

Gamow-Teller decay studies with the 2p-2h configurations

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1. The Skyrme interaction with the tensor terms is used in the particle-hole channel.

$$\begin{aligned}
\mathbf{V}_{12}^{\text{C}} = & \textcolor{red}{t}_0(1 + \textcolor{red}{x}_0 P_\sigma) \delta(\mathbf{r}) + \\
& (1/2) \textcolor{red}{t}_1(1 + \textcolor{red}{x}_1 P_\sigma) [\mathbf{k}'^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{k}^2] + \textcolor{red}{t}_2(1 + \textcolor{red}{x}_2 P_\sigma) \mathbf{k}' \delta(\mathbf{r}) \mathbf{k} + \\
& (1/6) \textcolor{red}{t}_3(1 + \textcolor{red}{x}_3 P_\sigma) \rho^\alpha(\mathbf{R}) \delta(\mathbf{r}) + i \textcolor{red}{W}_0 [\mathbf{k}' \times \delta(\mathbf{r}) \mathbf{k}] (\sigma_i + \sigma_j),
\end{aligned}$$

$$\begin{aligned}
\mathbf{V}_{12}^{\text{T}} = & \frac{\textcolor{red}{T}}{2} \{ [(\sigma_1 \cdot \mathbf{k}')(\sigma_2 \cdot \mathbf{k}') - \frac{1}{3}(\sigma_1 \cdot \sigma_2) \mathbf{k}'^2] \delta(\mathbf{r}) \\
& + \delta(\mathbf{r}) [(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) - \frac{1}{3}(\sigma_1 \cdot \sigma_2) \mathbf{k}^2] \} \\
& + \textcolor{red}{U} \{ (\sigma_1 \cdot \mathbf{k}') \delta(\mathbf{r}) (\sigma_1 \cdot \mathbf{k}) - \frac{1}{3}(\sigma_1 \cdot \sigma_2) [\mathbf{k}' \cdot \delta(\mathbf{r}) \mathbf{k}] \}
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{r} = & \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2, \quad P_\sigma = (1 + \sigma_1 \sigma_2)/2, \\
\mathbf{k} = & (\overrightarrow{\nabla}_1 - \overrightarrow{\nabla}_2)/2i, \quad \mathbf{k}' = -(\overleftarrow{\nabla}_1 - \overleftarrow{\nabla}_2)/2i.
\end{aligned}$$

particle-hole channel

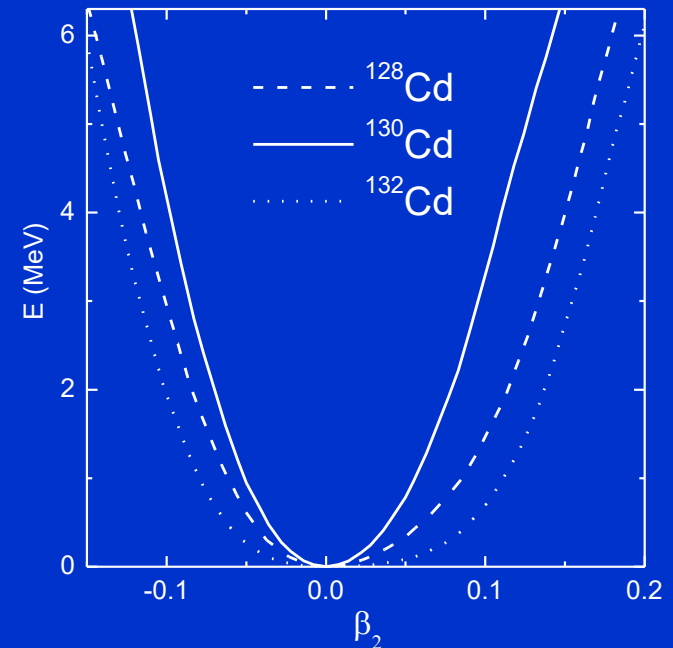
particle-particle channel

Skyrme interaction



HF-BCS calculations

$$V_0 \left(1 - \eta \frac{\rho(r_1)}{\rho_0} \right) \delta(\vec{r}_1 - \vec{r}_2)$$



$$Q_\beta = \Delta M_{n-H} + B(Z+1, N-1) - B(Z, N)$$

$$S_{xn} = B(Z+1, N-1) - B(Z+1, N-1-X)$$

$\Delta M_{n-H}=0.782\text{MeV}$ is the mass difference between the neutron and the hydrogen atom

QRPA calculations $Q_\nu^+ |0\rangle$

$$Q_\nu^+ = \sum_{n,p} X_{np}^\nu A^+(n,p;JM) - (-1)^{J-M} Y_{np}^\nu A(n,p;J-M)$$

$$A^+(n,p;JM) = \sum_{m_n m_p} \langle j_n m_n j_p m_p | JM \rangle \alpha_{j_n m_n}^+ \alpha_{j_p m_p}^+$$

Using the equation-of-motion approach one can get

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

Making use of the finite rank separable approximation for the residual interaction enables one to perform the calculations in very large configuration spaces

Nguyen Van Giai, Ch. Stoyanov, and V. V. Voronov, Phys. Rev. C57,1204 (1998).

A.P.S., Ch. Stoyanov, V. V. Voronov, and Nguyen Van Giai, Phys. Rev. C66,034304 (2002).

A.P.S., V. V. Voronov, and Nguyen Van Giai, Prog. Theor. Phys. 128 (2012) 489.

