS and a compound nucleus (CN), respectively.

The potential energy  $U_{TNS}$  of a trinuclear system is calculated as:

$$U_{TNS} = V_{int,12} + V_{int,13} + V_{int,23} + B_1 + B_2 + B_3 - B_{CN}$$

where  $B_1$ ,  $B_2$ ,  $B_3$  and  $B_{CN}$  are mass excesses of three nuclei forming the TNS and a compound nucleus (CN), respectively.

The total interaction potential  $V_{int}$  is the sum of the nuclear  $V_N$  and Coulomb potential  $V_C$  energies:

$$V_{int} = V_N + V_C$$

# Potential energy of Dinuclear system

$$V_c(R_{12}, Z_1, Z_2) = \frac{Z_1 Z_2 e^2}{R_{12}} + \frac{Z_1 Z_2 e^2}{R_{12}^3} \sqrt{\frac{9}{20\pi}} (R_{01}^2 \beta_1 + R_{02}^2 \beta_2) + \frac{Z_1 Z_2 e^2}{R_{12}^3} \frac{3}{7\pi} (R_{01}^2 \beta_1^2 + R_{02}^2 \beta_2^2)$$

Here,  $\beta_1$  and  $\beta_2$  are the quadrupole deformation parameters,  $R_{01}$  and  $R_{02}$  are the radii of spherical nuclei,  $R_{12}$  is the relative distance between their centers, and  $Z_1$  and  $Z_2$  are their charge numbers.

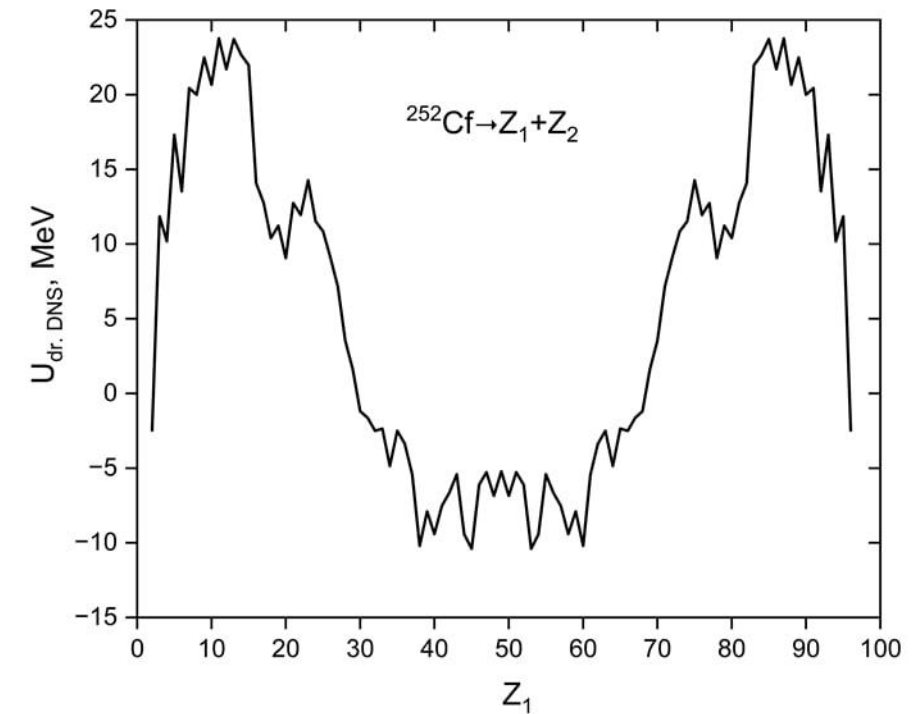
The nuclear part of the interaction is calculated using the double folding formalism with the density dependent effective nucleon-nucleon forces  $f_{eff}$  by Migdal

$$V_{N,12}(R_{12}) = \int \rho_1(r) f_{eff}(r, R_{12}) \rho_2(r - R_{12}) dr$$

where  $f_{eff}(r, R_{12}) = C_0 \left[ F_{in} + (F_{ex} - F_{in}) \frac{\rho_0 - \rho(r, R_{12})}{\rho_0} \right]$ , and  $\rho(r, R_{12}) = \rho_1(r) + \rho_2(r - R_{12})$ .

