Study of the excited state spectrum of 46Ti in the proton pickup reaction $^{45}Sc(^{3}He, d)^{46}Ti$

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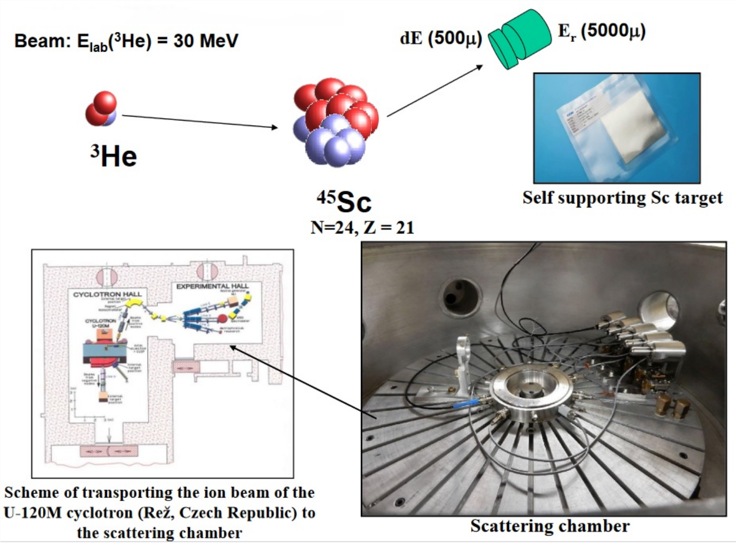
Cluster Degrees of Freedom

- It is important to take into account cluster degrees of freedom, when describing nuclei near the magic core.
- Clustering is essential in explaining excitations associated with the formation of superdeformed states in atomic nuclei.
- In the proton pickup reaction $^{45}Sc(^{3}He, d)^{46}Ti$ at a beam energy of 30 MeV, new levels with excitation energies greater than 10 MeV were discovered.
- The possibility of interpreting these states as a cluster system of $^{42}Ca +^{4} He$ was analyzed.

$^3He + ^{45}Sc \longrightarrow ^{46}Ti + d$



Self supporting Sc target

Scheme of transporting the ion beam of the U-120M cyclotron (Rež, Czech Republic) to the scattering chamber

Scattering chamber

Experimental results

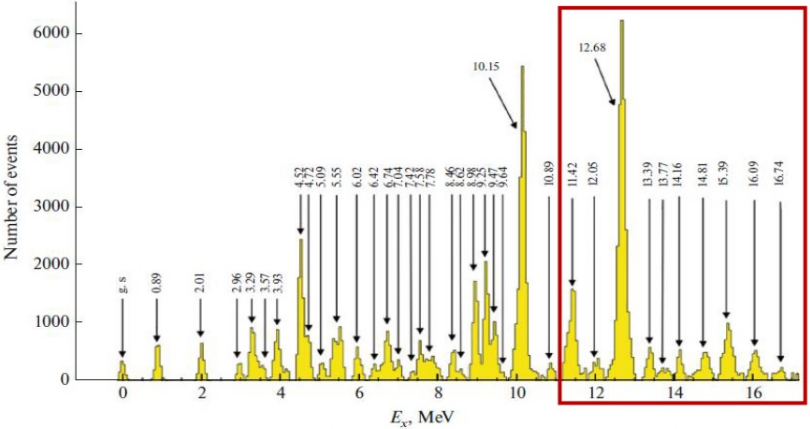


Fig. 4 Energy spectrum of excited states in the ^{46}Ti nucleus recovered from the spectrum of deuterons produced in the $^{45}Sc(^3He, d)^{46}Ti$ reaction and registered at an angle of 15° in the lab. system.

Excited states with energies from 11.4 to 16.7 MeV were observed for the first time in the (^{3}He, d) reaction and were populated with a high probability.