A.P.S., and H. Sagawa, Prog. Theor. Exp. Phys. 2013 (2013)103D03.

Two-phonon structures

The coupling between one- and two-phonon terms in the wave functions of excited states are taken into account

$$|\psi_{JM\nu}\rangle = \left(\sum_i R_i(J\nu) Q_{JMi}^+ + \sum_{\lambda_1 i_1 \lambda_2 i_2} P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) [Q_{\lambda_1 \mu_1 i_1}^+ \bar{Q}_{\lambda_2 \mu_2 i_2}^+]_{JM} \right) |0\rangle$$

$J^\pi = 1^+$ is constructed from the charge-exchange QRPA phonons with the following multipolarities:

$$\lambda^\pi = 1^+, 1^-, 2^-, 3^+ \quad (1)$$

and the vibrational QRPA phonons with the following multipolarities:

$$\lambda'^{\pi'} = 1^-, 2^+, 3^-, 4^+. \quad (2)$$

Two-phonon structures

Using the variational principle, one obtains a set of linear equations for the unknown amplitudes $R_i(J\nu)$ and $P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)$

$$(\Omega_{\lambda i} - E_\nu)R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji)P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) = 0$$

$$(\Omega_{\lambda_1 i_1} + \bar{\Omega}_{\lambda_2 i_2} - E_\nu)P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) + \sum_i U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji)R_i(J\nu) = 0$$

These equations have the same form as the equations of the quasiparticle-phonon model (QPM)[*V. A. Kuzmin, V. G. Soloviev, J. Phys. G 10, 1507 (1984)*], but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme forces.

In the allowed GT approximation, the β -decay rate is expressed by summing the probabilities of the energetically allowed GT transitions ($E_m^{GT} \leq Q_\beta$) weighted with the integrated Fermi function

$$T_{1/2}^{-1} = \sum_m \lambda_{if}^m = D^{-1} \left(\frac{G_A}{G_V} \right)^2 \sum_m f_0(Z+1, A, E_m^{GT}) B(GT)_m$$

$$E_m^{GT} = Q_\beta - E_{1_m^+}$$

where λ_{if}^m is the partial β -decay rate, $\frac{G_A}{G_V} = 1.25$, $D = 6147$ s. The transition energies E_m^{GT} refer to the ground state of the parent nucleus. $E_{1_m^+}$ denotes the excitation energy of the daughter nucleus.

$$E_{1_m^+} \approx E_m - E_{2qp,lowest}$$

$E_{2qp,lowest}$ corresponds the lowest two-quasiparticle energy.

E_m and $B(GT)_m$ are the solutions either of the QRPA equations, or of the equations taking into account the two-phonon configurations.

The difference in the characteristic time scales of the β -decay and subsequent neutron emission processes justifies an assumption of their statistical independence.

The probability of the βxn emission accompanying the β decay to the one- and two-phonon excited states in the daughter nucleus can be expressed as

$$P_{xn} = T_{1/2} D^{-1} \left(\frac{G_A}{G_V} \right)^2 \sum_{m'} f_0(Z+1, A, E_{m'}^{GT}) B(GT)_{m'}$$