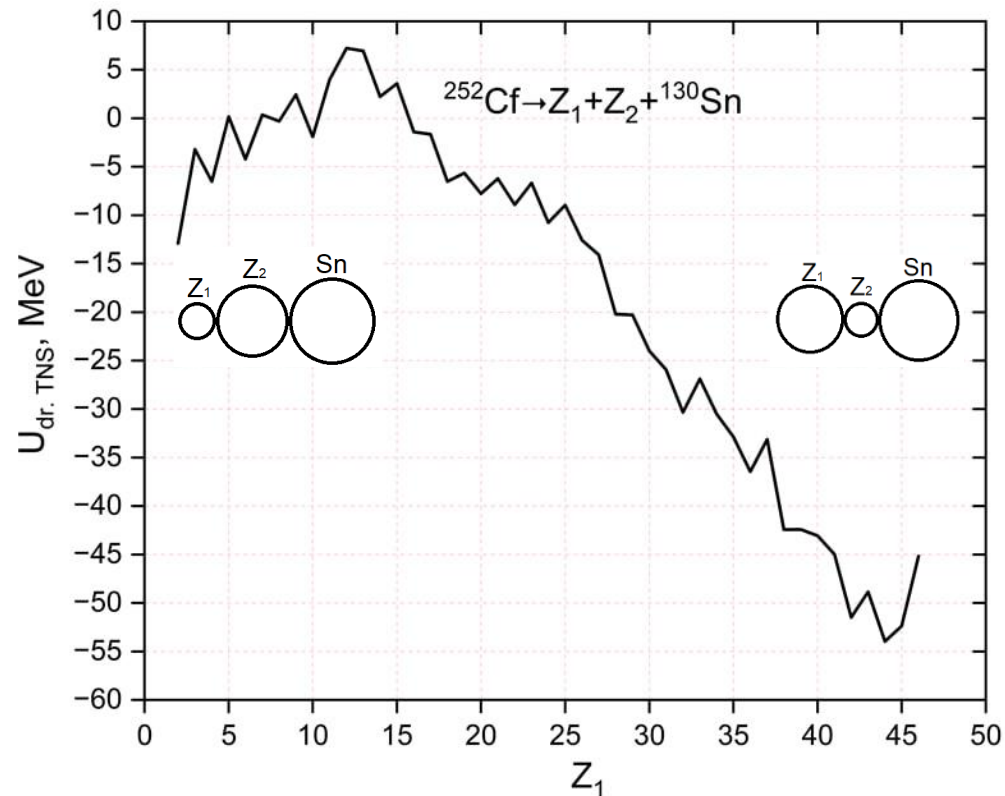
The following set of constants is used for the Migdal forces:  $C_0 = 300 \text{ MeV fm}^3$ ,  $F_{ex,in} = f_{ex,in} + f_{ex,in} \frac{(A_1 - 2Z_1)(A_2 - 2Z_2)}{(A_1 + 2Z_1)(A_2 + 2Z_2)}$ ,  $f_{in} = 0.09$ ,  $f_{ex} = -2.59$ ,  $f_{in} = 0.42$ ,  $f_{ex} = 0.54$ .

Based on the driving potential diagram for **252Cf (Californium-252)** spontaneous fission, the **minimum points** labeled Sr, Zr, Rh, Cd, Sn, I, Se, and Nd represent **preferred fragmentation paths** where the potential energy is lowest during the fission process. This explains the observed **asymmetric mass yield** in experiments, where fragments cluster around light ( $Z=35-45$ ) and heavy ( $Z=55-65$ ) groups. The deepest minima (e.g., Sr–Nd) correspond to the most probable fission products.



The calculated driving potential of cluster + heavy nucleus for  $^{252}\text{Cf}$  as a function of the charge number  $Z_1$  of one nucleus in the DNS.