Hyperdeformed state in ^{46}Ti as an α-cluster state

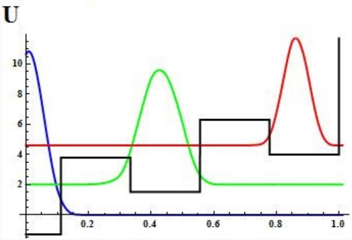
$$\Psi(^{45}\mathrm{Sc}) = \alpha_1(^{45}\mathrm{Sc}) + \beta_1(^{42}\mathrm{Ca} + t) + \ldots$$

$$\Psi(^{46}\mathrm{Ti}) = \alpha_2(^{46}\mathrm{Ti}) + \beta_2(^{42}\mathrm{Ca} + {}^{4}\mathrm{He}) + \ldots$$

spherical shell component

highly deformed cluster component

$\Psi(^{46}\mathrm{Ti})=\alpha($  $) + \beta($  $)$



Degrees of Freedom of DNS

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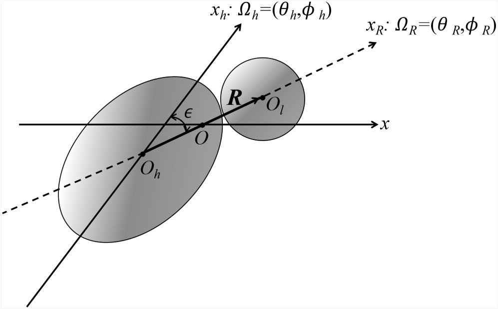


- rotation of the system as a whole $\Omega_R = (\theta_R, \phi_R)$
- rotation of the deformed fragment $\Omega_h = (\theta_h, \phi_h)$
- relative motion in R

Daughter nucleus can be deformed. It is assumed it has axially-symmetric quadrupole β_2 and octupole β_3 deformations.

Hamiltonian for the α-particle DNS

The classical expression for the kinetic energy:

$$T = \frac{1}{2}B(\xi)\dot{\xi}^2 + \frac{1}{2}\mu(\xi)\dot{R}^2 + \frac{1}{2}\mu(\xi)R^2(\xi)(\dot{\theta}_R^2 + \theta_R^2\dot{\phi}_R^2) + \Im_h(\xi)(\dot{\theta}_h^2 + \theta_h^2\dot{\phi}_h^2)$$

The quantum kinetic energy operator:

$$T = -\frac{\hbar^2}{2B}\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} - \frac{\hbar^2}{2\mu(\xi)}\frac{1}{R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R} + \frac{\hbar^2}{2\mu(\xi)R^2(\xi)}L_R^2 + \frac{\hbar^2}{2\Im_h(\xi)}L_h^2,$$

Angular momentum operators:

$$L_i^2 = -\frac{1}{\sin\theta_i}\frac{\partial}{\partial\theta_i}\sin\theta_i\frac{\partial}{\partial\theta_i} - \frac{1}{\sin^2\theta_i}\frac{\partial^2}{\partial\phi_i^2}, \qquad (i = R, h)$$

Hamiltonian for the α-particle DNS

As a function of angular variables, the interaction energy in the DNS can be approximated with good accuracy as:

$$V(\eta, \epsilon) = C_0(\xi) + C_2(\xi)\sqrt{\frac{5}{2}}P_2(\cos\epsilon),$$

where ϵ is the angle between the vector **R** and the symmetry axis of the deformed fragment. This angle is related to the Euler angles Ω_H and Ω_R as:

$$P_l(\cos\epsilon) = \frac{4\pi}{2l+1}\left(Y_l(\Omega_H) \cdot Y_l(\Omega_R)\right).$$

$$H = H_\xi + H_{rot} + H_{int}$$

$$H_\xi = \frac{\hbar^2}{2B}\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + C_0(\xi)$$

$$H_{rot} = \frac{\hbar^2}{2\mu(\xi)R^2(\xi)}L_R^2 + \frac{\hbar^2}{2\Im_h(\xi)}$$

$$H_{int} = C_2(\xi)\sqrt{\frac{5}{2}}P_2(\cos\epsilon)$$

T.M. Shneidman et al., Phys.Rev. C 92, 034302 (2015).

Wave functions

In the trial wave function, the coordinate ξ can be separated from the angular coordinates:

$$\Psi(\xi, \Omega_H, \Omega_R) = \psi(\xi) G(\Omega_H, \Omega_R).$$

To determine the wave function $\psi(\xi)$, we obtain the Schrödinger equation in the form:

$$\left(-\frac{\hbar^2}{2\mu(\xi)\sqrt{B(\xi)}}\frac{\partial}{\partial\xi}\mu(\xi)B(\xi)\frac{\partial}{\partial\xi} + U(\xi)\right)\Psi_n(\xi) = E_n\Psi_n(\xi),$$