$$P_{1n} : \quad Q_\beta - S_{2n} \leq E_{m'}^{GT} \leq Q_\beta - S_n$$

$$P_{2n} : \quad Q_\beta - S_{3n} \leq E_{m'}^{GT} \leq Q_\beta - S_{2n}$$

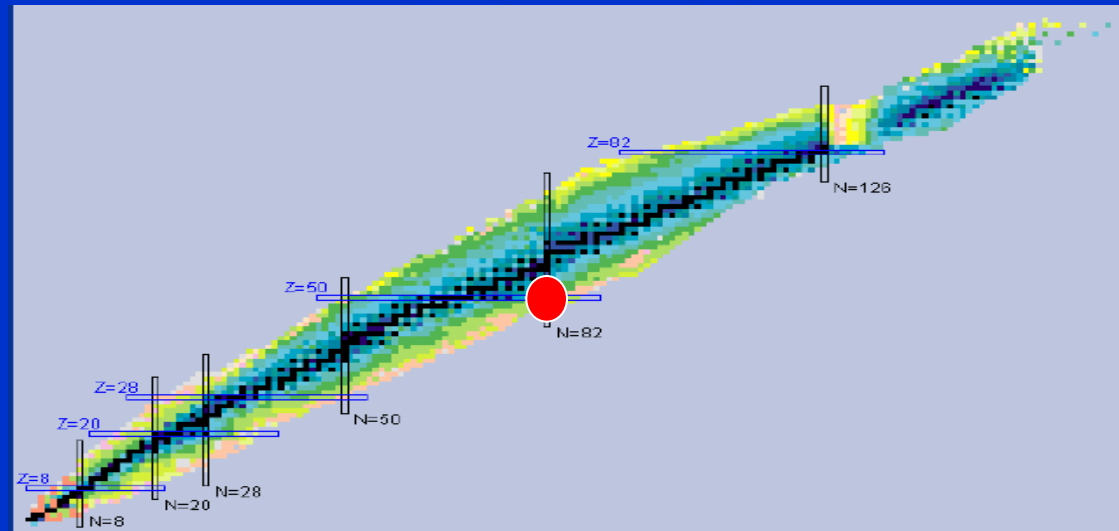
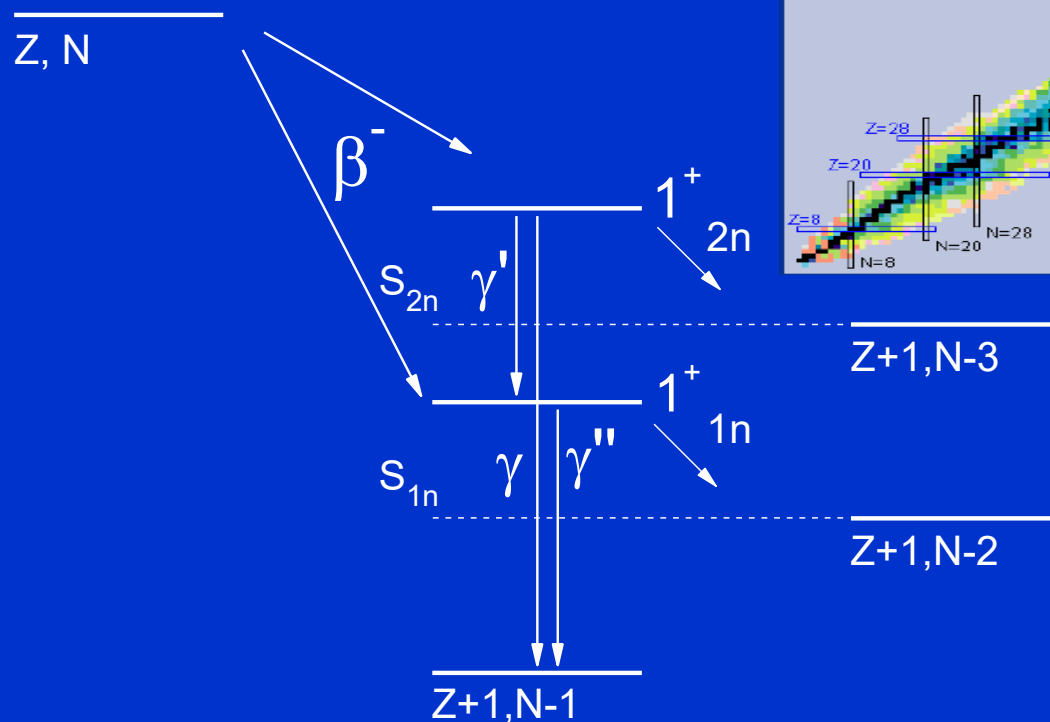
$$P_{3n} : \quad Q_\beta - S_{4n} \leq E_{m'}^{GT} \leq Q_\beta - S_{3n}$$