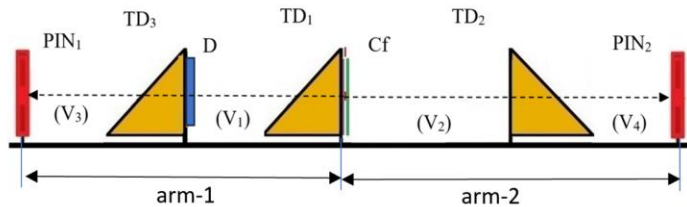
# Trinuclear system



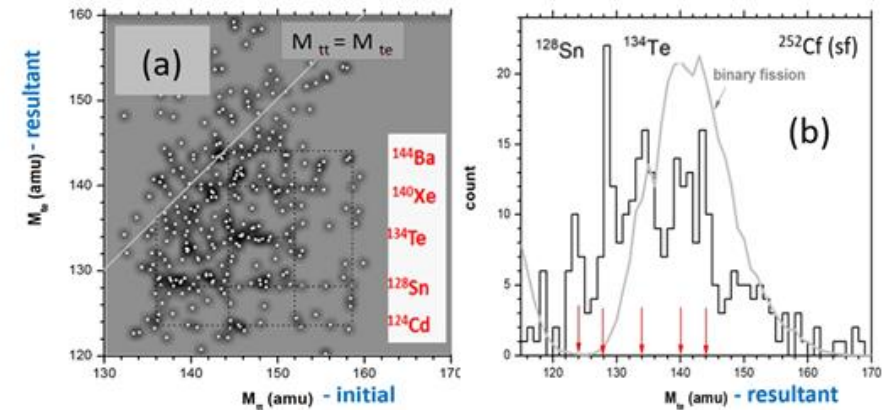
The driving potential of TNS formed from a nearly symmetric DNS state  $^{122}\text{Cd} + ^{130}\text{Sn}$ , as a function of the left border nucleus charge number  $Z_1$ .

The stability of the magic nucleus Sn ensures that nucleon transfer during ternary nuclear system (TNS) evolution does not change the identity of the right border nucleus (nucleus 2), which remains Sn. For a TNS derived from the nearly symmetric  $^{122}\text{Cd} + ^{130}\text{Sn}$  dinuclear system (DNS), Figure 4 presents the TNS driving potential as a function of the left border nucleus charge number  $Z_1$ . Transitioning to more symmetric TNS states requires surmounting the potential barrier, characterized by the height of the TNS Businaro-Gallone (B.G.) point (indicated in Fig. 4). It is essential to note that the process of TNS formation competes with binary decay of the precursor  $^{122}\text{Cd} + ^{130}\text{Sn}$  DNS configuration. The outcome of this competition is the principal determinant of the ternary-to-binary fission yield ratio.

# LIS set-up (measuring the mass of the fragment twice: before and after passing through the Ti foil)



**Fig. 1.** Layout of the LIS spectrometer. Triangles depict MCP based timing detectors TD<sub>1</sub>, TD<sub>2</sub>, TD<sub>3</sub>. The TD<sub>1</sub> “start” detector includes a source of spontaneous fission <sup>252</sup>Cf(sf). The TD<sub>3</sub> detector includes a Cu degrader foil D, 4 u thick. The FFs velocities V<sub>1</sub>, V<sub>2</sub>, V<sub>3</sub>, and V<sub>4</sub> are measured at four flight-paths, each 150 mm long. The aperture for fission fragments detected in coincidence in the opposite PIN diodes does not exceed  $\pm 1.5^\circ$ .



Correlation mass distribution  $M_{tr}-M_{te}$  for the “initial”  $M_{tr}$  masses from the heavy mass peak in binary fission <sup>252</sup>Cf(sf) (a) and its projection onto the  $M_{te}$  axis (b). The spectrum of fragments of conventional binary fission is shown in grey. Positions of the magic and semi magic nuclei are marked by the arrows. Corresponding isotopes are listed in the column in (a).

# Kinetic Energy Distribution in TNS Decay and Maximum Approaching Distance

According to law of conservation of impulse and energy was calculated impulse and kinetic energy of DNS and third nuclei:

$$\begin{cases} V_{TNSmax} = V_{TNSmin} + E_{kDNS} + E_{k3} \\ 0 = (m_1 + m_2)v_{DNS} + m_3v_3 \end{cases}$$

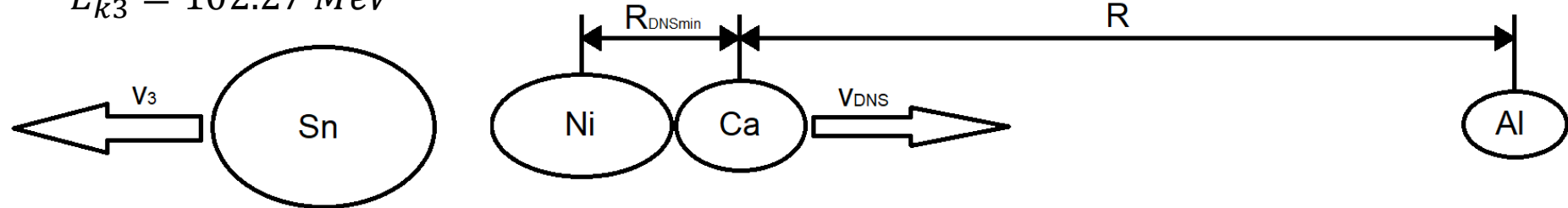
The TNS Ca+Ni+Sn decays into two distinct subsystems and Kinetic Energies after TNS Decay:

**DNS** → Ca+Ni

$$E_{kDNS} = 108.97 \text{ MeV}$$

Isolated Fragment → Sn

$$E_{k3} = 102.27 \text{ MeV}$$



To know the maximum approaching distance DNS to the foil were solved this system of equations:

$$\begin{cases} E_{kDNS} = V_{C14}[\mathbf{R}] + V_{C24}[\mathbf{R} + \mathbf{R}_{DNSmin}] + E'_{kDNS} + E_{k4} \\ (m_1 + m_2)v_{DNS} = (m_1 + m_2)v'_{DNS} + m_4v_4 \end{cases}$$

With its kinetic energy of 108.97 MeV, the DNS (Ca+Ni) can approach an Al foil to a maximum distance of:

$$R_{max.app.} = 40.1 \text{ fm}$$

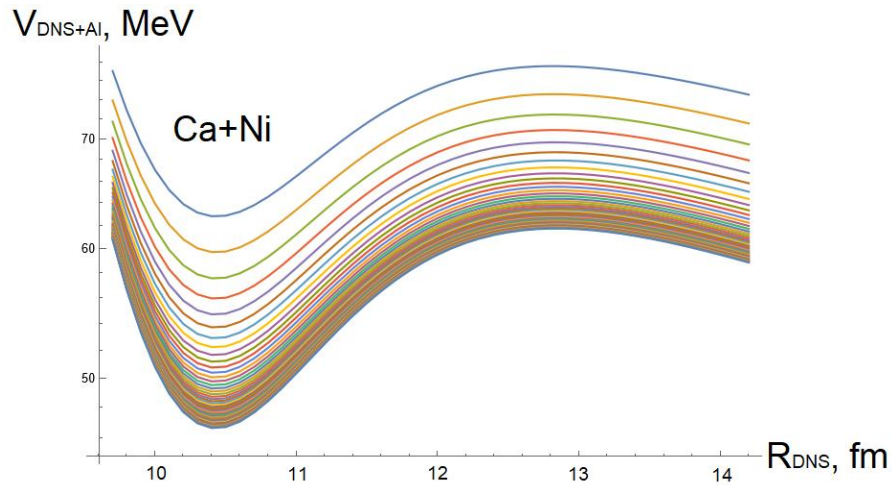
Illustration of interaction DNS and Foil. The first nuclei is Ca, second one is Ni and fourth is foil's nuclei, in this case foil is Al.

# Interaction between DNS and the third nucleus as a foil in the Coulomb field.

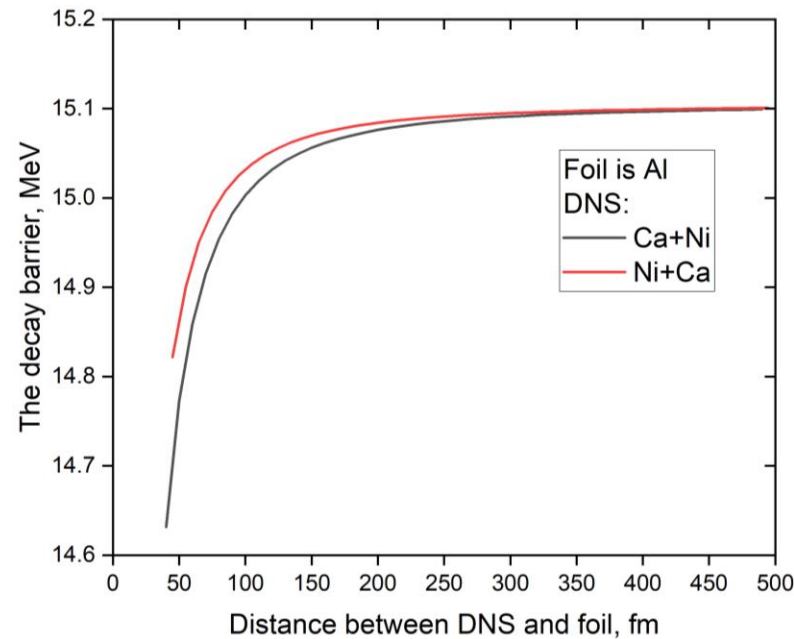
The potential energy  $V_{DNS+Al}$  of a DNS and foil is calculated as:

$$V_{DNS+Al} = V_{C12} + V_{N12} + V_{C14} + V_{C24}$$

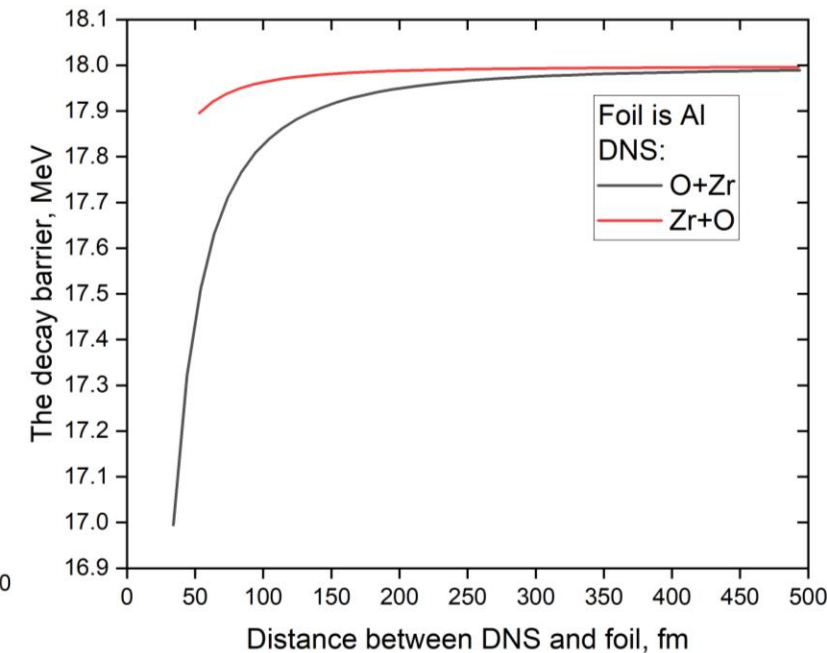
where the nuclear  $V_N$  and Coulomb potential  $V_C$  energies.



Changing Potential Barrier DNS when DNS approaching to the foil nuclei.



Changing the decay barrier of DNS according to the distance between DNS and foil



# Vibration of DNS

$$\begin{cases} E_{kDNS} = V_{C14}[\mathbf{R}] + V_{C24}[\mathbf{R} + (\mathbf{R}_{DNSmin} + \Delta x)] + E_{kDNS} + E_{k4} \\ (m_1 + m_2)v_{DNS} = (m_1 + m_2)v_{DNS} + m_4v_4 \end{cases}$$

Where  $\Delta x$  is difference of distance between DNS.

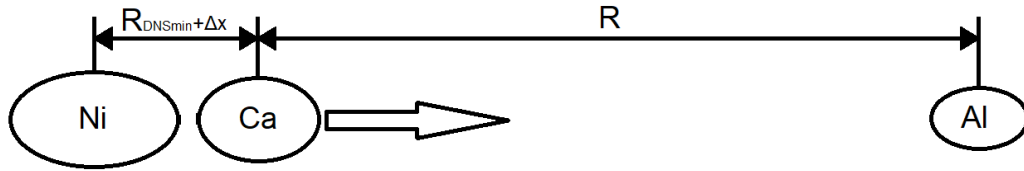
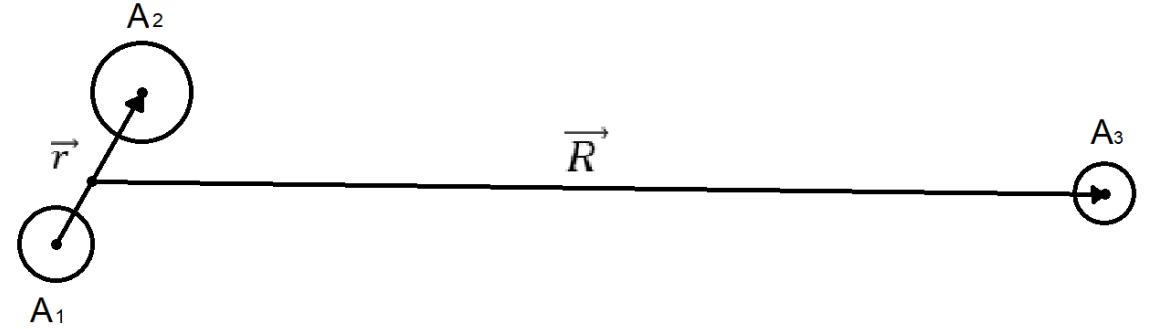


Illustration of interaction DNS and Foil in vibration case. The fourth nucleus, that is, due to the Coulomb field of foil, exerts two different effects on the two nuclei in the DNS structure and causes a change in the distance between them.