For the wave function $G(\Omega_H, \Omega_R)$, the Schrödinger equation takes the form:

$$\left(\frac{\hbar^2 \hat{L}_R^2}{2\mu(\bar{\xi})R^2(\bar{\xi})} + \frac{\hbar^2 \hat{L}_H^2}{2\Im_h(\bar{\xi})} + C_2(\bar{\xi})\sqrt{\frac{5}{2}}P_2(\cos\epsilon)\right) G_{IM}(\Omega_H, \Omega_R) = (E_{nIM} - E_n) G_{IM}(\Omega_H, \Omega_R).$$

Collective motion in mass asymmetry

$$\Psi(\mathbf{r_1},\mathbf{r_2},....) = \Psi_m(\mathbf{r_1},\mathbf{r_2},....)+S_\alpha^{1/2}\Psi_\alpha(\mathbf{r_1},\mathbf{r_2},....)+...+S_C^{1/2}\Psi_C(\mathbf{r_1},\mathbf{r_2},....)$$

Mass asymmetry coordinate: $x = \pm\frac{2A_2}{A}$

$$H = -\frac{\hbar^2}{2}\frac{d}{dx}\frac{1}{B(x)}\frac{d}{dx} + U(x)$$

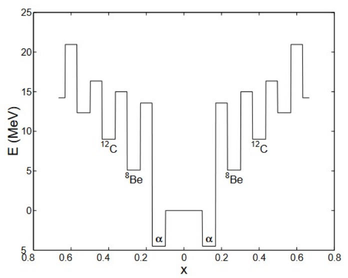
T.M. Schneidman et al, PRC **67**, 014313 (2003).

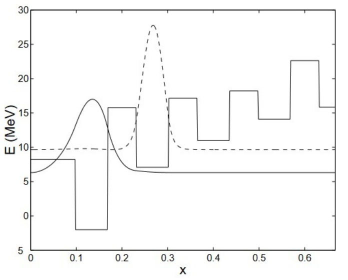
$$U(x) = B_1(x)+B_2(x)-B+V_N(x)+V_C(x)$$

G. G. Adamian et al, IJMPE **5**, 191 (1996).

$$B^{-1}(x) = \frac{1}{m_0}\frac{A_{neck}}{2\sqrt{2\pi}b^2A^2}$$

G.G. Adamian et al., NPA **584**, 205 (1995).





Potencial energy

Potential energy

$$U(\xi) = V(\xi) - (B(A,Z) - B(A_1,Z_1) - B(A_2,Z_2))$$

The nucleus-nucleus potential $V(\xi)$ is calculated as:

$$V(\xi) = V_C + V_N$$

The Coulomb potential:

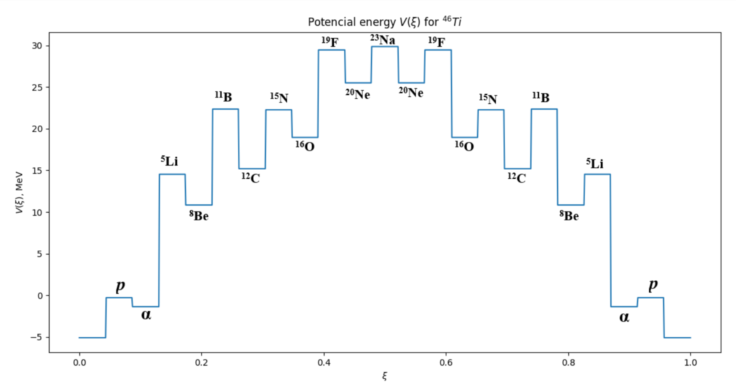
$$V_C(R,\epsilon,\beta_2,\beta_3) = \frac{e^2 Z_1 Z_2}{R_m} + \frac{3eZ_1Z_2R_m^2}{5R_m^3}\beta_2 Y_{20}(\epsilon,0) + \dots$$

The interaction between nucleons and nuclei:

$$V_N(R,\epsilon,\beta_2,\beta_3) = \int \rho_1(\mathbf{r}_1)\rho_1(\mathbf{R}_m - \mathbf{r}_2)F(\mathbf{r}_1 - \mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2$$

$$\rho_i = \frac{\rho_0}{1 + \exp\left(\frac{r_0 - R_i(\theta_i,\phi_i)}{a'}\right)}$$

Potential energy



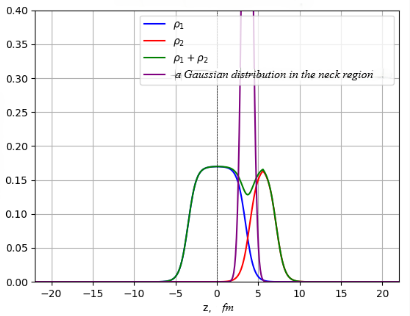
Mass parameters

$$(B^{-1})_{\eta\eta} = \frac{1}{m} \frac{A_{\text{neck}}}{2\sqrt{2\pi} b^2 A^2}$$