$$P_{4n} : \quad Q_\beta - S_{5n} \leq E_{m'}^{GT} \leq Q_\beta - S_{4n}$$

$$T_{1/2} : \quad 0 \leq E_m^{GT} \leq Q_\beta$$

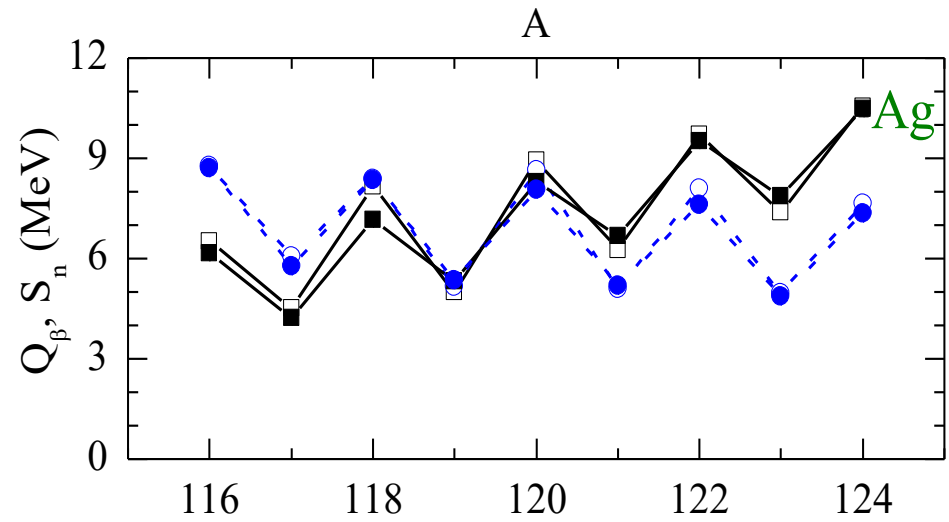
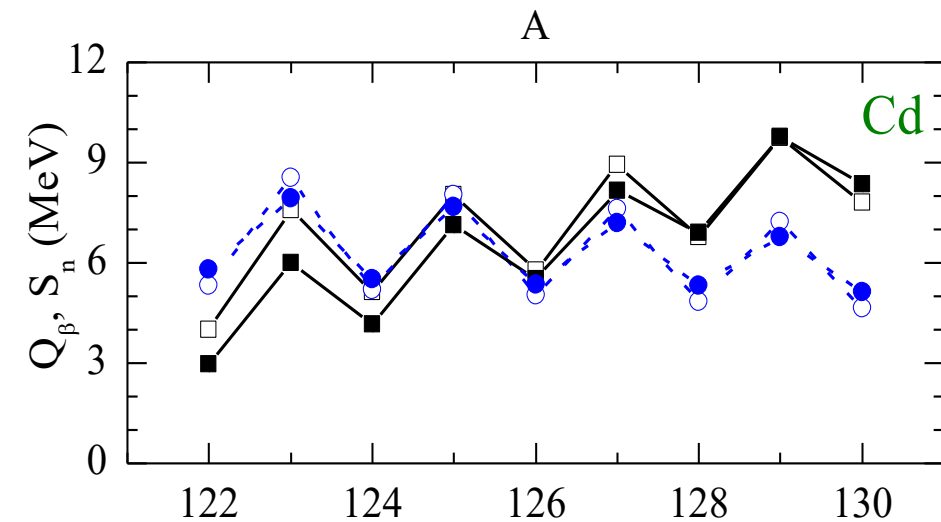
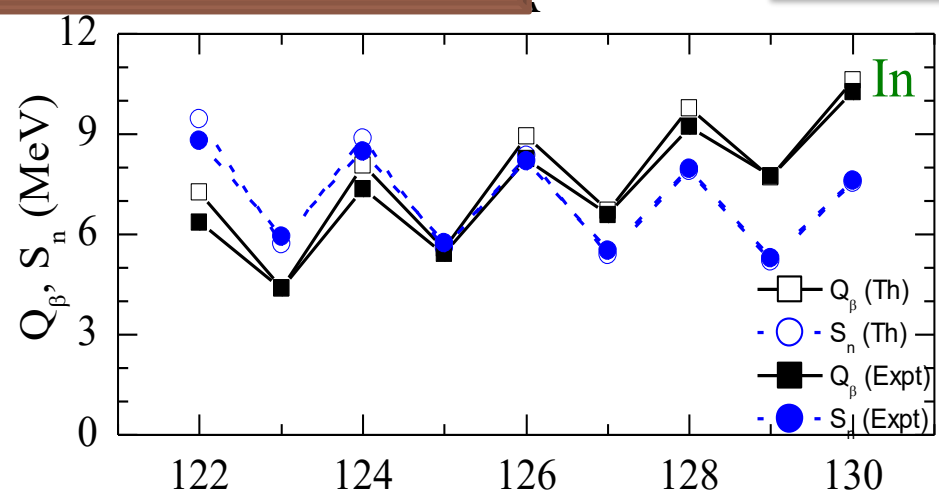
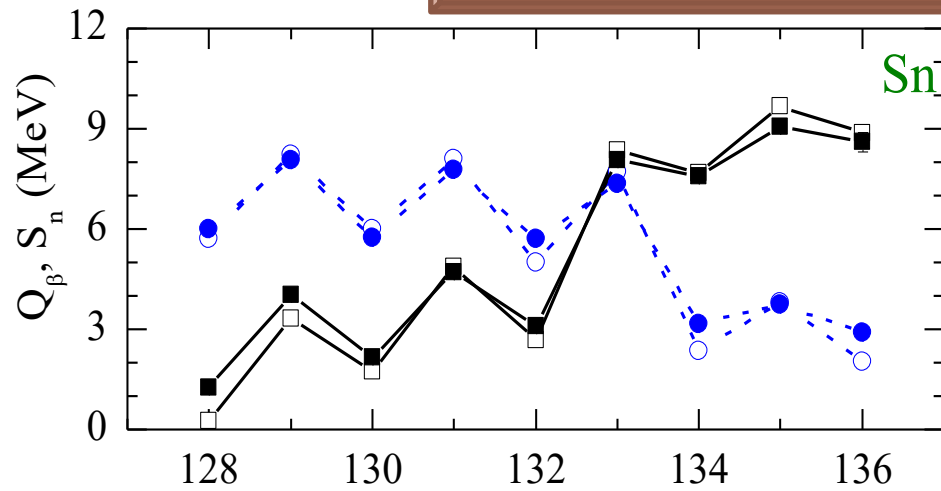
The β -decay properties of r-process “waiting-point nuclei” ^{129}Ag , ^{130}Cd , ^{131}In

We concentrate on the delayed multi-neutron emission in the region below the neutron-rich doubly magic nucleus ^{132}Sn . The β -decay rates help to reconstruct the Gamow-Teller strength distribution. Our theoretical investigation taking into account the phonon-phonon coupling for the study of low-energy spectrum populated in the β -decay of unstable nuclei.



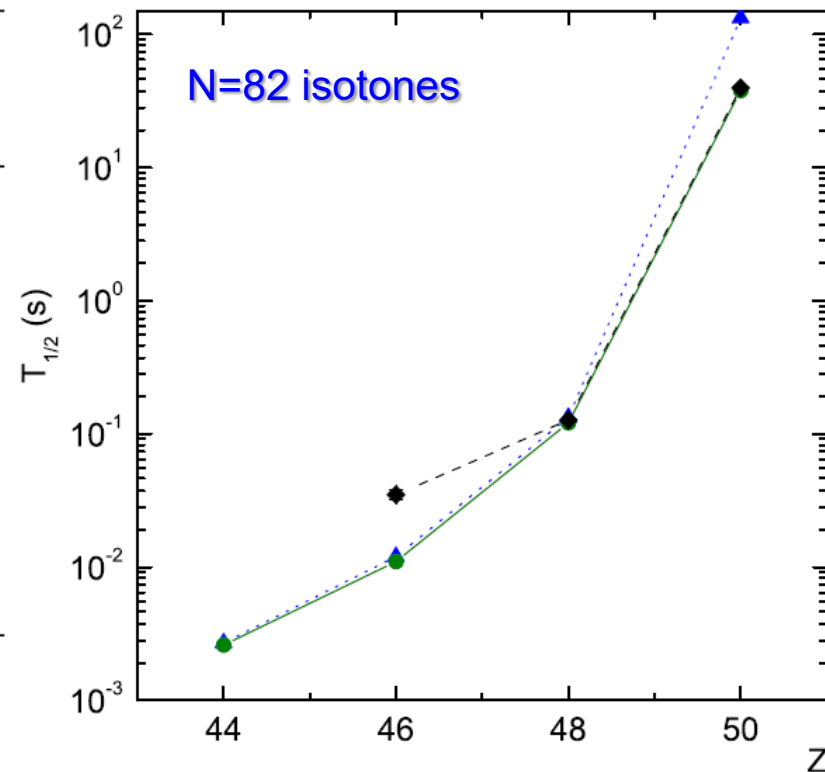
βn -decay window

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The evolution of the β -decay half-lives