When DNS Ca+Ni maximum approaches to Al foil the distance of nuclei in DNS changes to  $\Delta x = 0.2$  fm and it depends 0.96 MeV.



$$\begin{cases} T = \frac{M_r(\dot{\vec{r}})^2}{2} + \frac{M_R(\dot{\vec{R}})^2}{2} = \frac{\vec{p}_r^2}{2M_r} + \frac{\vec{p}_R^2}{2M_R} \\ V = \frac{Z_1Z_2e^2}{\vec{r}} + \frac{Z_1Z_3e^2}{|\vec{A}_{r1}\vec{r} + \vec{R}|} + \frac{Z_2Z_3e^2}{|\vec{R} - \vec{A}_{r2}\vec{r}|} \end{cases}$$

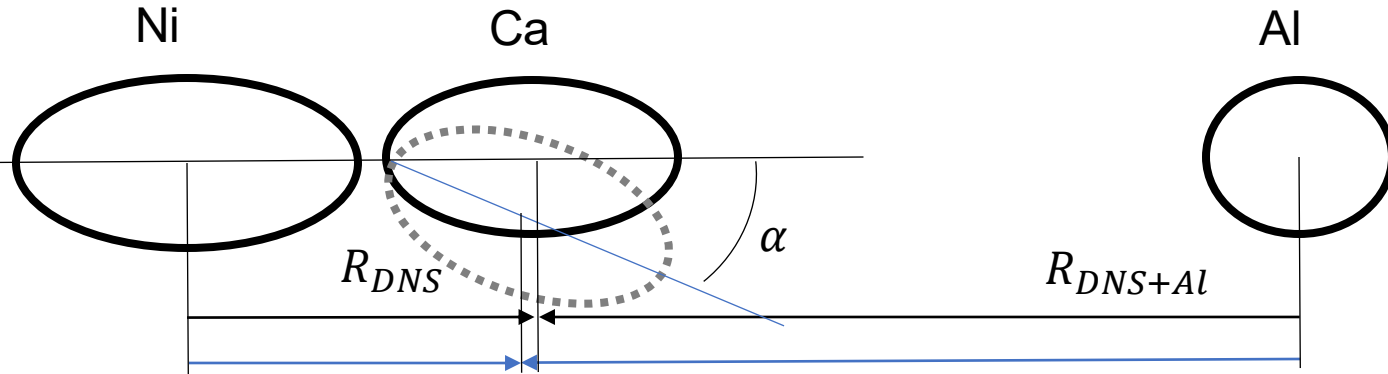
Where  $M_r = \frac{m_1m_2}{m_1+m_2}$ ,  $M_R = \frac{(m_1+m_2)m_3}{m_1+m_2+m_3}$ ,  $A_{r1} = \frac{A_2}{A_1+A_2}$ ,  $A_{r2} = \frac{A_1}{A_1+A_2}$

$$\begin{cases} M_r\ddot{\vec{r}} = -\lambda M_r(\dot{\vec{r}})^2 + A_{r1}\frac{Z_1Z_3e^2}{(\vec{A}_{r1}\vec{r} + \vec{R})^2} - A_{r2}\frac{Z_2Z_3e^2}{(\vec{R} - \vec{A}_{r2}\vec{r})^2} - \frac{\partial V_{int,DNS}}{\partial r} \\ M_R\ddot{\vec{R}} = \frac{Z_1Z_3e^2}{(\vec{A}_{r1}\vec{r} + \vec{R})^2} + \frac{Z_2Z_3e^2}{(\vec{R} - \vec{A}_{r2}\vec{r})^2} \end{cases}$$

$$\hbar\lambda = 1 \text{ MeV}$$

Energy dissipation was only  $6.28 \times 10^{-8} \text{ MeV}$ .

# Angular oscillation



The interaction between the DNS, in other words first and second nuclei, is expressed through the distance and angle between them as follows:

$$V_{int,DNS} = V_{C,DNS}[R_{DNS}, \alpha] + V_{N,DNS}[R_{DNS}, \alpha]$$

The interaction between the DNS and foil is expressed as follows:

$$V_{int,DNS+Al} = V_{C,14}[R_{DNS+Al}, \alpha]$$

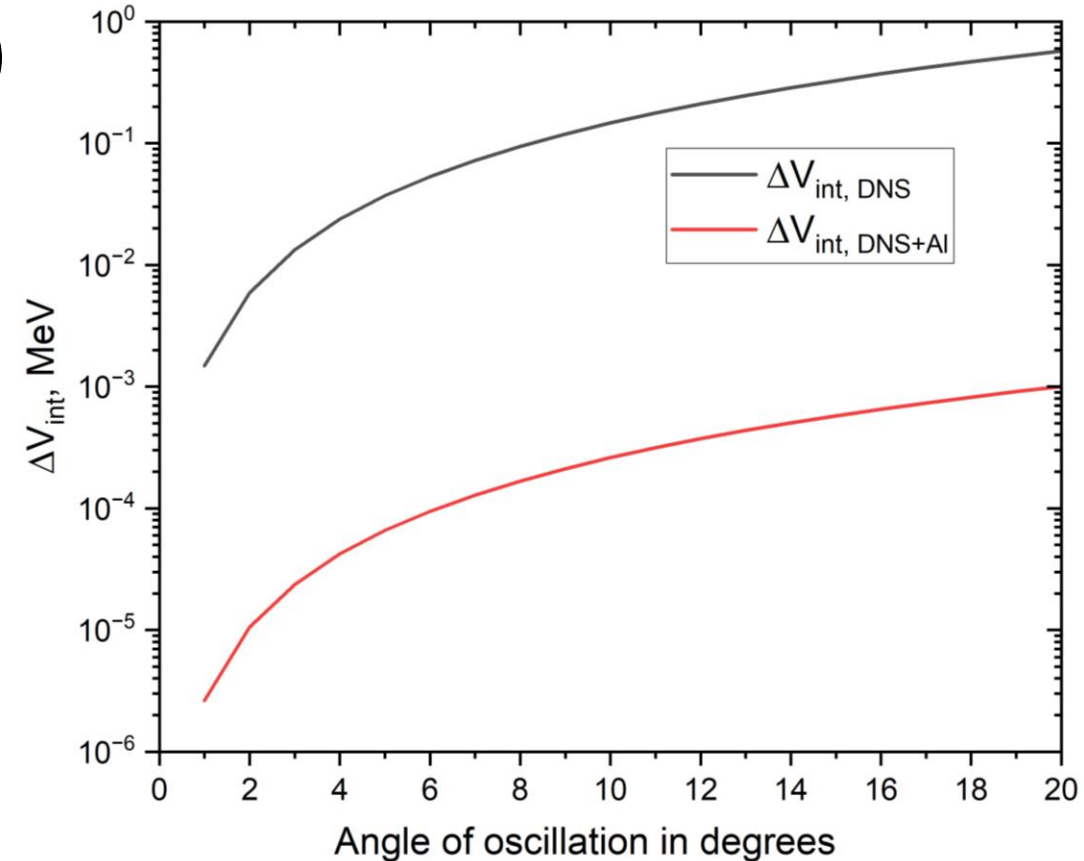
Calculation of the energy used to bend the first nucleus at some angle:

- For DNS

$$\Delta V_{int,DNS} = V_{C,DNS}[R_{DNS}, 0] + V_{N,DNS}[R_{DNS}, 0] - V_{C,DNS}[R_{DNS}, \alpha] - V_{N,DNS}[R_{DNS}, \alpha]$$

- For DNS and foil

$$\Delta V_{int,DNS+Al} = V_{C,DNS+Al}[R_{DNS+Al}, 0] - V_{C,DNS+Al}[R_{DNS+Al}, \alpha]$$