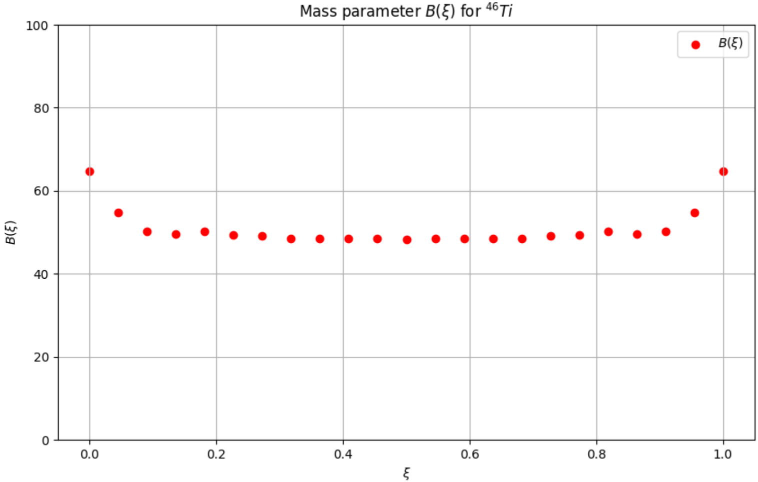
,

$$A_{\text{neck}} = \int dr \rho(r) \exp\left(-\frac{z^2}{b^2}\right)$$

G.G. Adamian et al., NPA **584**, 205 (1995).



Mass parameters



Wave equation

$$\left(-\frac{\hbar^2}{2\mu(\xi)\sqrt{B(\xi)}}\frac{\partial}{\partial\xi}\mu(\xi)B(\xi)\frac{\partial}{\partial\xi}+U(\xi)\right)\Psi_n(\xi)=E_n\Psi_n(\xi)$$

Numerical differentiation: finite differences

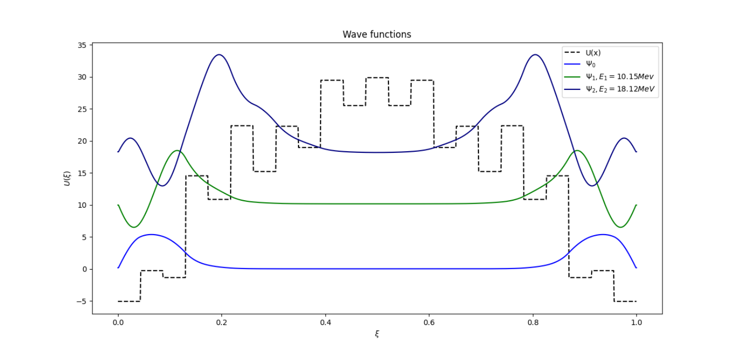
$$\left.\frac{df}{dx}\right|_{x=n\Delta x}=\frac{f_{n+1}-f_{n-1}}{2\Delta x}$$
$$\left.\frac{d^2f}{dx^2}\right|_{x=n\Delta x}=\frac{f_{n+1}-2f_n+f_{n-1}}{\Delta x^2},$$

Discrete form of Wave equation

$$\left(-\frac{1}{2B(x)\mu(x)2dx^2}-\frac{1}{2B(x)dx^2}+\frac{1}{4B^2(x)2dx}(\frac{dB}{dx})\right)\psi_{n+1}$$
$$+\left(\frac{2}{2B(x)dx^2}+U(x)\right)\psi_n,$$
$$+\left(\frac{1}{2B(x)\mu(x)2dx^2}-\frac{1}{2B(x)dx^2}-\frac{1}{4B^2(x)2dx}\frac{dB}{dx}\right)\psi_{n-1}=E\Psi,$$

Diagonalization of matrix of Hamiltonian

$$\begin{pmatrix} d & a_1 & 0 & \cdots & 0 & a_2 & 0 \\ a_2 & d & a_1 & 0 & \cdots & 0 & 0 \\ 0 & a_2 & d & a_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & a_2 & d & a_1 \\ 0 & a_1 & 0 & \cdots & 0 & a_2 & d \end{pmatrix} \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \vdots \\ \psi_{N-1} \\ \psi_N \end{pmatrix} = E \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \vdots \\ \psi_{N-1} \\ \psi_N \end{pmatrix}$$