Nucleus	Half-life (ms)		
	QRPA	PPC	Expt.
^{126}Cd	334	263	513 ± 6
^{128}Cd	212	180	245 ± 5
^{130}Cd	133	121	127 ± 2
^{132}Cd	42	38	82 ± 4
^{134}Cd	36	32	65 ± 15

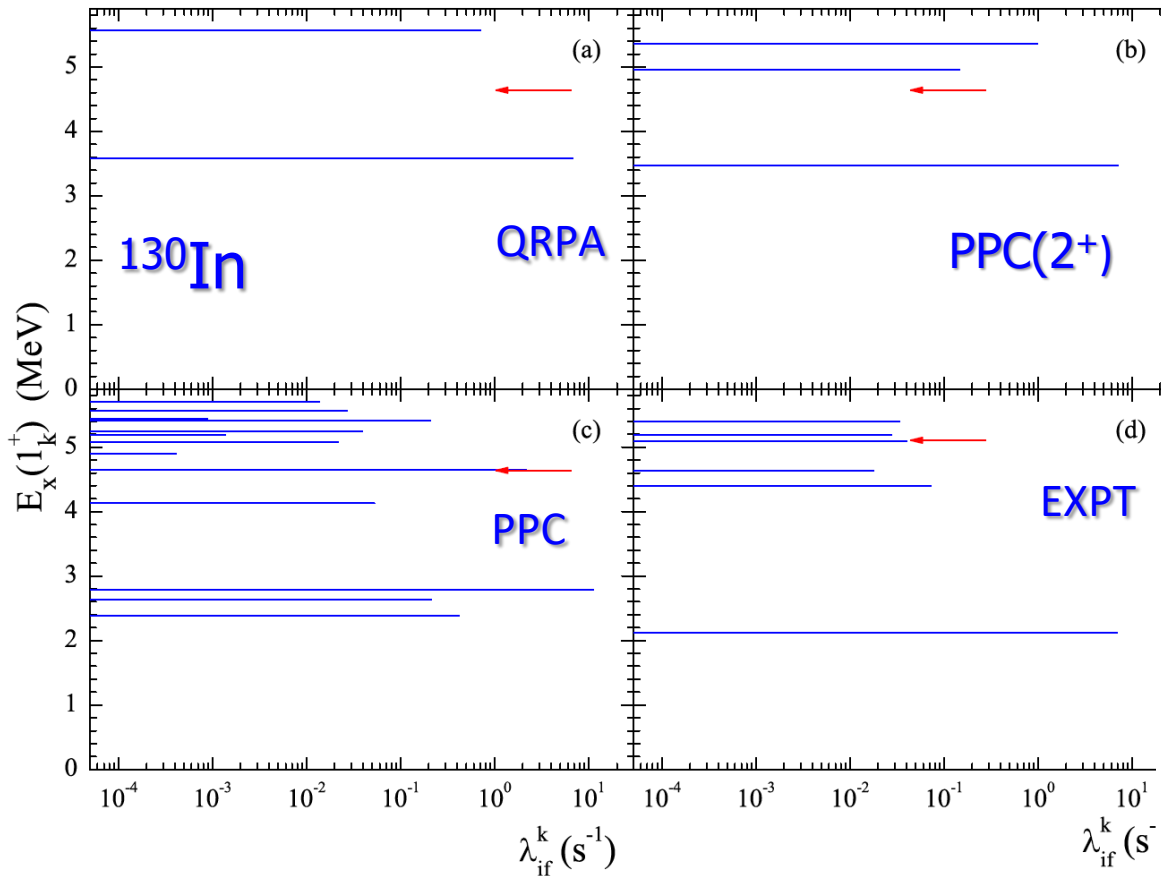


$$T_{1/2}^{-1} = \sum_m \lambda_{if}^m = D^{-1} \left(\frac{G_A}{G_V} \right)^2 \times \sum_m f_0(Z+1, A, E_m^{GT}) B(GT)_m$$

The largest contribution in the calculated β -decay half-life comes from the $[1_1^+]_{QRPA}$ configuration, but the two-phonon contributions are appreciable. The inclusion of the two-phonon terms results in an increase of the transition energy.