# Half-life of DNS

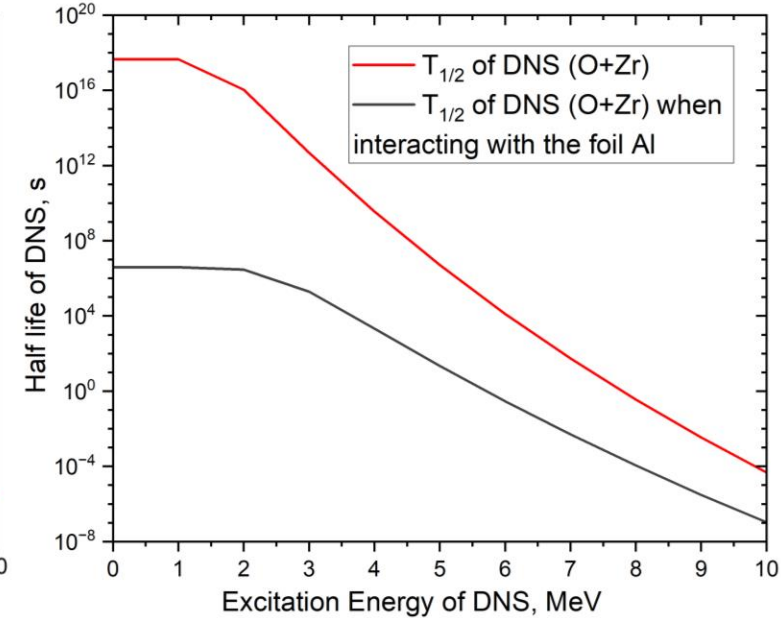
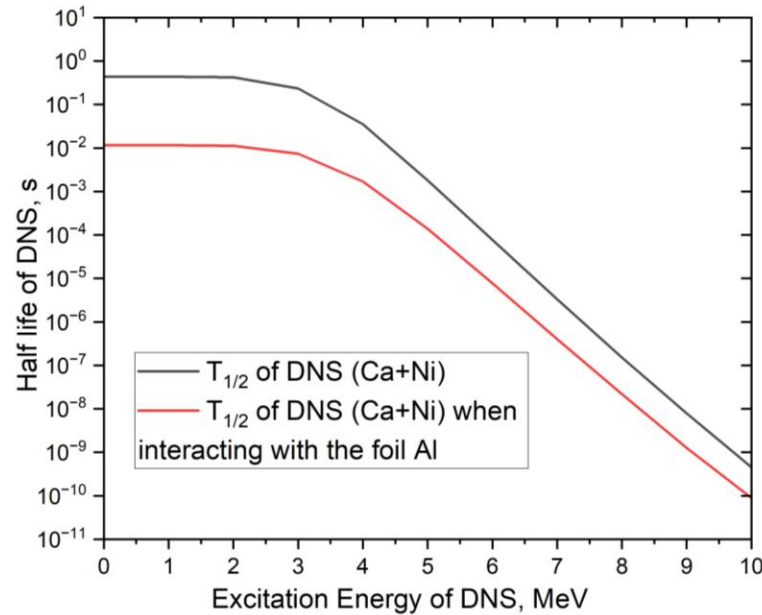
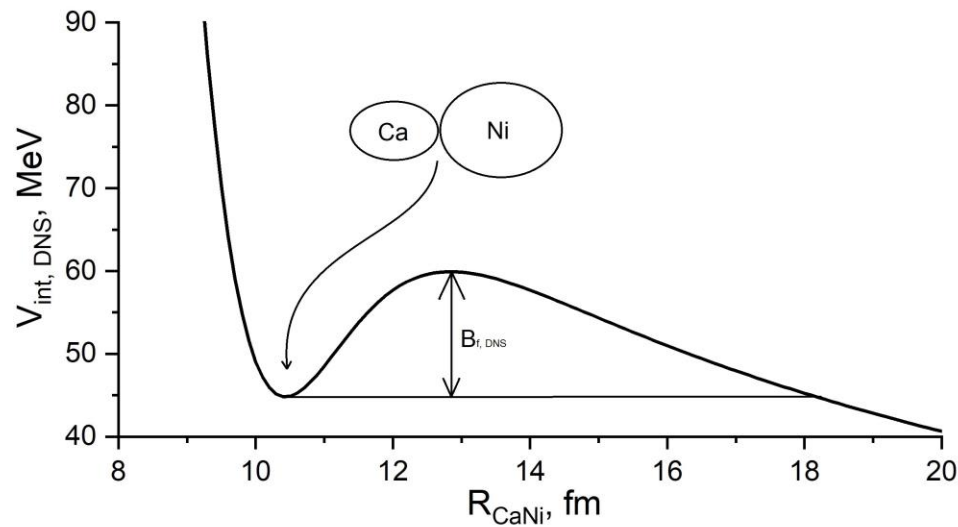
Half-life of DNS is calculated with in WKB approximation

Reduced mass:  $m = \frac{m_1 m_2}{m_1 + m_2}$

Probability of tunneling (from WKB approximation)  $P = \frac{1}{1 + \text{Exp}(\frac{2}{\hbar^2} \sqrt{\int_{R_{ex}}^{R_{en}} m(V_{int} - E) dR})}$

Potential pocket frequency  $\hbar w_0 = \sqrt{\frac{V_{int}'' \cdot (\hbar c)^2}{m}}$

Half – life  $T = \frac{\pi \cdot \log 2}{w_0 \cdot P}$



# Conclusion

This work investigates the collinear cluster tripartition (CCT) in spontaneous fission of  $^{252}\text{Cf}$ , emphasizing the critical role of a third nucleus (e.g., a foil) in modifying the decay dynamics. Key insights include:

1. The formation of the TNS in spontaneous ternary fission is critically dependent on the evolution of the DNS.
2. Based on the DNS driving potential minima, the formation of isomers corresponding to fragments like Sr, Zr, Rh, Cd, Sn, I, Se, and Nd is probabilistically favored during the fission process.

# Future Work

1. Kinetic energy profiles of DNS fragments post-foil
2. Angular distribution of DNS break-up events

Thank you for your attention