Angular oscillations

To describe angular oscillations in the alpha-cluster dinuclear system (DNS), it is necessary to solve the Schrödinger equation

$$\left(\frac{\hbar^2\hat{L}_R^2}{2\mu(\bar{\xi})R^2(\bar{\xi})}+\frac{\hbar^2\hat{L}_H^2}{2\Im_h(\bar{\xi})}+C_2(\bar{\xi})\sqrt{\frac{5}{2}}P_2(\cos\epsilon)\right) \quad G_{IM}(\Omega_H,\Omega_R)=$$

$$(E_{nIM}-E_n)G_{IM}(\Omega_H,\Omega_R).$$

with the Hamiltonian:

$$\hat{H}=\frac{\hbar^2\hat{L}_R^2}{2\mu(\bar{\xi})R^2(\bar{\xi})}+\frac{\hbar^2\hat{L}_H^2}{2\Im_h(\bar{\xi})}+C_2(\bar{\xi})\sqrt{\frac{5}{2}}P_2(\cos\epsilon)+E_n.$$

The Hamiltonian is diagonalized over a set of basis functions

$$\Phi_{LM,\pi}^{l_1,l_2,n}=F_n(\xi)\left[Y_{l_1}(\Omega_h)\times Y_{l_2}(\Omega_R)\right]_{LM},$$

where n=0,1,2,..., l_1=0,2,4,..., l_2=0,1,2... Since the heavy fragment is assumed to have only axially symmetric quadrupole deformation, the wave function should remain invariant under the transformation: $\theta_h\to\pi-\theta_h$, $\phi_h\to\pi+\phi_h$,and the quantum number l_1 can only take even values.

Angular oscillations

$$H = H_{\rm rot} + H_{\rm bend} + V_{\rm int},$$

$$H_{\rm rot} = \frac{\hbar^2}{2\mu R_{\rm m}^2}(L^2 - 2L_3'),$$

$$H_{\rm bend} = \frac{\hbar^2}{2\Im_b}\frac{1}{\epsilon}\frac{\partial}{\partial\epsilon}\epsilon\frac{\partial}{\partial\epsilon} + \frac{\hbar^2}{2\Im_b\epsilon^2}L_3'^2 + \frac{C}{2}\epsilon^2,$$

$$V_{\rm int} = \frac{\hbar^2}{2\mu R_{\rm m}^2}\left[\frac{1}{\epsilon}(L_1'L_3' + L_3'L_1') + 2iL_2'\frac{1}{\sqrt{\epsilon}}\frac{\partial}{\partial\epsilon}\sqrt{\epsilon}\right]$$

(

Analysis of the results

Energy, MeV	I^{π}
10.15	0^+
10.53	1^-
11.22	2^+
11.87	3^-
12.53	4^+
13.04	5^-
13.39	6^+
12.73	0^+
12.00	1^-
11.92	2^+
12.82	3^-
13.28	4^+
13.99	5^-
14.35	6^+

Calculated lowest excited states of an alpha-cluster system $^{42}Ca + ^{4}He$ and their characteristics I^{π}. The ground state is assumed to have an energy of 10.15 MeV

Energy, MeV (experiment)	Energy, MeV (theory)
10.15	10.15
10.89	10.53
11.42	11.22
12.05	11.87 (12.00)
12.68	12.73 (12.82)
13.39	13.39
13.77	
14.16	
14.81	
15.39	
16.09	
16.74	

Highly excited levels and calculated levels in ^{46}Ti, obtained in the framework of the cluster model of a dinuclear system, measured in the $^{45}Sc(^{3}He, d)^{46}Ti$ reaction

Conclusion

- Using the dinuclear system (DNS) model, the wave functions and potential energy of the system were calculated.
- These results enabled the interpretation of the observed highly excited states as superdeformed states associated with the formation of an alpha-cluster configuration ($^{42}Ca +^{4} He$).
- The findings can be applied to further studies of superdeformed and hyperdeformed nuclear states, as well as to the development of new theoretical models of nuclear structure.
- The study supports the hypothesis of a cluster nature in the highly excited states of ^{46}Ti. However, to conclusively determine the existence of a deformed alpha-cluster structure in these states, additional experiments measuring deuteron angular distributions for the $^{45}Sc(^{3}He, d)^{46}Ti$ reaction over a wide angular range are required.

Thank You!