The phonon-phonon coupling effect on the β -decay rates

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^{130}Cd

$P_{n\text{ tot}}=13.7\%$



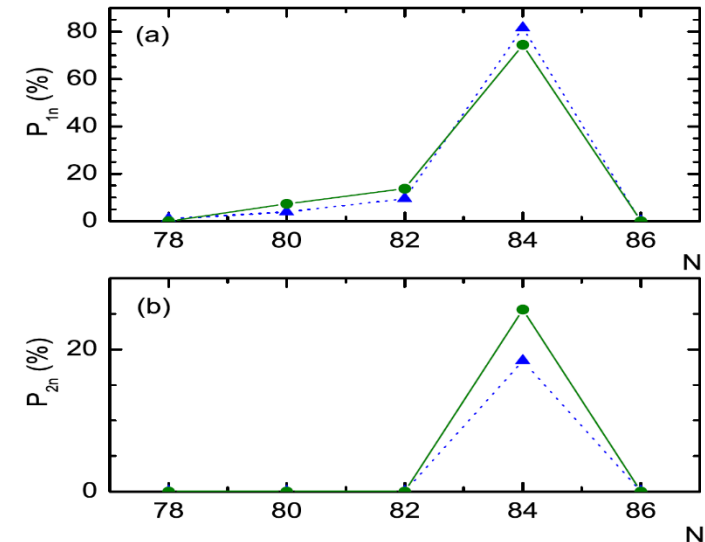
EXPT $3.5 \pm 1\%$

^{132}Cd

$P_{n\text{ tot}}=100\%$

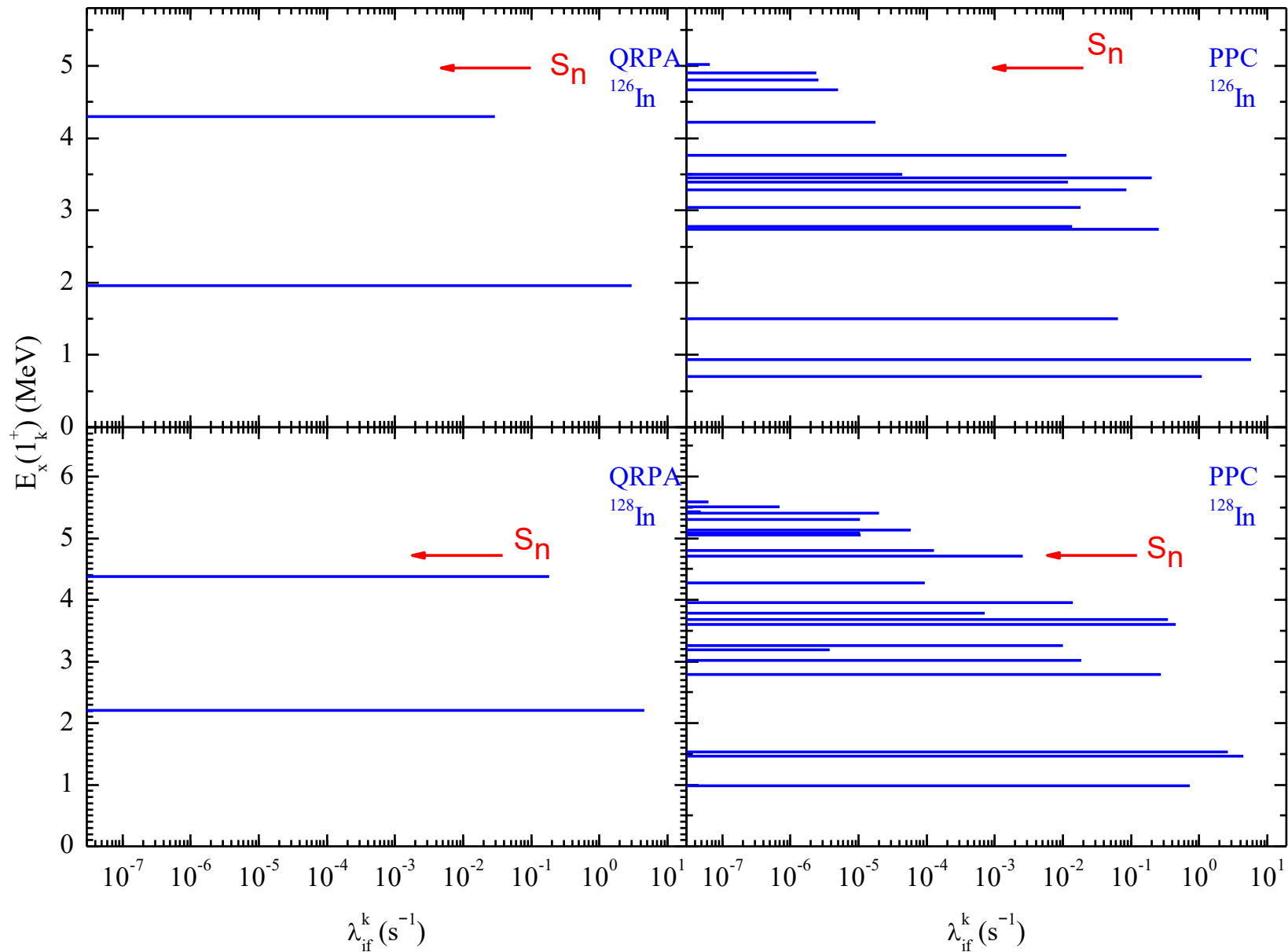


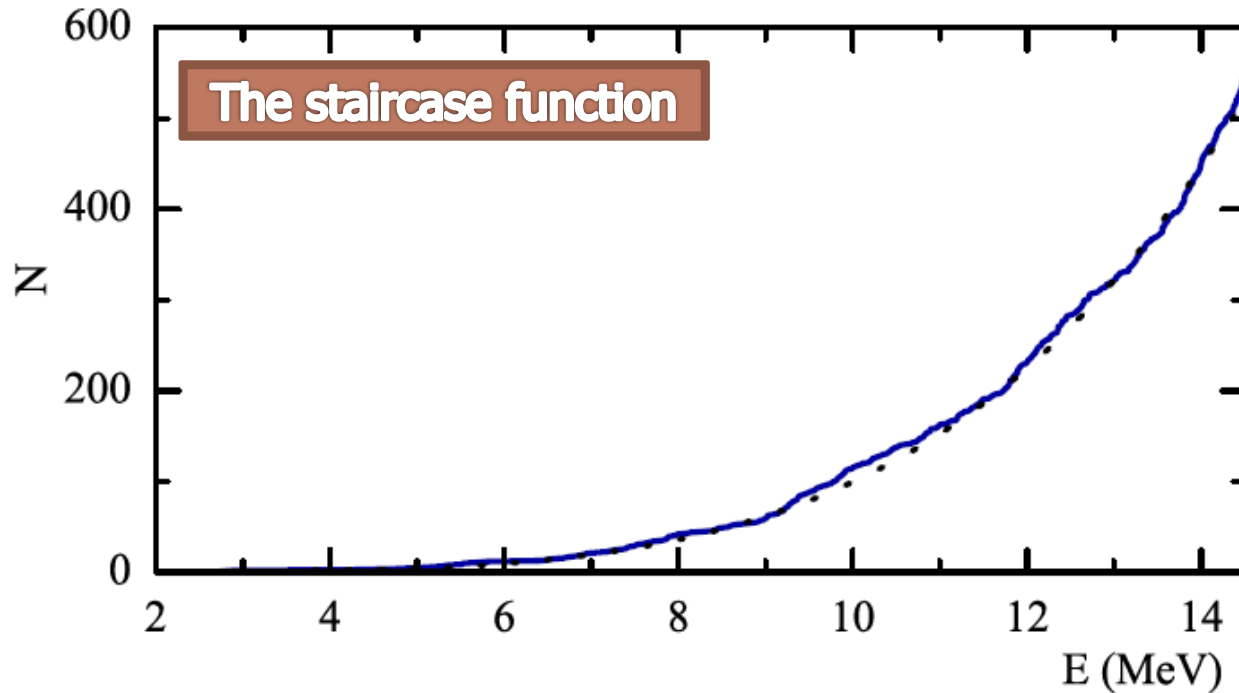
EXPT $60 \pm 15\%$



The phonon-phonon coupling effect on the β -decay rates

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The staircase function

$J^\pi = 1^+$ is constructed from the charge-exchange QRPA phonons with the following multipolarities:

$$\lambda^\pi = 1^+, 1^-, 2^-, 3^+ \quad (1)$$

and the vibrational QRPA phonons with the following multipolarities:

$$\lambda'^\pi = 1^-, 2^+, 3^-, 4^+. \quad (2)$$

The function $N(E)$ can be described by the following level density:

$$\rho(E) = \alpha(E - E_0)^\beta.$$

We find that $E_0 = 0.43$ MeV, $\beta = 3.30$ and $\alpha = 2.6 \times 10^{-2} \text{ MeV}^{-\beta-1}$. Notice that in the Fermi-gas model with equidistant single-particle spectrum the exponent is $2n-1$ for the density of $np - nh$ excitations, and the value $\beta = 3$ for $2p - 2h$ excitations, which is quite close to the fitted values of β .

β-delayed γ-spectrum

TABLE I: The calculated energies, log *ft*, transition probabilities, partial widths and dominant components of phonon structures of the low-lying 1⁺ states in ¹³⁰In.

1 _k ⁺	Energy (MeV)	Structure	log <i>ft</i>	1 _k ⁺ →1 _{g.s.} ⁻		1 _k ⁺ →2 ₁ ⁻		1 _k ⁺ →3 ₁ ⁺	
				<i>B</i> (<i>E</i> 1) (W.u.)	Γ(<i>E</i> 1) (eV)	<i>B</i> (<i>E</i> 1) (W.u.)	Γ(<i>E</i> 1) (eV)	<i>B</i> (<i>E</i> 2) (W.u.)	Γ(<i>E</i> 2) (eV)
1 ₁ ⁺	2.4	99%[3 ₁ ⁺ ⊗2 ₁ ⁺]	4.7	4.2×10 ⁻⁷	1.1×10 ⁻⁵	4.5×10 ⁻⁵	1.1×10 ⁻⁴	5.8	7.2×10 ⁻⁴
1 ₂ ⁺	2.6	98%[3 ₁ ⁺ ⊗4 ₁ ⁺]	4.9	5.3×10 ⁻⁷	1.7×10 ⁻⁵	0.7×10 ⁻⁶	2.3×10 ⁻⁵	0.1	2.6×10 ⁻⁵
1 ₃ ⁺	2.8	77%[1 ₁ ⁺]	3.1	0.9×10 ⁻⁵	3.6×10 ⁻⁴	1.2×10 ⁻⁵	4.5×10 ⁻⁴	0.1	4.8×10 ⁻⁵
1 ₄ ⁺	4.1	99%[2 ₁ ⁻ ⊗3 ₁ ⁻]	4.8	1.5×10 ⁻⁵	1.9×10 ⁻³	2.2×10 ⁻⁵	2.7×10 ⁻³	0.1	8.2×10 ⁻⁴
1 ₅ ⁺	4.7	57%[1 ₂ ⁺]+ 32%[3 ₁ ⁺ ⊗2 ₂ ⁺]	2.8	0.7×10 ⁻⁵	1.4×10 ⁻³	2.0×10 ⁻⁴	3.8×10 ⁻²	1.0	2.0×10 ⁻²

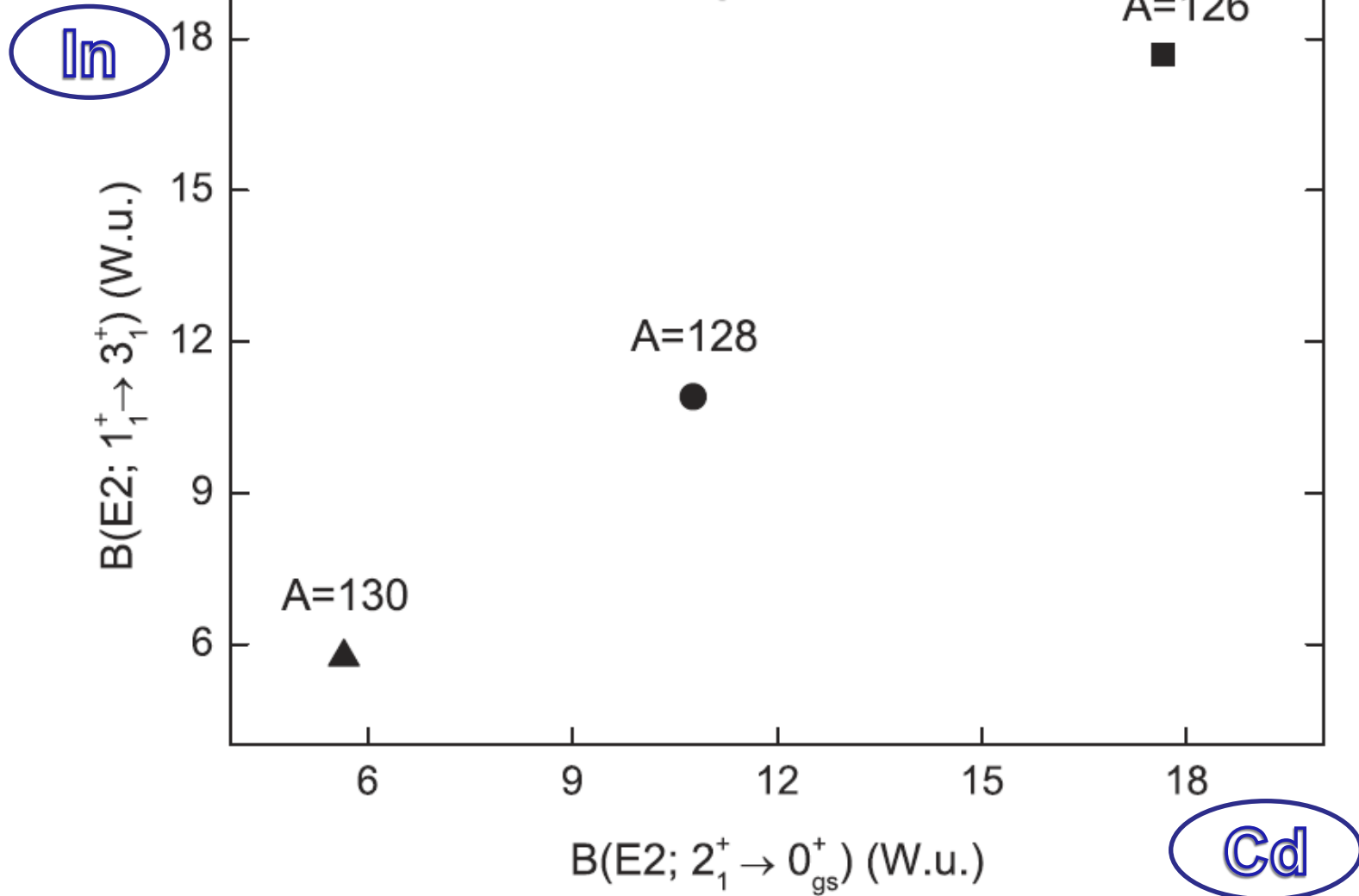
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$$\frac{\Gamma(E2; 1_3^+ \rightarrow 3_1^+)}{\Gamma(E1; 1_3^+ \rightarrow 1_{g.s.}^-)} = 0.13.$$

This fact indicates that indeed the calculated third 1+ state corresponds to the experimental level located at 2120 keV which was also reproduced by the shell model.

It is shown for the first time that the structure the additional 1+ states is mostly dominated by the two-phonon configuration built on the charge-exchange first 3+ phonon.

The correlation between the low-lying E2 transition strengths of the parent and daughter isobaric companions



Summary

- Starting from the Skyrme mean-field calculations, we have studied the effects of the phonon-phonon coupling on the β -delayed multi-neutron emission. Our results represent the successful comparison between experimental β -delayed γ -spectra and those calculated with the Skyrme EDF.
- We predict a non-zero probability of the neutron emission in the case of ^{126}Cd . Among our initial motivation was the search for multi-neutron emission in the case ^{132}Cd in comparison to the N=82 isotone ^{130}Cd . The inclusion of the phonon-phonon coupling results in the 55% increase of the ratio P_{2n}/P_{1n} .
- It is shown that the space extension of the charge-exchange phonons enriches the 1^+ spectrum of ^{130}In . An important increase of the level density near the neutron threshold is achieved. It is shown that the structure the additional 1^+ states is mostly dominated by the two-phonon configuration built on the charge-exchange first 3^+ phonon.
- The results have shown the correlation between the low-lying E2 transition strengths of the parent and daughter isobaric companions as compared to the β -decay data of $^{126,128,130}\text{Cd}$.

Спасибо за внимание !